International Reserves for Emerging Economies: A Liquidity Approach

Kuk Mo Jung*
University of California-Davis†

November 2013

Abstract

The massive stocks of foreign exchange reserves, mostly held in the form of U.S. T-bills by emerging economies are still an important puzzle. Why do emerging economies continue to willingly loan to the United States despite low rates of return? This paper argues that a model incorporating international capital markets, characterized by a non-centralized trading mechanism and U.S. T-bills as facilitators of trade, can provide an answer to this question. Declining financial frictions in these particular markets would generate rising liquidity premiums on U.S. T-bills. Meanwhile, the higher liquidity properties of the U.S. T-bills would induce recipients of foreign investments, namely emerging economies, to hold more of the liquidity (i.e., the U.S. T-bills) in equilibrium. Dynamic panel estimation results provide empirical support for the implications of this model.

JEL classification: E44, E58, F21, F31, F36, F41

Keywords: international reserves, liquidity, dynamic panel estimation

*I am grateful to David Andolfatto, Paul Bergin, Athanasios Geromichalos, Guido Menzio, Ju-Hyun Pyun, Martine Quinzii, Katheryn Russ, Kevin Salyer, Ina Simonovska, and Alan Taylor for useful comments and suggestions. I am indebted to Seungduk Lee and Bank of Korea for sharing the EPFR data as well. I would also like to thank participants at the 5^{th} South West Search and Matching Conference, the 39^{th} Eastern Economic Association Annual Meetings, and UC Davis seminars for their valuable feedback. All errors are mine.

†Corresponding Email Address: kmjung@ucdavis.edu

1. Introduction

The international reserves, which are mostly held in the form of U.S. T-bills by emerging economies, are thought to have played a major role in shaping global financial flows and real interest rates over the last decade. However, economists are still unclear about the root causes of the rapid growth in reserve holdings by emerging economies. Most economists studying this topic point to either risk or policy-related factors. The risk approach stresses a hedging role of reserve assets against random sudden stops, whereas the policy approach focuses on the reserve assets as a tool in a policy of currency undervaluation.[1]

While these explanations admittedly provide important insights, one major challenge with them is that the calibrated versions and/or forecasts of their models usually fail to match the sheer size and trend of many emerging markets' reserve accumulation by a large margin. Some even call this failure an *excess reserve accumulation puzzle* (Summers, 2006; Jeanne and Ranciere, 2011). Therefore, the aim of this paper is to offer a solution for this puzzle. To that end, this paper first renews a very important but largely neglected attribute of the reserve assets (i.e., U.S. T-bills) in the literature: *liquidity*.[2] In a world where emerging economies are in need of sustained foreign capital inflows, reserve assets could play a crucial role in facilitating foreign capital inflows by serving as primary collateral similar to Dooley, Folkerts-Landau, and Garber (2004). Alternatively, reserve assets could alleviate the loss from the repatriation of foreign capital by serving as a means of buying back foreign capital. What is of importance is that despite these different roles, reserve assets could effectively serve as liquidity services to the extent that holding more of these assets attracts more foreign capital inflows.

I argue that the sustained enhancement of reserve assets' liquidity holds a key to understanding the recent upward trend in reserve accumulation by many emerging markets. Therefore, the first task in supporting this argument is to account for what drives this liquidity property enhancement. For this reason, I bring insights from a new branch of monetary economics—with Lagos (2010) at the front—that pioneers a new asset pricing model for which assets, in addition to the discounted value of future dividend streams, can be valued for their endogenous liquidity properties.[3] I take this insight further and apply it to a global portfolio choice problem with endogenous change in reserve assets' liquidity property.

The explanation for the endogenous and simultaneous change in reserve assets liquidity as well as for reserve hoarding is simple and intuitive: emerging economies seek to make contracts with developed countries to bring foreign capital through international capital mar-

[1] For a more comprehensive literature review, see Bernanke (2005); Lane and Milesi-Ferretti (2007a); Ghironi, Lee, and Rebucci (2007); Gourinchas and Rey (2007); McGrattan and Prescott (2007); Warnock and Warnock (2009); and Obstfeld, Shambaugh, and Taylor (2010).

[2] A recent empirical study by Krishnamurthy and Vissing-Jorgensen (2012) demonstrates that U.S. Treasury bonds have superb liquidity that is akin to the U.S. dollar's liquidity.

[3] With *ad hoc* assumptions, such as a cash-in-advance constraint, accounting for endogenous change in asset liquidity is virtually impossible. This is where the monetary search framework's usefulness comes in.

kets. Importantly, in the present model, these markets do not use a centralized trading mechanism, such as an exchange. Instead, agents from emerging and developed countries meet in a bilateral fashion and negotiate the terms of trade. Due to imperfect credit and limited commitment, reserves assets are assumed to effectively serve as a sole medium of exchange for foreign capital inflows.[4]

Within this framework, reserve assets can carry a *liquidity premium*, which reflects assets' ability to facilitate transactions in international capital markets. A process of declining friction (e.g., financial deregulation) in these markets expedites trade between agents, which makes the *liquidity premium* on the reserve assets higher and thus leads to low rates of return in equilibrium. In this context, agents from emerging markets value the reserve assets' enhanced liquidity attributes more than their counterparts. This is due to the fact that the agents from developed countries do not require any liquidity services from the reserve assets in the international capital markets.[5] Eventually, this leads to the equilibrium level of reserve hoarding by emerging economies to increase.

This new liquidity approach for analyzing the recent buildup of emerging markets' international reserves relies on the premise that international capital markets are characterized by decentralized trading. This assumption is by no means a pure theoretical abstraction. Over the last decade, the global economy witnessed the emergence of foreign capital inflows into emerging economies, especially those associated with newly developed financial instruments, such as hedge fund investment, leveraged buyout funds by private equity firms, wholesale funding by multinational investment banks, and so on. What is crucial is that these new types of private investment inflows (consistent with the present model's assumptions) are mostly carried out through over-the-counter (OTC) markets, such as those described in Duffie, Gârleanu, and Pedersen (2005).

To further justify the model framework, this study also includes quantitative analysis. The main testable implication of the model is that foreign capital inflows through OTC markets should be tightly linked to the recent upsurge in emerging markets' reserve holdings. This hypothesis is tested on a collection of 53 emerging economies using data from 1997 to 2007. Dynamic panel estimation results strongly support the hypothesis. In addition, the extraordi-

[4]This assumption of reserve assets (i.e., U.S. T-bills) being used as a direct medium of exchange instead of serving a more practically plausible role, such as collateral, can be justified on the grounds of recent monetary search literature. Lagos (2011); Venkateswaran and Wright (2013); Geromichalos and Herrenbrueck (2013) demonstrate that assets can effectively act as media of exchange despite multiple contractual differences (e.g., collateral, REPO, and secondary OTC assets). Furthermore, employing the model with assets as a direct medium of exchange can avoid unnecessary complexities.

[5]I admit the importance of the role of emerging market's public sectors (i.e., central banks) in the emergence of rapid reserve accumulation. Nonetheless, analyzing the phenomena purely from the perspective of a private sector's portfolio choice is not implausible either. This is because most emerging economies are channeling their private sector's foreign asset savings through the official sector. In other words, the reserve assets held by the central bank in emerging economies are indirectly controlled by private sector decisions through capital controls, the issuance of quasi-collateralized sterilization bonds, and so on. See Caballero, Farhi, and Gourinchas (2008) for more empirical observations that justify this approach.

nary rise of East Asia's international reserve holdings, which are often regarded as anomalous in conventional risk-based models, is in fact found to be quantitatively consistent with out-of-sample forecasts based on the liquidity-based model specification. In short, this study not only provides a new liquidity-based perspective of the phenomena, but it also demonstrates evidence for the theory's empirical relevance.

Analyzing reserve accumulation through the lens of the liquidity channel adds new insights to the existing literature. Some prominent studies (e.g., Caballero, Farhi, and Gourinchas (2008); Mendoza, Quadrini, and Rios-Rull (2009)) emphasize local assets' lack of pledgeability or emerging markets' financial underdevelopment as major sources of the excess global demand for U.S. assets (i.e., U.S. T-bills). If this argument holds universally, then the recent turmoil in the U.S. financial market along with the U.S. debt fiasco that damaged U.S. assets' pledgeability should have coincided with at least a slowdown of reserve accumulation. However, the data clearly show no sign of such a comedown. By restricting the two-country model to a symmetric financial asset case, this study does not suffer the same issue. The liquidity-based theory provides a much more natural way of supporting sustained reserve accumulation even in the aftermath of the crisis.

Another advantage of this paper is that it provides a framework that is not specific to East Asia. Aizenman and Marion (2002); Durdue, Mendoza, and Terrones (2007); Jeanne and Ranciere (2011) argue that a series of emerging market crises in the 1990s gave rise to East Asia's extraordinary demand for foreign reserve assets. Meanwhile, Summers (2006) and Dooley, Folkerts-Landau, and Peter (2005) suggest that reserve accumulation is a direct consequence of East Asia's industrial policies aiming to achieve undervalued currencies. However, China and India were not hit by the Asian financial crisis, and many East Asian countries switched to an almost fully flexible exchange regime after the crisis.[6] In this regard, the present model complements East Asian-based explanations by providing an extra liquidity channel through which demand for reserve assets (i.e., U.S. T-bills) can be boosted.

Having drawn on a multiple-asset version of Lagos and Wright (2005), this paper also lays a foundation for addressing an unresolved issue in international macroeconomics. Kehoe, Ruhl, and Steinberg (2013) highlight that the international macro literature does not provide micro-founded explanations for why emerging economies bias their international portfolios toward U.S. assets. By introducing the international capital market with U.S. assets (i.e., U.S. T-bills) as the sole medium of exchange, this paper provides such a micro-foundation for U.S. asset biased portfolio composition.[7]

[6]Furthermore, empirical evidence is not in favor of their arguments on many occasions. See Aizenman and Lee (2007) for the mixed empirical evidence on the conventional explanations.

[7]Deeper insights into why the U.S. assets may be a superior means of payment in transactions have been offered in the literature. Devereux and Shi (2013) construct a dynamic general equilibrium model of a vehicle currency where agents prefer an indirect trade using the U.S. dollar to a direct trade using their own currencies. One could also refer to the intuition of Rocheteau (2011) and Lester, Postlewaite, and Wright (2012) where

As already pointed out, this study attempts to bring intuitive insights from the growing monetary search literature that studies the broader notion of assets as facilitators of trade (e.g., Lagos and Rocheteau (2008); Lagos (2011); Lester, Postlewaite, and Wright (2012)). In line with the present paper, some of these studies take the insights further by applying this notion of asset liquidity to traditional macro puzzles related to asset pricing and portfolio choice theory. Lagos (2010) proposes a framework enriched with risk shocks to resolve the *equity premium* puzzle, which the consumption-based capital asset-pricing model usually finds hard to reconcile. Geromichalos and Simonovska (2011) also bring the monetary search literature closer to questions related to international portfolio diversification. Similar to the present paper's setup, they consider a two-country environment characterized by assets' role as media of exchange, which plays a crucial role in rationalizing the home asset bias puzzle. Nonetheless, this study is the first to explore the implications from the OTC international investment market on the asymmetric distribution of asset holdings across countries.

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 explains the optimal behavior of agents in the economy. Section 4 characterizes the equilibrium. Section 5 sets up a testable hypothesis from the model, formulates empirical specifications for emerging markets' international reserve holdings, and presents the estimation results. Concluding remarks follow in Section 6.

2. The Model

Time is discrete, and the horizon is infinite. There are two countries, home and foreign. The home country represents a developing country (i.e., China), while the foreign country represents a developed country (i.e., the United States). Each country has a unit measure of agents who live forever. For notational simplicity, I shall henceforth call home and foreign agents H and F, respectively.

Each period is divided into three sub-periods for which economic activities differ. During the first sub-period, every agent, regardless of where he/she resides, is endowed with a production technology that allows him/her to transform each unit of labor into a unit of *numeraire goods*. These goods are identical, so all agents are basically self-sufficient in *numeraire goods* consumption. In this regard, this model abstracts away from international goods trade. However, the agents can trade these goods for two different financial assets (i.e., home and foreign assets). What is critical here is that trade for financial assets in this sub-period takes place in one centralized, or Walrasian, market (CM).

The two financial assets (i.e., home and foreign) are perfectly divisible and meant to represent emerging market debt and U.S. T-bills, respectively. For tractability, Lucas (1978) trees

asymmetric information on different assets would give rise to one particular asset as a sole medium of exchange.

are adopted. In each country, a new set of trees is born in the CM every period. Each unit of the tree delivers one unit of *numeraire goods* in the next period's CM, and then it dies. This simplifies the maturity of both bonds to one period.[8] Any agent can purchase and trade shares of these trees at the ongoing market price (i.e., ψ and ψ^* for home and foreign assets, respectively). The supply of these trees for each country is fixed over time and is denoted by T and T^*, respectively.

During the second sub-period, H and F both visit the foreign investment market (FIM) to engage in anonymous bilateral trade with search frictions. Importantly, it is assumed that only F is endowed with a technology to produce *capital goods*, and H obtains utility from consuming them, which gives H and F motives for trading with each other. Real-world examples of *capital goods* could be foreign investment projects for building infrastructure, enhancing local firms' management skills and knowhow, developing financial market structure, and so on. Emerging economies benefit from these projects for a variety of reasons, such as employment opportunities and positive technology spillovers. These benefits explain why H seeks to acquire *capital goods* from the F in the model. In this sense, the recipient of foreign investment projects (i.e., H) is identified as a buyer of *capital goods*, whereas F is labeled a seller in this FIM. Most importantly, it is assumed that exchange in this market requires a medium of exchange (MOE). Motivated by the theoretical arguments and empirical evidence given earlier, only foreign assets (i.e., U.S. T-bills) can serve as a direct medium of exchange. Lastly, for the sake of this paper's objective, the model abstracts from bargaining considerations. Accordingly, the buyer (i.e., H) is assumed to make a *take-it-or-leave-it offer* to the seller (i.e., F) in any bilateral meeting.

In the third sub-period, all agents are back in their local country and visit their own decentralized markets, where bilateral and anonymous trade for specialized goods only among local agents takes place. Again, this restriction precludes the model from an international goods trade channel. Following Lagos and Wright (2005), the local decentralized market is termed DM henceforth, and goods here shall be called as *special goods*. Since the key liquidity mechanism of this model is derived from the FIM, the DMs are set up to be as simple as possible. The two DMs are symmetric. Analogous to the FIM, exchange has to be *quid pro quo*, and only local assets can serve as the means for payment in the DM.[9] Furthermore, the *take-it-or-leave-it offer* by the buyer of the special goods is again assumed.

Time preference with a parameter $\beta \in (0, 1)$ applies only between periods but not between sub-periods, and H consumes in all sub-periods, and she supplies labor in the first and third sub-periods. Let $\mathcal{U}^H(x, X, l, L, t)$ represent H's preferences, where x and X are consumption

[8]One could instead introduce multi-period bonds, which would not change the qualitative implications of model. The one-period bond assumption is imposed purely for simplification.

[9]This assumption is not just motivated by empirical evidence. Geromichalos and Simonovska (2011) actually show that local assets can indeed endogenously arise as a superior medium of exchange in local markets with an introduction of tiny transaction costs associated with local trade using foreign assets.

in the DM and CM, respectively while l and L denote labor hours in the DM and CM respectively. Lastly, t captures the amount of capital goods obtained from the F in the FIM. Agent F consumes only in the first and third periods, and she supplies labor in the second and third sub-periods. Let $\mathcal{U}^F(x, X, l, L, h)$ represent F's preferences, where the only new variable is h, which stands for labor units employed in the FIM. Following the traditional monetary search literature, the quasi-linear utility functional form is adopted as

$$\mathcal{U}^H(x, X, l, L, t) = U(X) - L + u(t) + u(x) - l$$
$$\mathcal{U}^F(x, X, l, L, h) = U(X) - L - c(h) + u(x) - l$$

Thus, the usual assumption for the utility and cost functions in the literature applies: $u' > 0, U' > 0, u'' < 0, U'' < 0, U'(0) = u'(0) = +\infty, c' > 0, c'' > 0$.[10]

Next, search frictions in the FIM and DM need to be defined. First in the FIM, a random match between H and F is assumed with a matching function, $M(B, S)$, which indicates the total number of matches in the FIM when the mass of buyers (i.e., H) and sellers (i.e., F) equal B and S, respectively. This matching function, M, is assumed to be increasing in both arguments and homogeneous of degree one. As a result, the arrival rate of buyers (sellers) to an arbitrary seller (buyer), χ_f and χ_h, respectively, can be expressed as

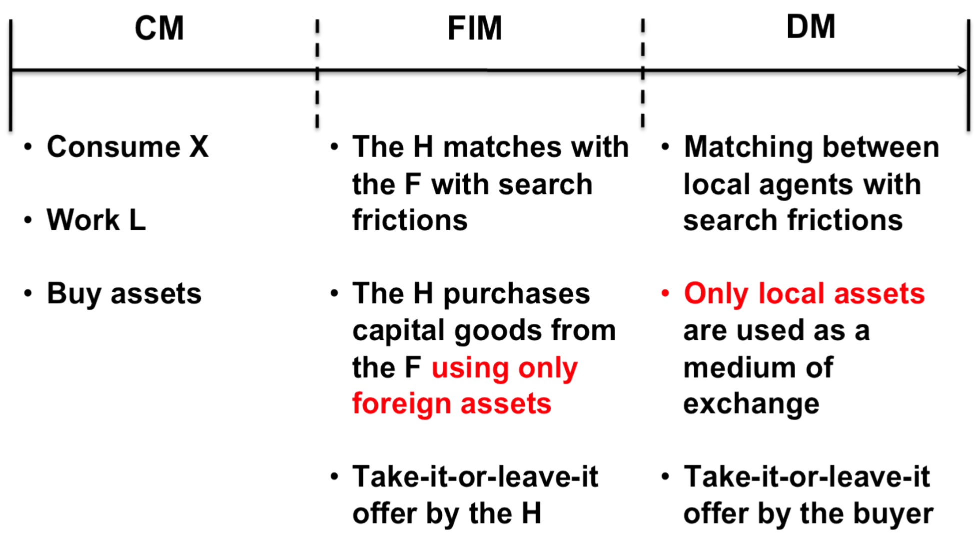
$$\chi_f = \frac{M(B, S)}{S} = M\left(\frac{B}{S}, 1\right) \equiv f(\theta)$$
$$\chi_h = \frac{M(B, S)}{B} = M\left(1, \frac{S}{B}\right) \equiv \theta\chi_f$$

where θ, market tightness equals $\frac{S}{B}$. It is finally assumed that buyers (H) always visit the FIM and sellers (F) get to visit the FIM with a probability $\delta \in (0, 1)$ which is meant to capture a degree of international financial market integration. Under this assumption, the market tightness is then given by $\theta = \frac{\delta}{1} = \delta$.

Search frictions in the DM closely follow Lagos and Wright (2005). In each of the DMs, two agents i and j are drawn at random. This leads to three possible events. The probability that i consumes what j produces but not vice versa (i.e., a single coincidence) is denoted as σ. Symmetrically, the probability that j consumes what i produces but not vice versa is also σ. In a single-coincidence meeting, the agent who wishes to consume is called the buyer and the agent who produces is called the seller. The probability neither wants anything the other produces is $1 - 2\sigma$, which implies $\sigma \leq 1/2$.[11] Lastly, the σ is assumed to be symmetric across the two countries. Figure 1 summarizes the timing of events in this model economy.

[10]For simplicity, it is assumed that $c(l) = l$, which is of no importance for main model implications later.
[11]The last potentially possible case (i.e., when both parties like what the other produces) is ignored for simplicity.

Figure 1: Timing of Events



3. Value Functions and Optimal Behavior

3.1. Value Functions

For presentation convenience this section begins by describing an agent's value function in the CM. Consider the H who enters this market with a portfolio of home and foreign assets (a, a^*). The Bellman's equation is then expressed by

$$W^H(a, a^*) = \max_{\{X, L, \widehat{a}, \widehat{a}^*\}} \left\{ U(X) - L + \Omega^H(\widehat{a}, \widehat{a}^*) \right\}$$
$$s.t. \;\; X + \psi\widehat{a} + \psi^*\widehat{a}^* = L + a + a^*$$

where ψ and ψ^* stand for the prices of home and foreign asset respectively and variables with hats denote next period's choices. It can be easily verified that, at the optimum, $X = \bar{X}$. Replacing L from the H's budget constraint into $W^H(a, a^*)$ yields

$$W^H(a, a^*) = U(\bar{X}) - \bar{X} + (a + a^*) + \max_{\{\widehat{a}, \widehat{a}^*\}} \left\{ -\psi\widehat{a} - \psi^*\widehat{a}^* + \Omega^H(\widehat{a}, \widehat{a}^*) \right\} \tag{1}$$

It is important to note that no wealth effects for the H's portfolio choice exists following from the quasi-linearity of $\mathcal{U}$. Furthermore, $W^H(a, a^*)$ is linear in state variables and therefore can be simplified as

$$W^H(a, a^*) = \Lambda^H + a + a^* \tag{2}$$

Next, consider the F's Bellman's equation. Regarding her asset holdings, she is assumed to carry no home assets as she leaves the CM for simplicity.[12] Thus, when the F enters the CM, she can only hold foreign assets that she received during trade in either of the preceding FIM and DM (i.e., a^* is the only state variable for the F). This gives the CM value function of F as

$$W^F(a^*) = \max_{\{X,L,\widehat{a}^*\}} \big\{U(X) - L + \Omega^F(\widehat{a}^*)\big\}$$
$$s.t.\ \ X + \psi^* \widehat{a}^* = L + a^*$$

Since $X = \bar{X}$ at the optimum again, the Bellman's equation for the F can be rewritten as

$$W^F(a^*) = U(\bar{X}) - \bar{X} + a^* + \max_{\{\widehat{a}^*\}} \big\{ - \psi^* \widehat{a}^* + \Omega^F(\widehat{a}^*)\big\} \tag{3}$$

The quasi-linearity assumption also simplifies the e.q.(3) as an affine function[13]

$$W^F(a^*) = \Lambda^F + a^* \tag{4}$$

Once the CM closes, both proceed to the FIM. The matching probabilities (or arrival rates) for the two types of agents (H and F) are exogenously given by χ_h and χ_f. Let t denote the amount of *capital goods* transferred from the F to the H in the FIM, while b^* stands for the total units of foreign assets received by the F in exchange for the t given to H. These terms will be determined through bargaining which will be dealt in details later. For now, it is understood that b^* and t will, in general, be functions of the foreign asset holdings of both H and F within a match. Let $\widetilde{a}^*$ denote the amount of foreign asset holdings that an agent expects a potential counterparty to carry. The following then shows the value functions for the H and F during the FIM.

$$\Omega^H(a, a^*) = \chi_h\big\{u(t) + V^H(a, a^* - b^*)\big\} + (1 - \chi_h)V^H(a, a^*) \tag{5}$$
$$\Omega^F(a^*) = \chi_f\big\{V^F(a^* + b^*) - c(t)\big\} + (1 - \chi_f)V^F(a^*) \tag{6}$$

where V^H and V^F denote a value function for the H and F respectively in the following local DMs, and $t = t(a^*, \widetilde{a}^*)$, $b^* = b^*(a^*, \widetilde{a}^*)$.

Finally consider value functions in the last sub-period. For the home country's local DM, let $F(\widetilde{a})$ be the distribution of home asset holdings among home agents. Let q also be the

[12] Intuitively, the F does not require any liquidity service from home assets in any of the FIM and her local DM. However, there may be a case where the cost of carrying home assets becomes zero in equilibrium. In this case, the F may choose to hold home assets purely as a savings instrument. In order to avoid this situation, one could possibly introduce an infinitesimally small cost of participating the CM in line with Chiu and Molico (2010). This would ensure no home asset holdings by the F all the time. One can also refer to Rocheteau and Wright (2005) for a careful proof of the result that sellers do not hold any means of payment.

[13] The definition of Λ^H and Λ^F are obvious from e.q.(1) and (3) respectively.

quantity of *special goods* produced by the seller, and n the total payment in the units of home assets, made to the seller by the buyer. These terms will also be determined through bargaining explained later. The Bellman's equation then is

$$\begin{aligned} V^H(a,a^*) = {} & \sigma\big\{u(q(a)) + \beta W^H(a-n(a),a^*)\big\} \\ & + \sigma\int\Big\{-q(\widetilde{a}) + \beta W^H(a+n(\widetilde{a}),a^*)\Big\}dF(\widetilde{a}) \\ & + (1-2\sigma)\beta W^H(a,a^*) \end{aligned} \tag{7}$$

where $q = q(a)$ and $n = n(a)$.

The first line captures the payoff from buying $q(a)$ and going to the next period's CM with asset holdings of $(a - n(a), a^*)$. The second line means the expected payoff from selling $q(\widetilde{a})$ and going to the next period's CM with $(a + n(\widetilde{a}), a^*)$. It is easy to see that only the amount of assets that the buyer brings into the DM matters for the determination of the terms of trade. The last line is the payoff from going to the next period's CM with no trade history in the current DM. The F's value function in the DM can be computed in a similar way. Using the same intuition, the Bellman's equation for the F can be expressed as

$$V^F(a^*) = \sigma\big\{u(q(a^*)) + \beta W^F(a^* - n(a^*))\big\} \tag{8}$$

$$+ \sigma\int\Big\{-q(\widetilde{a}^*) + \beta W^F(a^* + n(\widetilde{a}^*))\Big\}dF(\widetilde{a}^*) \tag{9}$$

$$+ (1-2\sigma)\beta W^F(a^*)$$

Having figured out the value functions for all agents, the next section describes the terms of trade in a bilateral trading session in the DM and FIM respectively.

3.2. Bargaining in the DM

Since the DM follows after the FIM, the terms of trade in the FIM should be critically affected by the terms of trade in the DMs. For this reason, backward induction is employed. Following the previous section, the terms of trade in any bilateral meeting within the home DM are $\{q(a), n(a)\}$, where a is the amount of home asset holdings that the buyer has brought into the bargaining. With *take-it-or-leave-it* offers by the buyer, the bargaining problem is then

$$\begin{aligned} \max_{\{q,n\}} & \Big\{u(q) + \beta\Big[W^H\big(a-n,a^*\big) - W^H\big(a,a^*\big)\Big]\Big\} \\ s.t.\ \ & 1.\ \ q \le \beta\Big[W^H\big(a+n,a^*\big) - W^H\big(a,a^*\big)\Big] \\ & 2.\ \ n \le a \end{aligned}$$

The buyer tries to maximize the *special goods* consumption utility (i.e. $u(q)$). At the same time, she also needs to minimize the loss from giving up n in exchange for q in a present discounted form. This is what the objective function in the bargaining problem describes. By the same logic, gains in exchange for q must be greater than or equal to the cost of producing q for the seller. Moreover, the amount of assets handed over to the seller can not exceed what the buyer owns at the time of negotiation due to the limited commitment problem. These explain the two budget constraints. Exploiting the linearity of the $W^H(a, a^*)$ in e.q.(1), the bargaining problem can be rewritten as

$$\max_{\{q,n\}} \left\{u(q) - \beta n\right\}$$
$$s.t.\ \ 1.\ \ q = \beta n$$
$$2.\ \ n \leq a$$

By the symmetric DM assumption across the countries, the bargaining problem in the foreign country's local DM can be written almost identically.

$$\max_{\{q^*,n^*\}} \left\{u(q^*) + \beta\Big[W^F\big(a^* - n^*\big) - W^F\big(a^*\big)\Big]\right\}$$
$$s.t.\ \ 1.\ \ q^* \leq \beta\Big[W^F\big(a^* + n^*\big) - W^F\big(a^*\big)\Big]$$
$$2.\ \ n^* \leq a^*$$

The linearity of the $W^F(a^*)$ in e.q.(3) again simplifies the problem as

$$\max_{\{q^*,n^*\}} \left\{u(q^*) - \beta n^*\right\}$$
$$s.t.\ \ 1.\ \ q^* = \beta n^*$$
$$2.\ \ n^* \leq a^*$$

The following lemma describes the bargaining solutions during the two DMs in detail.

Lemma 1. *Define* $\tilde{q} = \left\{q : u'(q) = 1\right\}$ *and* $\check{a} = \frac{\tilde{q}}{\beta}$. *The the solution to the bargaining problem of* H *is given by*

$$\text{If} \quad a \geq \check{a} \quad \text{then} \begin{cases} q = \tilde{q} \\ n = \check{a} \end{cases} \quad \text{If} \quad a < \check{a} \quad \text{then} \begin{cases} q = \beta a \\ n = a \end{cases}$$

By the same token, the solution for the foreign country's DM bargaining is:

$$\text{If} \quad a^* \geq \breve{a} \quad \text{then} \begin{cases} q^* = \tilde{q} \\ n^* = \breve{a} \end{cases} \quad \text{If} \quad a^* < \breve{a} \quad \text{then} \begin{cases} q^* = \beta a^* \\ n^* = a^* \end{cases}$$

Proof. It can be easily verified that the suggested solution satisfies the necessary and sufficient conditions for maximization. □

With no hold-up problem understanding the lemma above is straightforward. What determines the solutions critically depends upon the buyer's local asset holdings brought into the bargaining. When her local asset holdings are short of the threshold level, $\breve{a}$, she would purchase as much q as her local assets holdings allow. On the contrary, if her local asset holdings are greater than or equal to $\breve{a}$ then, she would only spend a portion of the assets such that she could purchase only up to the optimal amount of $\tilde{q}$.

3.3. Bargaining in the FIM

Consider a meeting in the FIM between the H with foreign asset holdings of a_h^* and the F with a_f^*. Assuming again the *take-it-or-leave-it* offer by the buyer (H), the bargaining problem is written as

$$\begin{aligned} &\max_{\{t,b^*\}} \left\{ u(t) + \left[V^H\left(a, a_h^* - b^*\right) - V^H\left(a, a_h^*\right) \right] \right\} \\ &s.t. \;\; 1. \;\; c(t) \leq \left[V^F\left(a_f^* + b^*\right) - V^F\left(a_f^*\right) \right] \\ &\qquad\; 2. \;\; b^* \leq a_h^* \end{aligned}$$

Intuition of this bargaining problem is identical to the DM's case. The two constraints stand for the usual participation constraint for the seller (F) and resource constraint for the buyer (H) respectively. Notice that the threat point adjusted surplus or loss is not discounted here due to no time discount between sub-periods. If one substitutes the value functions V^H and V^F from e.q.(7) and e.q.(8) into the expression above, the bargaining problem can be rewritten as

$$\begin{aligned} &\max_{\{t,b^*\}} \left\{ u(t) - \beta b^* \right\} \\ &s.t. \;\; 1. \;\; c(t) \leq \beta b^* + \sigma\left[u(q(a_f^* + b^*)) - \beta n(a_f^* + b^*)\right] - \sigma\left[u(q(a_f^*)) - \beta n(a_f^*)\right] \\ &\qquad\; 2. \;\; b^* \leq a_h^* \end{aligned}$$

It is understood that the $q(\cdot), n(\cdot)$ are described by the solutions to the DM bargaining problem described earlier. The participation constraint for the F in this problem deserves some intuitive explanation. Unlike the DM's bargaining case, the seller (F)'s gain in exchange for *capital goods* comes from two sources; the asset's store-of-value (i.e., βb^*) and medium-of-exchange-value in the subsequent DM $\big($i.e., $\sigma\big[u(q(a_f^*+b^*))-\beta n(a_f^*+b^*)\big]-\sigma\big[u(q(a_f^*))-\beta n(a_f^*)\big]\big)$. This is what makes the FIM trade essentially drive the liquidity mechanism of the model later. Lemma 2 summarizes the bargaining solution.

Lemma 2. *Define* $t^{op}=\big\{t:\frac{u'(t)}{c'(t)}\big\}$, $\bar{a}_f^*=\big\{a_f^*:\sigma u(\beta a_f^*)+(1-\sigma)\beta a_f^*=\beta\check{a}+\sigma\big[u(\tilde{q})-\tilde{q}\big]-c(t^{op})\big\}$, $\tilde{a}_h^*=\big\{a_h^*:c(t^{op})=\sigma u(q(a_h^*))+(1-\sigma)\beta a_h^*\big\}$, $\grave{a}_h^*=\big\{c(t^{op})-\sigma[u(\tilde{q})-u(\beta a_f^*)+\tilde{q}-\beta a_f^*]\big\}/\beta$ *and* $a_{h,Min}^*=\big\{a_h^*:c(t^{op})=\sigma\big[u(q(a_f^*+a_h^*))-u(q(a_f^*))\big]+(1-\sigma)\beta a_h^*\big\}$ *where* $\tilde{q}$ *and* $\check{a}$ *are defined in Lemma.1. Under a parameter space such that* $\beta\check{a}<c(t^{op})<\beta\check{a}+\sigma\big[u(\tilde{q})-\tilde{q}\big]$*, the bargaining solution is as follows.*[14]

$$\text{If}\quad\begin{cases}a_h^*\geq\frac{c(t^{op})}{\beta}\\ a_f^*\geq\check{a}\end{cases}\quad\text{then}\quad\begin{cases}t=t^{op}\\ b^*=\frac{c(t^{op})}{\beta}\end{cases}\quad\Rightarrow\text{Region 1}$$

$$\text{If}\quad\begin{cases}a_h^*\leq\frac{c(t^{op})}{\beta}\\ a_f^*\geq\check{a}\end{cases}\quad\text{then}\quad\begin{cases}t=\big\{t:\beta a_h^*=c(t)\big\}\\ b^*=a_h^*\end{cases}\quad\Rightarrow\text{Region 2}$$

$$\text{If}\quad\begin{cases}a_h^*\geq\grave{a}_h^*\\ \bar{a}_f^*\leq a_f^*\leq\check{a}\end{cases}\quad\text{then}\quad\begin{cases}t=t^{op}\\ b^*=\grave{a}_h^*\end{cases}\quad\Rightarrow\text{Region 3}$$

$$\text{If}\quad\begin{cases}\check{a}-a_f^*\leq a_h^*\leq\grave{a}_h^*\\ \bar{a}_f^*\leq a_f^*\leq\check{a}\end{cases}\quad\text{then}\quad\begin{cases}t=\big\{t:c(t)=\beta a_h^*+\sigma\big[u(\tilde{q})-u(\beta a_f^*)-\tilde{q}+\beta a_f^*\big]\big\}\\ b^*=a_h^*\end{cases}\quad\Rightarrow\text{Region 4}$$

$$\text{If}\quad\begin{cases}a_h^*\geq a_{h,Min}^*\\ a_f^*\leq\bar{a}_f^*\end{cases}\quad\text{then}\quad\begin{cases}(t,b^*)=\big\{(t,b^*):t>t^{op},b^*<a_h^*,b^*<\check{a}-a_f^*,\\ \qquad c(t)=(1-\sigma)\beta b^*+\sigma\big[u(\beta(a_f^*+b^*)-u(\beta a_f^*)\big]\big\}\end{cases}\quad\Rightarrow\text{Region 5}$$

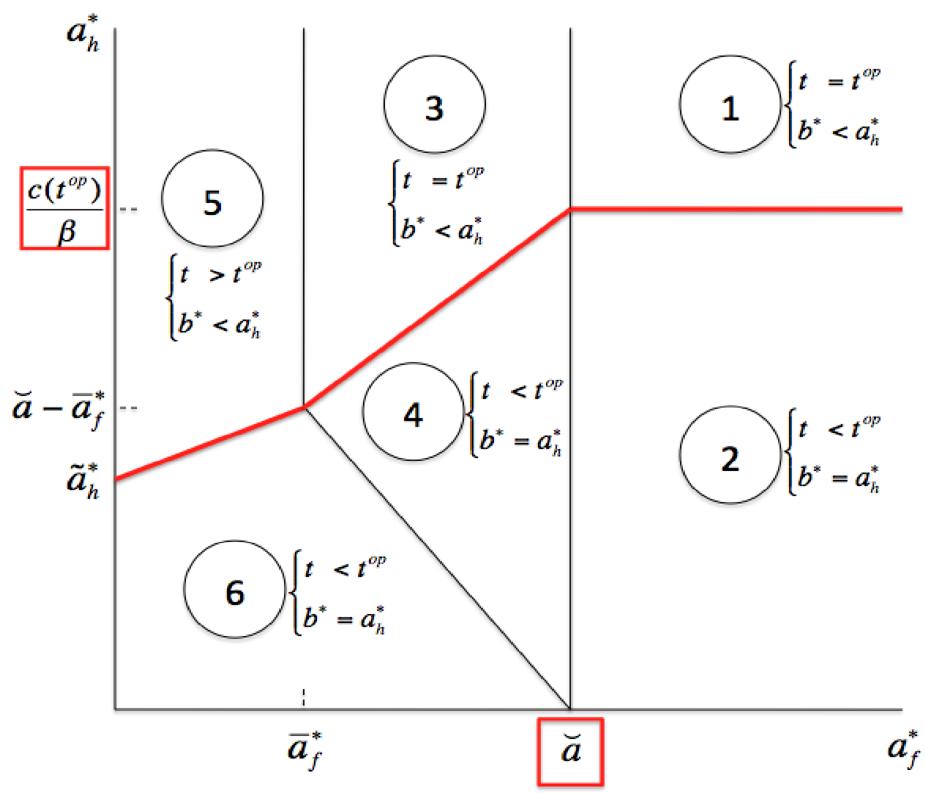
$$\text{If}\quad\begin{cases}a_h^*\leq a_{h,Min}^*\\ a_f^*\leq\check{a}-a_h^*\end{cases}\quad\text{then}\quad\begin{cases}t=\big\{t:c(t)=(1-\sigma)\beta a_h^*+\sigma\big[u(\beta(a_f^*+a_h^*))-u(\beta a_f^*)\big]\big\}\\ b^*=a_h^*\end{cases}\quad\Rightarrow\text{Region 6}$$

Proof. See appendix □

Lemma 2 can be intuitively interpreted with an assistance of Figure 2. First thing to notice here is that the bargaining solution is affected not only by the a_h^* but also the a_f^*. This feature again stems from a specific timing of the FIM and DM introduced in this model. After the FIM, the seller (F) needs to come back home and to visit the local DM using foreign assets as means of payment. Knowing this, F's participation constraint for accepting the offer from the H within the FIM bargaining has to be linked to the liquidity constraint in the subsequent local DM. For instance, there are 6 regions of different bargaining solutions depending on the

[14] The other potential parameter space case where $\beta\check{a}+\sigma\big[u(\tilde{q})-\tilde{q}\big]\leq c(t^{op})$ is relegated to appendix. As a matter of fact, this second case becomes nested by the first one. For this reason, only the latter will be considered for the rest of the analysis.

Figure 2: Regions of the FIM Bargaining Solution



combination of $\{a_h^*, a_f^*\}$ at the time of bargaining.

Consider the situation where the liquidity holdings of F are plentiful (i.e., the amount of foreign assets held by the F already satisfies the first best liquidity amount, $\breve{a}$ in the subsequent local DM). In this case, the H's foreign asset holdings solely determine the terms of trade because the F's expected surplus from obtaining foreign assets during the FIM trade is derived only from a store value of the asset (i.e., dividend payment in the next period's CM). Given this observation, if the H's foreign asset holdings happen to be greater than or equal to the amount required to purchase the first best (t^{op}), then she would only give up the amount just enough to cover t^{op}. On the other hand, if she is short of the amount for the t^{op} then, she would give up all her foreign asset holdings and purchase t as much as possible. These two situations are respectively illustrated in region 1 and 2 of Figure 2.

When the F's foreign asset holdings fall short of $\breve{a}$, then the liquidity factor kicks in to alter the participation constraint for the F during the FIM bargaining. Suppose $\{a_h^*, a_f^*\}$ initially lies in region 1 and all of sudden a_f^* falls below $\breve{a}$. This would enforce the F's liquidity constraint in the subsequent DM to bind. As a result, the F would appreciate the acquisition of foreign assets during the FIM trade more. This in turn would bring about more favorable terms of trade for the H, and therefore the t^{op} with less amount of foreign assets changing hands compared to the region 1 case. Exactly same reasoning applies to the shift from region 2 to region

4. Given the same amount of foreign asset holdings transferred to the F, she would agree to provide more *capital goods* (t) in region 4 than region 2. Again, the difference between t in the two regions reflects on the different liquidity constraints that the F would face in the subsequent local DM.

Region 5 represents a situation where the discrepancy between agents' foreign asset holdings is somewhat extreme. In this case, the F faces a severe liquidity constraint in the following local DM while the H holds a lot of foreign assets. Hence the F's rather extreme desperation for liquidity basically drives up the liquidity property of foreign assets to the point where she would be willing to accept very bad terms of trade $\big($i.e., $\{t > t^{op}, b^* < a^*_h\}\big)$. Notice here that there even exist some part of region 5 where the sum of foreign asset holdings by both H and F falls short of $\check{a}$ and yet the H would get more than t^{op}.

Lastly, region 6 implies the situation in which the liquidity in the economy dries up most. The H would give up all of her foreign assets to acquire t as much as possible. Since the first best liquidity amount for the F can not be met in anyway (i.e., $a^*_h + a^*_f < \check{a}$), the amount of goods produced would be strictly less than the first best outcome (i.e., t^{op}).

3.4. Objective Function and Optimal Behavior

With bargaining solutions of the FIM and DMs in place, one can proceed to derive the objective function of the representative agent and describe optimal behavior. The aim of the objective function is to figure out agent's optimal portfolio choice. Consider the objective function for the H first. To that end, substitute e.q.(7) into e.q.(5) and lead the emerging expression for $\Omega^H(a, a^*)$ by one period. This would generate the following.

$$\begin{aligned}\Omega^H(\widehat{a}_h, \widehat{a}^*_h) =& \beta\widehat{a}^*_h + \chi_h\big[u(t(\widehat{a}^*_h, \widehat{a}^*_f)) - \beta b^*(\widehat{a}^*_h, \widehat{a}^*_f)\big] \\ &+ \beta\widehat{a}_h + \sigma\big[u(q(\widehat{a}_h)) - \beta n(\widehat{a}_h)\big] \\ &+ \beta\Lambda^H + \sigma\int\Big\{-q(\widetilde{a}) + \beta W^H(n(\widetilde{a}), 0)\Big\}dF(\widetilde{a})\end{aligned} \tag{10}$$

The three lines of this expression represent different benefits at different sub-periods for the H who enters the FIM with a portfolio of $(\widehat{a}_h, \widehat{a}^*_h)$. The first line corresponds to the benefit from holding foreign assets. The first term indicates the discounted vale of dividends at the next period's CM while the second term shows the net surplus from the FIM trade. Notice that this net surplus depends upon her belief on the F's foreign asset holdings $(\widehat{a}^*_f)$ since they would affect the terms of trade during the FIM bargaining. The second line analogously stands for the discounted value of home asset dividends as well as the net surplus from participating in the subsequent local DM as a buyer. Last line implies the constant benefit that does not depend on the H's portfolio choice (i.e., next period CM's net consumption utility gain plus the net surplus in the subsequent local DM as a seller).

The next step is to plug e.q.(10) into $W^H(a, a^*)$ in e.q.(1). Focusing on the terms inside the maximum operator of e.q.(1) (i.e., ignoring the terms that do not affect the choice variables), one can derive the H's objective function as follows.

$$\begin{aligned} J^H(\widehat{a}_h, \widehat{a}_h^*) =& \big[-\psi + \beta\big]\widehat{a}_h + \big[-\psi^* + \beta\big]\widehat{a}_h^* \\ &+ \sigma\big[u(q(\widehat{a}_h)) - \beta n(\widehat{a}_h)\big] \\ &+ \chi_h\big[u(t(\widehat{a}_h^*, \widehat{a}_f^*)) - \beta b^*(\widehat{a}_h^*, \widehat{a}_f^*)\big] \end{aligned} \tag{11}$$

Maximization of the above function with respect to $(\widehat{a}_h, \widehat{a}_h^*)$ therefore fully describes the optimal asset holdings of the H in every period. Conforming with the literature, the interpretation of e.q.(11) is standard. The first line represents the net cost of carrying one unit of home and foreign assets respectively from today's CM into tomorrow's CM. The second and third line express the expected surplus from carrying the home and foreign assets into the DM and FIM respectively.

What is worth noting here is that the third line in e.q.(11) depends on the terms t and b^*, which in turn depend on the the bargaining protocol in the FIM. Given the H's choice of $\widehat{a}_h^*$ and beliefs on the $\widehat{a}_f^*$, she can end up in different branches of the bargaining solution as shown in lemma 2 and figure 2. This leads to different functional forms for the $J^H(\widehat{a}_h, \widehat{a}_h^*)$ with respect to the different regions. Lemma 3 presents an auxiliary result that highlights some important properties of the region-specific $J^H(\widehat{a}_h, \widehat{a}_h^*)$.

Lemma 3. *Define $J_i^H(\widehat{a}_h, \widehat{a}_h^*), i = 1, \cdots, 6$ as the H's objective function in region i. Then the partial derivative with respect to the second argument, $\widehat{a}_h^*$ in each region can be expressed as follows.*

$$\begin{aligned} &\frac{\partial J_1^H(\widehat{a}_h, \widehat{a}_h^*)}{\partial \widehat{a}_h^*} = \frac{\partial J_3^H(\widehat{a}_h, \widehat{a}_h^*)}{\partial \widehat{a}_h^*} = \frac{\partial J_5^H(\widehat{a}_h, \widehat{a}_h^*)}{\partial \widehat{a}_h^*} = -\psi^* + \beta \\ &\frac{\partial J_2^H(\widehat{a}_h, \widehat{a}_h^*)}{\partial \widehat{a}_h^*} = \frac{\partial J_4^H(\widehat{a}_h, \widehat{a}_h^*)}{\partial \widehat{a}_h^*} = -\psi^* + \beta + \chi_h\beta\left\{\frac{u'(t)}{c'(t)} - 1\right\} \\ &\frac{\partial J_6^H(\widehat{a}_h, \widehat{a}_h^*)}{\partial \widehat{a}_h^*} = -\psi^* + \beta + \chi_h\beta\left\{\frac{u'(t)}{c'(t)}\big[(1-\sigma) + \sigma u'(\beta(\widehat{a}_h^* + \widehat{a}_f^*))\big] - 1\right\} \end{aligned}$$

Proof. See the appendix □

The F's objective function should take a simpler form than the H's since her optimal choice would not depend upon the H's choice at all. Following the same steps as in the H's case, substitute e.q.(8) into e.q.(6) and lead the emerging expression for $\Omega^F(a^*)$ by one period to get

$$\Omega^F(\widehat{a}_f^*) = \beta\widehat{a}_f^* + \sigma\big[u(q(\widehat{a}_f^*)) - \beta n(\widehat{a}_f^*)\big] + \beta\Lambda^F + \sigma\int\Big\{ -q(\widetilde{a}^*) + \beta W^F(n(\widetilde{a}^*))\Big\} dF(\widetilde{a}^*) \tag{12}$$

This expression can be interpreted in a similar manner to the e.q.(10). Notice here that unlike the H's case, the F does not appreciate the liquidity properties of foreign assets within the FIM trade since the H would exploit the whole surplus from the *take-it-or-leave-it* offer. This fact leads to a lot more concise form of the objective function for the F. Plugging e.q.(12) into e.q.(3) and focusing on the terms inside the maximum operator, one can derive the F's objective function as

$$J^F(\widehat{a}_f^*) = \big[-\psi^* + \beta\big]\widehat{a}_f^* + \sigma\big[u(q(\widehat{a}_f^*)) - \beta n(\widehat{a}_f^*)\big] \tag{13}$$

where the first term stands for the cost of carrying foreign assets into the local DM and the second term captures the expected surplus term as in e.q.(11).

Based on the two objective functions, one can consider equilibrium characteristics of home and foreign asset prices. As a matter of fact, it would be easy to verify whether the cost of carrying asset terms in e.q.(11) and (13) are non-negative or not in equilibrium. The next lemma does this verification by stating important results regarding the sign of the cost terms in equilibrium.

Lemma 4. *In any equilibrium, the following two conditions must hold.*

$$\psi \geq \beta \quad \text{and} \quad \psi^* \geq \beta$$

Proof. See appendix □

Notice that the β is the so-called *fundamental* value of the asset in the literature (i.e., the price that agents would be willing to pay for one unit of the asset if neither FIM nor DMs existed). The non-negative sign of the cost terms assigns a very intuitive meaning to the objective functions of agents. Agents wish to bring assets into either of the FIM and DM in order to facilitate trade. However, they face a trade-off because carrying these assets is not free (i.e. the first line in e.q.(11) and (13)). This eventually gives rise to the optimal portfolio choice problem of agents. In what follows, each agent's optimal portfolio choice problem is rigorously studied.

First, consider the F's problem which is easier. The following lemma describes the optimal portfolio choice of the F taking ψ and ψ^* as given.

Lemma 5. *A foreign agent's optimal choice of foreign asset holdings satisfies the following. If $\psi^* = \beta$, then the optimal foreign asset holdings of the F should be greater than or equal to $\breve{a}$. On the other hand, if $\psi^* > \beta$, then there exists an **unique** level of foreign asset holdings, $\widetilde{a_f^*}$ such that $\widetilde{a_f^*} \in (0, \breve{a})$ and*

$$\psi^* - \beta = \sigma\beta\big\{u'(q(\widetilde{a_f^*})) - 1\big\}$$

Proof. See appendix □

A standard marginal cost-benefit analysis can be applied to interpret the optimal condition in lemma 5. The F at the optimum must choose to hold the amount of foreign assets such that the marginal benefit $\big($i.e., $\sigma\beta\big\{u'(q(\widetilde{a_f^*})) - 1\big\}\big)$ equals to the marginal cost of holding additional unit of foreign assets (i.e., $\psi^* - \beta$). This optimal condition in turn implies the usual downward sloping asset demand curve $\big($i.e., a negative relationship between ψ^* and $\widetilde{a_f^*}\big)$ due to $u'(\cdot) < 0$. For instance, when the cost of carrying foreign assets falls to zero (i.e., $\psi^* = \beta$), the optimality requires that the F should hold the maximum possible amount of the foreign asset, $\breve{a}$.

The H's optimal portfolio choice is nontrivial because her own belief on the F's foreign asset holdings would critically affect the objective function, $J^H(\widehat{a}_h, \widehat{a}_h^*)$ as in lemma 3. Thus, one ought to build on lemma 3 in order to study the optimal behavior of the H in details. Lemma 6 summarizes the results.

Lemma 6. *A home agent's optimal choice of home asset holdings is simple and satisfies the following. If $\psi = \beta$, then the optimal home asset holdings of the H should be greater than or equal to $\breve{a}$. On the other hand, if $\psi > \beta$, then there exists an **unique** level of home asset holdings, $\widetilde{a}$ such that $\widetilde{a} \in (0, \breve{a})$ and*

$$\psi - \beta = \sigma\beta\big\{u'(q(\widetilde{a})) - 1\big\}$$

Taking ψ^* given, the H's optimal choice of foreign asset holdings ($\widetilde{a_h^*}$) can be categorized into three different regimes depending on her beliefs on the F's foreign asset holdings.[15]

Belief 1*:* $a_f^* > \breve{a}$

If $\psi^* = \beta$, then $\widetilde{a_h^*} = \mathbb{R}_{++} \geq \frac{c(t^{op})}{\beta}$. If $\psi^* > \beta$, then $\exists! \widetilde{a_h^*} \in \Big(0, \frac{c(t^{op})}{\beta}\Big)$ such that $\widetilde{a_h^*} = \frac{c(t)}{\beta}$ and $\psi^* - \beta = \chi_h \beta\big\{\frac{u'(t)}{c'(t)} - 1\big\}$.

Belief 2*:* $\bar{a}_f^* < a_f^* \leq \breve{a}$

1. *If $\psi^* = \beta$, then $\widetilde{a_h^*} = \mathbb{R}_{++} \geq \grave{a}_h^*$.*
2. *If $\beta < \psi^* \leq \overline{\psi^*}$, then $\exists! \widetilde{a_h^*} \in [\breve{a} - a_f^*, \grave{a}_h^*)$ such that $\beta\widetilde{a_h^*} = \big\{c(t) - \sigma[u(\tilde{q}) - u(\beta a_f^*) + \tilde{q} - \beta a_f^*]\big\}$ and $\psi^* - \beta = \chi_h \beta\big\{\frac{u'(t)}{c'(t)} - 1\big\}$.*

[15] $\bar{a}_f^*, a_{h,Min}^*, \grave{a}_h^*, t^{op}$ and $\widetilde{q}$ are defined in lemma 1 and 2.

3. *If* $\overline{\psi^*} < \psi^*$*, then* $\exists! \widetilde{a_h^*} \in (0, \check{a} - a_f^*)$ *such that* $\beta \widetilde{a_h^*} = c(t) - \sigma\big[u(\beta(a_f^* + \widetilde{a_h^*})) - u(\beta a_f^*) + \beta \widetilde{a_h^*}\big]$ *and* $\psi^* - \beta = \chi_h \beta \big\{ \frac{u'(t)}{c'(t)} \big\{(1-\sigma) + \sigma u'(\beta a_f^*)\big\} - 1 \big\}$.

where $\overline{\psi^*}$ *is such that* $\overline{\psi^*} - \beta = \chi_h \beta \big\{ \frac{u'(t)}{c'(t)} - 1 \big\}$ *and* $c(t) = \sigma\big[u(\tilde{q}) - \tilde{q}\big] - \sigma\big[u(q(a_f^*)) - q(a_f^*)\big] + \beta(\hat{a} - a_f^*)$.

Belief 3*:* $a_f^* \leq \bar{a}_f^*$

If $\psi^* = \beta$*, then* $\widetilde{a_h^*} = \mathbb{R}_{++} \geq a_{h,Min}^*$*. If* $\psi^* > \beta$*, then* $\exists! \widetilde{a_h^*} \in (0, a_{h,Min}^*)$ *such that* $\beta \widetilde{a_h^*} = c(t) - \sigma\big[u(\beta(a_f^* + \widetilde{a_h^*})) - u(\beta a_f^*) + \beta \widetilde{a_h^*}\big]$ *and* $\psi^* - \beta = \chi_h \beta \big\{ \frac{u'(t)}{c'(t)} \big\{(1-\sigma) + \sigma u'(\beta a_f^*)\big\} - 1 \big\}$.

Proof. See appendix □

Technical details of the H's optimization problem are relegated to appendix. In what follows, the most important properties of the H's choice are discussed in an intuitive way. First of all, her optimal home asset demand is trivial because she would only take the local DM's bargaining protocol into consideration. As a matter of fact, it is identical to the F's optimal foreign asset holdings since both H and F face a symmetric market structure in their own local DMs (i.e., same degree of search frictions and only local assets being used as media of exchange).

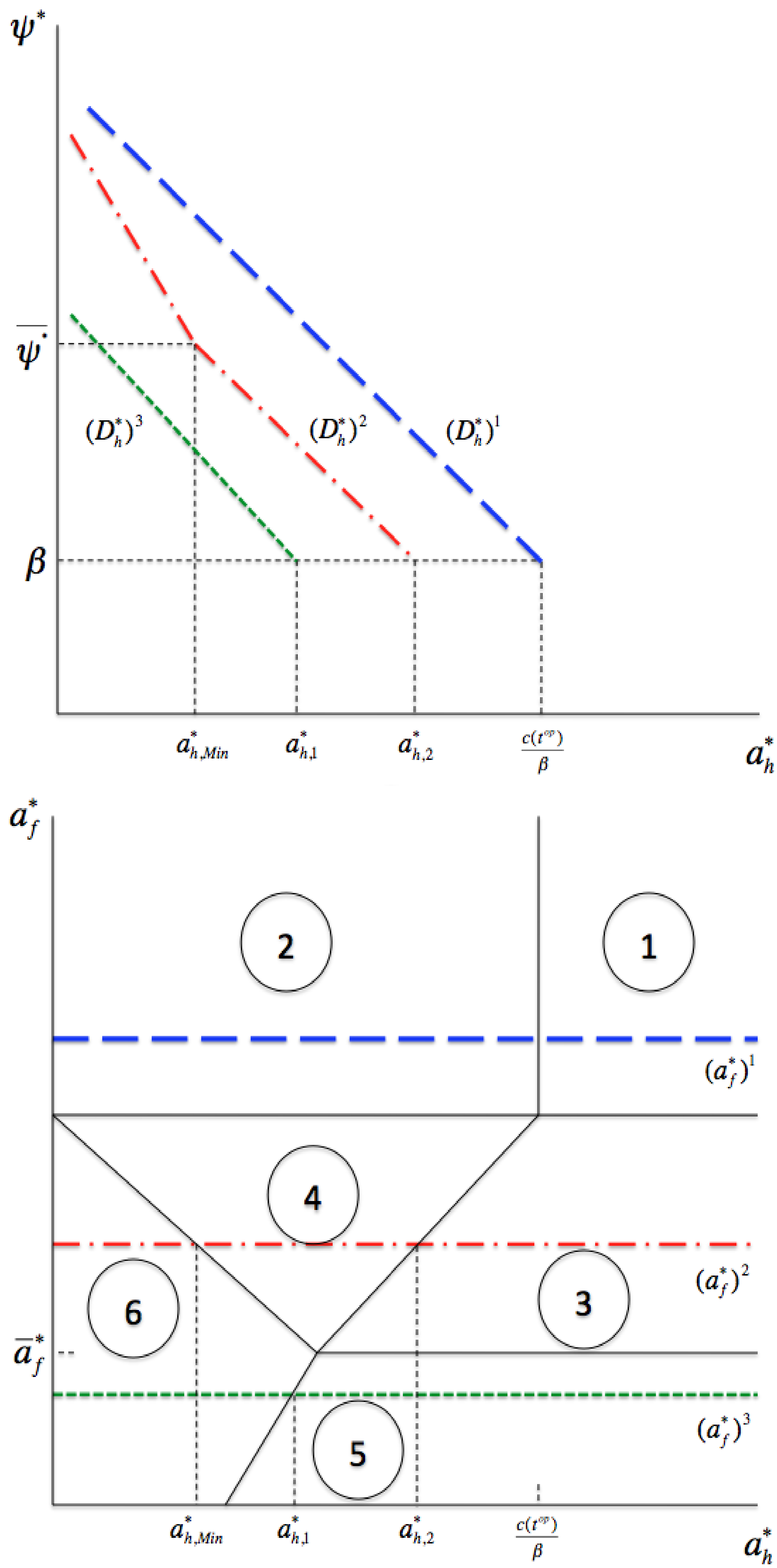
However, the H's optimal choice for foreign asset holdings would be a little more complex. Suppose the foreign asset price is at the fundamental level (i.e., $\psi^* = \beta$) for instance. Since the cost of carrying foreign assets becomes zero, it would not be optimal for the H to be in a region where her assets would not allow her to afford the optimal quantity of *capital goods*. In short, if $\psi^* = \beta$, then the H would never choose a portfolio in the interior of regions 2, 4 or 6 of figure 2.

In contrast, if $\psi^* > \beta$, carrying the asset becomes costly. The optimal choice of the H is then pinned down by the first-order conditions and, graphically, it lies within regions of either 2, 4 or 6 depending on her beliefs upon a_f^*. For instance, suppose the H's belief on the F's holdings of foreign assets happens to be greater than the first best liquidity amount (i.e., $\check{a}$). In this case, figure 2 confirms that the FOC associated with the region 2 always pins down the H's optimal choice of foreign asset holdings.

Interesting case happens when the H believes that a_f^* lies in between $\bar{a}_f^*$ and $\check{a}$. In this scenario, the relative size of the foreign asset price becomes crucial. When ψ^* is too high $\big($i.e., $\psi^* > \overline{\psi^*}\big)$, the cost of carrying asset becomes too burdensome for the H. Thus, she would typically choose to hold less foreign assets and settles in the interior of region 6 and the associated FOC determines the optimal level of foreign asset holdings. On the other hand, if the ψ^* stays in a rather moderate range $\big($i.e., $\beta < \psi^* < \overline{\psi^*}\big)$, then she would increase the asset holdings to the extent that the diminished cost of carrying asset is reflected (i.e., she would end up within the region 4).

To graphically sum up intuition, the foreign asset demand by the H, D_h^* is plotted in figure 3 against the price, ψ^*. In this graph, the H's belief on the level of F's asset holdings is kept

Figure 3: Home Agent's Foreign Asset Demand given Different Levels of a_f^*



fixed at the values $(a_f^*)^1$, $(a_f^*)^2$ and $(a_f^*)^3$. These values are indicated in the lower panel of figure 3, which replicates figure 2. The vertical alignment of the two plots enables one to find which regions (in terms of figure 2) the H finds herself in, for any choice of a_h^*, given the value of $(a_f^*)^j$, $j = 1, 2$ and 3. In essence, the greater F's foreign asset holdings are the more H demands the asset $\big($i.e., $(D_h^*)^1 > (D_h^*)^2 > (D_h^*)^3\big)$. This is attributed to the fact that the F who holds more foreign assets becomes less desperate for acquiring the asset during the FIM trade. Thus the H would have to give up more foreign assets to induce the F to accept the offer in the FIM bargaining.

Another important feature of the graph is the kinked demand curve for the moderate range of a_f^* $\big($i.e., $(a_f^*)^2\big)$. $(D_h^*)^2$ exhibits a kink at a threshold level of $\overline{\psi^*}$. To illustrate this property, one needs to recall the regime switch case (i.e., between region 4 and 6) in the neighborhood of $\overline{\psi^*}$ explained in the previous paragraph. Imagine a case where the foreign asset price steadily rises from its fundamental value β. Once the ψ^* pushes the H from the region 4 into 6, she would deal with more desperate F during the FIM bargaining. This would in turn make her less sensitive to the change in the foreign asset prices compared to the case in the region 4. Another way of putting it is that the F's willingness to provide more *capital goods* in exchange for the same amount of foreign assets would somewhat offset the effects of changes in the cost of carrying assets. In short, the H's elasticity of asset demand with respect to ψ^* should be lower in the region 6 than 4, consistent with the direction of a kink in $(D_h^*)^2$

4. Equilibrium

4.1. Definition and Existence of Equilibrium

Having established the optimal behavior of the representative agent, the next step is to discuss a recursive equilibrium of the economy. This paper only focuses on the steady state equilibrium and study the equilibrium property associated with effects of different degrees of search frictions in the FIM on equilibrium asset prices and portfolio composition. First, a steady state equilibrium in this model is defined as follows.

Definition 1. *For the two-country economy, a steady state equilibrium is a following list of an allocation* $\{X_i, L_i, a_h, a_i^*, i = \{h, f\}\}$, *together with value functions* $\{V^i, \Omega^i, W^i, i = \{H, F\}\}$, *a set of prices* $\{\psi, \psi^*\}$, *bilateral terms of trade* $\{t(a_h^*, a_f^*), b^*(a_h^*, a_f^*)\}$ *in the FIM, bilateral terms of trade* $\{q(a_h), n(a_h)\}$ *in the H's local DM, bilateral terms of trade* $\{q(a_f^*), n(a_f^*)\}$ *in the F's local DM when F was not matched in the preceding FIM, and bilateral terms of trade* $\{q(a_f^* + b^*(a_h^*, a_f^*)), n(a_f^* + b^*(a_h^*, a_f^*))\}$ *in the F's local DM when F was matched in the preceding FIM such that:*

- ✓ *Given prices, the value functions and decision rules satisfy e.q (1), (3), (5), (6) (7), and (8)*

✓ *Bargaining solutions in the FIM and DMs satisfy lemma 1 and 2*

✓ *The set of prices is such that all agents maximize their objective functions, e.q (11) and (13)*

✓ *Markets for the two assets clear and expectations are rational (i.e., $a_h = T$ and $a_h^* + a_f^* = T^*$).*

The definition of equilibrium is straightforward. Notice that the equilibrium quantity of *special goods* produced in the F's local DM depends on whether the F was matched in the preceding FIM or not. For instance, a foreign agent who did not get matched in the FIM can not purchase the first best amount of *special goods* (i.e., $\tilde{q}$) in her local DM unless she had brought more than $\breve{a}$ from the preceding CM. However, if she was matched in the FIM, then she would be, on some occasions, able to achieve the $\tilde{q}$ regardless of the *ex-ante* holdings of foreign assets less than $\breve{a}$ (i.e., the region 3 or 4). Obviously, on some other occasions when either her foreign asset holdings fall short of $\breve{a}$ to a great extent (i.e., the region 5) or H's foreign asset holdings are too small (i.e., the region 6), she would not be able to obtain the $\breve{a}$ even with the FIM matching. Next, the following lemma guarantees existence of equilibrium and states the conditions under which the equilibrium is unique.

Lemma 7. *If the exogenous foreign asset supply is relatively scarce, more specifically when $T^* < \breve{a} + \frac{c(t^{op})}{\beta}$, then a list of steady state equilibrium objects defined in the Definition 1 exists and also becomes unique. However, when $T^* \geq \breve{a} + \frac{c(t^{op})}{\beta}$, an indeterminacy arises in the portfolio choice of $\{a_h^*, a_f^*\}$ while prices $\{\psi, \psi^*\}$ remain unique.*[16]

Proof. See appendix □

Lemma 7 can be explained intuitively with the assistance of figure 2. If $T^* \geq \breve{a} + \frac{c(t^{op})}{\beta}$, then the figure 2 admits that the equilibrium portfolio of $\{a_h^*, a_f^*\}$ ought to lie within the region of 1, 2, 3, or 5. Suppose it lies in the interior of region 2. Here the F owns the first best amount of liquidity for her DM trade and therefore she would not pay anything more than the fundamental value of the foreign asset, β. Yet, the H would still like to pay *liquidity premium* on the asset to get closer to the t^{op}. This would cause a violation of no arbitrage condition for the foreign asset trade and therefore no equilibrium portfolio can be achieved in this region.

Similar reasoning applies to the region 3 and 5. In these regions, the F lacks liquidity in reference to the first best choice, $\breve{a}$, and therefore must be willing to pay liquidity premium on the foreign asset. The H, on the other hand, would not value the asset more than its fundamental value since she always accomplishes the t^{op} in this region. Again, no equilibrium would exist in these regions.

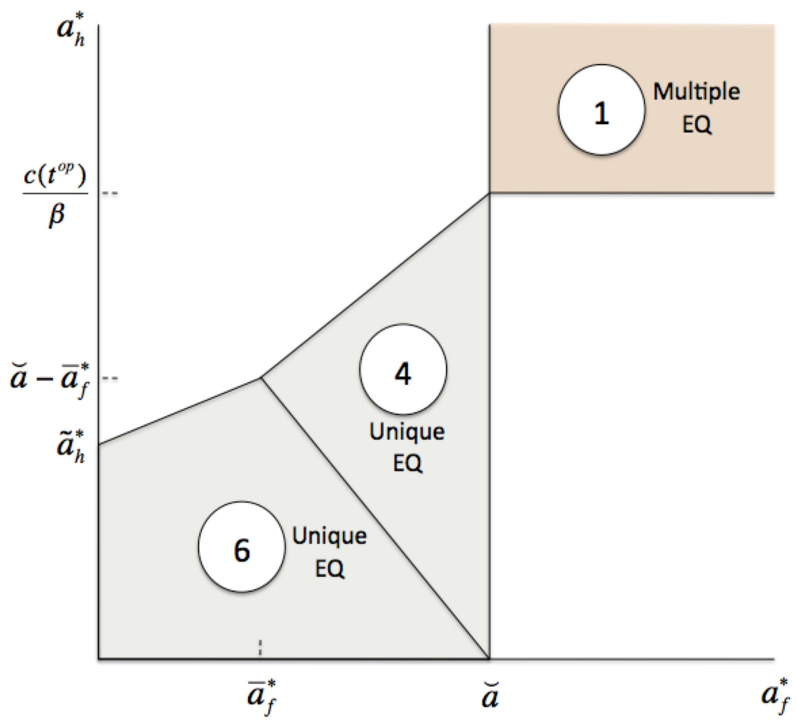
Lastly, if the equilibrium portfolio, $\{a_h^*, a_f^*\}$ stayed in the region 1, every agents would achieve the first best amount of liquidity for both of the FIM and DM. Hence, the price should

[16] The irrelevance of T, the home asset supply, for the uniqueness of the equilibrium is obvious. Intuitively, only home agents purchase home assets and the T does not affect the FIM bargaining protocol at all.

settle at the fundamental value, β and any combination of $\{a_h^*, a_f^*\}$ should satisfy the optimality. This eventually gives rise to a multiple equilibrium of the economy.

When $T^* < \breve{a} + \frac{c(t^{op})}{\beta}$, it is understood from figure 2 that the equilibrium portfolio of $\{a_h^*, a_f^*\}$ could potentially lie anywhere except within the region 1. For the same reason described earlier, the region 2, 3, and 5 are easily ruled out, which leaves only the region 4 and 6 as an equilibrium region. As witnessed again from the figure 2, neither H nor F would find herself in the plentiful liquidity situation within the region 4 and 6. The H would always want more foreign assets to aim for t^{op}. Similarly the F *ex-ante* would like to purchase foreign assets more as well.[17] This opens up the possibility of a unique market clearing price ψ^*, which can be indeed pinned down by the first-order conditions for both H and F. The uniqueness of this price is simply associated with the well-behaved (i.e. strictly concave) utility functions of agents. Technical details are left in the appendix. Finally owing to this unique price level of ψ^* given T^*, the rest of equilibrium objects must be unique as well. To assist the intuition graphically, figure 4 also plots the regions of $\{a_h^*, a_f^*\}$ in equilibrium.[18]

Figure 4: Aggregate Regions of $\{a_h^*, a_f^*\}$ in Equilibrium



[17] *Ex-ante* here means 'before the FIM trade'. The fact that foreign agents would make up the liquidity loss *ex-post* (i.e., after the FIM trade) does not attenuate the F's appreciation for the foreign asset's liquidity property. This is simply because the F would have to suffer more labor disutility, required for the asset acquisition during the FIM bargaining.

[18] For notational convenience, these regions are referred to as "aggregate regions" as opposed to the "individual regions" described in figure 2.

4.2. Characterization of Equilibrium

Given the existence of equilibrium, the next task is to assess to what extent the foreign asset supply T^* affects the various equilibrium objects. Lemma 7 confirms a unique equilibrium under $T^* < \breve{a} + \frac{c(t^{op})}{\beta}$. This allows one to perform comparative statics analyses. Focusing on the unique equilibrium case, proposition 1 evaluates the effects of T^* changes on equilibrium prices as well as the equilibrium portfolio of $\{a_h^*, a_f^*\}$.

Proposition 1. *The effects of foreign asset supply changes on the equilibrium would differ depending the relative size of the T^*.*

Scenario 1: *If $\breve{a} \leq T^* < \breve{a} + \frac{c(t^{op})}{\beta}$, then*

✓ $\psi^* > \psi = \beta$ *when $T^* = T$: The equilibrium home asset price is at the fundamental value and yet the foreign asset price exceeds that value under the symmetric asset supply.*

✓ $\frac{\partial a_f^*}{\partial T^*} > 0$, $\frac{\partial a_h^*}{\partial T^*} > 0$, $\frac{\partial t}{\partial T^*} > 0$*: The equilibrium foreign asset holdings by the both country as well as the FIM trade volume are strictly increasing in the level of foreign asset supply.*

✓ $\frac{\partial \psi^*}{\partial T^*} < 0$*: The effects of foreign asset supply changes on the equilibrium foreign asset price are strictly negative.*

Scenario 2: *If $T^* < \breve{a}$, then*

✓ $\psi^* > \psi > \beta$ *when $T^* = T$: Under the symmetric asset supply, both asset prices exceed the fundamental value and at the same time the foreign asset price is always settled at a higher level than the home asset price.*

✓ $\frac{\partial a_f^*}{\partial T^*} > 0$, $\frac{\partial \psi^*}{\partial T^*} < 0$*: The equilibrium foreign asset price and foreign asset holdings by the foreign country are strictly decreasing and increasing respectively in the level of asset supply.*

✓ $\frac{\partial a_h^*}{\partial T^*} \lesseqgtr 0$, $\frac{\partial t}{\partial T^*} \lesseqgtr 0$*: The effects of foreign asset supply changes on the equilibrium foreign asset holdings by the home country, and the FIM trade volume are ambiguous.*

✓ $\frac{\partial a_h^*}{\partial T^*}_{Scenario\ 2} < \frac{\partial a_h^*}{\partial T^*}_{Scenario\ 1}$*: Should a positive relationship between T^* and a_h^* arise then, the home country's foreign asset demand is weaker when T^* is relatively less abundant.*

Proof. See appendix □

Proposition 1 reveals that the exogenous foreign asset supply is the driving force of equilibrium. If the asset is plentiful, in the precise sense that $\breve{a} \leq T^* < \breve{a} + \frac{c(t^{op})}{\beta}$, then the unique equilibrium must be reached within the aggregate region 4 as shown in figure 4. It is already explained earlier why the foreign asset carries the liquidity premium in this region. Interestingly, under $T^* = T$, the home asset does not exhibit the liquidity premium since the H always acquires $\tilde{q}$ in this equilibrium region. What is more important is the effect of T^* changes on the equilibrium composition of $\{a_h^*, a_f^*\}$. Given the observation that both H and F would desire more liquidity in this region, it is obvious that an increase in T^* would push up the equilibrium a_h^* and a_f^* simultaneously. This would in turn relieve the liquidity shortage for every agents and therefore the new equilibrium price (or liquidity premium) of the foreign asset should decline.[19] Finally the FIM trade volume, t, would increase in this case because otherwise the optimality for the home agent would imply a decrease in ψ^* which is a contradiction $\big($i.e., recall $\psi^* - \beta = \chi_h \beta \big\{ \frac{u'(t)}{c'(t)} - 1 \big\}$ in region 4 from lemma 3$\big)$.

The properties of equilibrium responses towards a shift in the foreign asset supply are richer if the asset is relatively scarce (i.e., $T^* < \breve{a}$). In this scenario, the equilibrium must be settled within the region 6 of figure 4. First, if $T^* = T$, then both home and foreign assets carry liquidity premium. Yet, the fact that the foreign asset exhibits liquidity properties in the FIM and the foreign country's local DM simultaneously makes $\psi^* > \psi$ in equilibrium. Second, an increase in T^* would for sure induce the F to demand foreign assets more. This is intuitive given the F's even more desperation for liquidity in the region 6. As a result of more foreign asset holdings by the F, her optimality must require lower costs of carrying the foreign asset and therefore lower ψ^* in a new equilibrium.

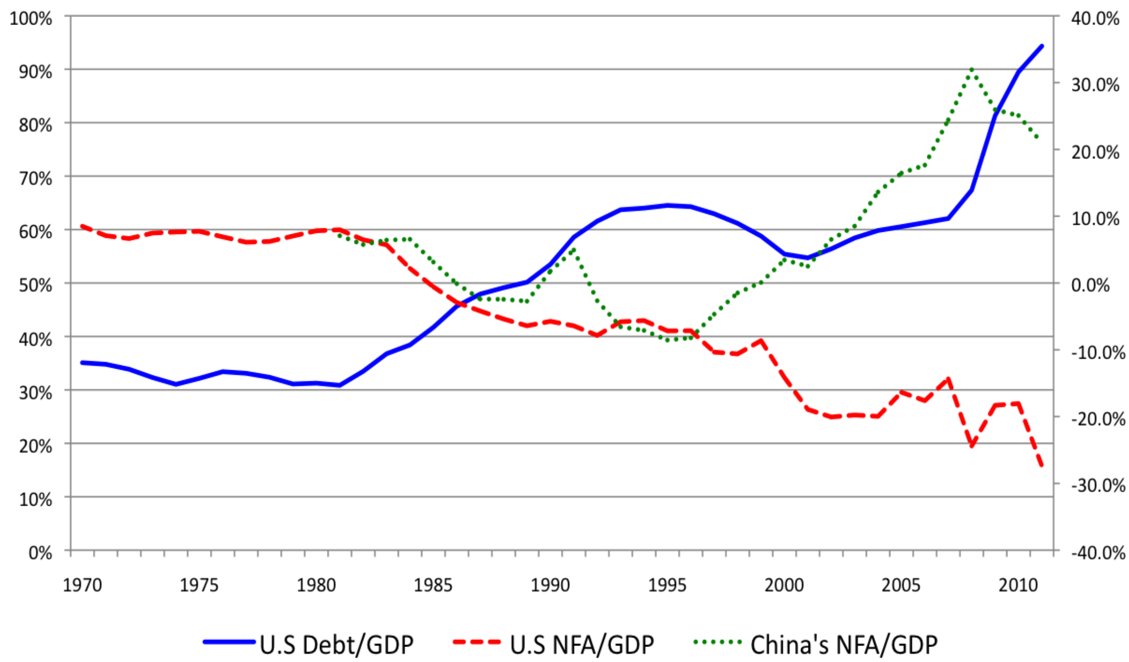
The equilibrium response of a_h^* and t would be, however, inconclusive. Yet this ambiguity can be intuitively understood with an assistance of the liquidity dependent participation constraint for the F during the FIM trade. At first, the decline in ψ^* (i.e., marginal cost of carrying the foreign asset) would generate an upward pressure for the H's foreign asset demand. However, as T^* rises, the F *ex-ante* anticipates a higher amount of liquidity carried from the FIM to her local DM. This would make the F become less desperate for the foreign asset during the FIM bargaining. Eventually, less favorable terms of trade (i.e., less liquidity properties of the foreign asset) for the H in the FIM would come about, and the initial upward pressure for the a_h^* would be somewhat mitigated. The sign of Δa_h^* and Δt would therefore depend upon the parameters of the economy in the end.

Nevertheless, it should be clear that the extent to which a_h^* increases in response to the rise in T^*, if true under some parameter values, must be smaller in the aggregate region 6 than 4. Again this is attributed to the additional incentive change for the F explained earlier. In fact, this less degree of positive relationship between T^* and a_h^* in the aggregate 6 is a mirror image of the kinked foreign asset demand curve, $(D_h^*)^2$ in figure 3. It illustrates how the H's demand exhibits the smaller price elasticity in the region 6 than 4. Since the ψ^* and T^* have a negative one to one relationship in equilibrium, the less responsiveness of a_h^* to T^* in the aggregate region 6 should come as no surprise.

Last but not least, this particular prediction of the proposition 1 has a certain degree of empirical relevance. According to the prediction, one should observe much stronger foreign asset demand by the home country and hence a deterioration (improvement) of the foreign

[19] A recent work by Krishnamurthy and Vissing-Jorgensen (2012) also demonstrates, bond supply does positively affect bond yields in the case of U.S. T-bill.

Figure 5: U.S. Debt/GDP versus U.S. and China's NFA/GDP, 1970-2011



(home) country's NFA position if the level of foreign asset supply exceeds a certain threshold level (i.e., $\breve{a}$). Figure 5 provides a supporting evidence for this prediction.[20] Up until the early 1980s, the U.S. Debt to GDP ratio was managed at a relatively lower level between 30 and 40 percentage. During the same period, the U.S. and China's NFA position remained stable and it is hard to see any significantly negative relationship between them. However, an interesting trend started to emerge between the three ratios from the early 1980s when the U.S. Debt/GDP really began to blow up. The graph clearly illustrates a strongly negative (positive) relationship between the U.S. Debt/GDP and the NFA/GDP of the U.S. (China), consistent with the prediction.[21] Now, it is ready to state the main results of the paper.

Proposition 2. *Under the unique steady state equilibrium, a decline in search frictions within the FIM generates asset price changes and a portfolio shift in line with a great deal of global imbalances observed over the last decade.*

- ✓ $\frac{\partial a_h^*}{\partial \chi_h} > 0$: *The equilibrium foreign asset holdings by the home country is strictly increasing in the degree of financial integration.*
- ✓ $\frac{\partial a_f^*}{\partial \chi_h} < 0$: *The equilibrium foreign asset holdings by the foreign country is strictly decreasing in the degree of financial integration.*

[20] Updated and extended version of dataset constructed by Lane and Milesi-Ferretti (2007b) is used for the U.S. and China's NFA position to GDP ratio

[21] It is, as a matter of fact, hard to verity if the U.S. Debt/GDP of approximately 40% in the early 80s precisely corresponds to the threshold level of $\breve{a}$ in the model. Nevertheless the qualitative aspect of the prediction should remain valid.

✓ $\frac{\partial \psi^*}{\partial \chi_h} > 0$*: An increase in the financial integration pushes up (down) the equilibrium foreign asset price (foreign interest rates).*

✓ $\frac{\partial t}{\partial \chi_h} > 0$*: The FIM trade volume and the financial integration is positively related.*

Proof. See appendix □

The most novel results of the paper, discussed in proposition 2 concern the effects of global financial integration on asset prices and the global portfolio composition. In essence, the extent to which *capital goods* trade with home agents through *OTC* markets is accessible to foreign agents, is suggested as a key driving force behind the recent upsurge in emerging markets' international reserve holdings. The intuition of the results are straightforward. An increase in χ_h or a decrease in search frictions in the FIM would first make the probability of matching in the FIM go higher which would in turn raise the marginal benefit of holding foreign assets for the home agent. Yet, the foreign agent would not be able to exploit this benefit due to her identity in the FIM trade (i.e., a seller). As a result, the global portfolio of $\{a_h^*, a_f^*\}$ should be more biased towards a_h^* at a new equilibrium. Since the home country would get to hold on to more liquidity during the FIM trade, the foreign country should be willing to produce more of *capital goods* at the new equilibrium. This would directly cause an increase in the FIM trade volume.

Finally, the less search friction in the FIM would generate higher liquidity properties of foreign assets through two channels. For one thing, the higher matching probability would surely make the liquidity value of foreign assets in the FIM higher. Also the fall in the amount of a_f^* would cause foreign agents to *ex-ante* appreciate the liquidity property of the asset in their local DM. To put it differently, with some of their foreign asset holdings taken by the home country during the CM, foreign agents would *ex-ante* fear the liquidity loss for the subsequent local DM. In short, the higher χ_h gets the higher agents appreciate the liquidity property of foreign assets in both FIM and the foreign country's local DM. This would eventually reduce (raise) the foreign asset's yield (price) at the new equilibrium.

5. Predictions of the Model and Empirical Evidence

This section explores the predictions of proposition 2 that are supported in the data. The main prediction of proposition 2 is clear: an increase in financial integration makes the liquidity property of foreign assets higher in the FIM trade. In consequence, the relative share of the T^* by the home country would increase, which would in turn boost the amount of foreign investment inflows, especially through OTC markets. In short, the level of foreign asset stocks held by the home country and the OTC-channeled foreign investment inflows must be positively linked according to proposition 2.

The goal of this section is to show empirically that this prediction holds true with real data. To that end, a panel regression analysis is performed. Before discussing the particular form of the regression equation and the various estimation methods, the next section describes the data and the construction of important variables.

5.1. Data

As a benchmark regression equation, a specification proposed in Obstfeld, Shambaugh, and Taylor (2010) is adopted. In all of the empirical equations, the dependent variable is the ratio of official international reserves to gross domestic product (GDP) for emerging economies, and the data is collected from updated and extended versions of the dataset constructed by Lane and Milesi-Ferretti (2007b) (LMF).[22] The dependent variable is supposed to correspond to the country size augmented by foreign asset holdings for the home country in the model. This premise can be justified on the grounds that emerging markets' foreign asset stocks are mostly owned by the official sector (i.e., central banks).

Obstfeld, Shambaugh, and Taylor (2010) include a measure of financial openness as an important regressor in order to support their financial stability model. Since financial openness also drives the home country's foreign asset holdings in this model, a financial openness measure is included in the regression specification as well. Almost all the other regressors chosen by Obstfeld, Shambaugh, and Taylor (2010), for example, the size of domestic financial liabilities (i.e., M2/GDP) and the trade-to-GDP ratio, are also included for the purpose of comparison with previous empirical studies. Table 1 summarizes these rather conventional explanatory variables used in the estimation.

Apart from these conventional regressors, one crucial independent variable namely, foreign (private) investment inflowsis added.[23] This variable is relatively new in the literature, and the motivation for including this variable comes strictly out of this model where an endogenous link between the foreign capital inflows and reserve holdings is a key distinctive feature. Therefore, one must take utmost care in estimating this particular variable from data. However, searching for data that matches the concept of t (i.e., the amount of foreign capital transacted through OTC markets) exactly would be a difficult task. In terms of balance of payments (BOP) accounting, the t in the model could be approximated to a change in privately held capital stocks by non-residents who operate under decentralized markets.

Given this accounting definition, most difficulties lie in how to partition foreign private investment inflows into those associated with either OTC or centralized trading. However,

[22] Neither sovereign wealth funds nor official gold holdings are included in the reserve measure.

[23] Public debt-to-GDP data for each country is also included as a regressor in order to reflect the T (i.e., total supply of asset). However, the model predicts that the home country's asset supply does not affect the foreign asset holding equilibrium of the home country. Data for the public debt-to-GDP ratio is hence collected from *IMF*s Historical Public Debt Database.

two possible ways can be devised. The first and more preferable measure for the t is to use a proxy variable. For instance, aggregate hedge fund inflows into each emerging country's OTC market and/or the aggregate private equity fund raised in each emerging market are good examples of bilateral international investment activities with search frictions.[24] However, a data excess problem precludes this study from taking this approach.

Thus, alternative measures have to be considered. Essentially one has to find a way to indirectly extract the bilaterally transacted investment inflows out of total private investment inflows. Luckily, data provided by the Emerging Portfolio Fund Research (EPFR) Global allows for the use of this method. EPFR provides weekly and monthly data on portfolio equity country flows since 1997. What is crucial is that the information in the EPFR data covers portfolio equity flows only from institutional investors and does not include flows from hedge funds, proprietary trading desks of foreign brokers and investment banks, foreign insurance companies investing their excess cash, or wealthy individuals.[25] Since these particular types of inflows (i.e., those not captured by the EPFR data) are likely to be transacted in a bilateral trading fashion, one can determine OTC-channeled foreign investment inflows by subtracting the EPFR data from the total portfolio equity inflows reported in the BOP statistics.[26]

Furthermore, one can also utilize the *errors and omissions* item from the BOP statistics when constructing bilaterally traded foreign investment inflows. In principle, this item accounts for the discrepancy between debit and credit entries in the BOP. More importantly, it has been widely used as a proxy variable for concealed and unrecorded private capital flows (i.e., the net of any concealed or unrecorded private capital inflows) in the capital flight literature.[27] Since most capital flows through centralized market clearing systems are officially recorded in the BOP, it is not implausible to assume that the item implicitly represents foreign investment inflows transacted through decentralized markets.

Based on these presumptions, the measure for the bilaterally traded foreign investment inflows is calculated as

✓ Foreign investment inflows through decentralized trading (DCF) = A + (B-C)

(i): A = Net errors and omissions from IFS's BOP statistics

(ii): B = Annual change in portfolio equity liabilities from LMF

[24] A few online databases, for example Hedge Fund Research (HFR), TASS, and the Center for International Securities and Derivatives Markets (CISDM), are expected to provide such hedge fund inflow data. For the private equity data, the Emerging Market Private Equity Association (EMPEA) provides one of the most comprehensive datasets.

[25] For more information on EPFR data coverage, access its website: http://www.epfr.com/.

[26] Notice here that only one particular type of private investment inflows (i.e., portfolio equity inflows) is considered. Another important type is undoubtedly FDI inflows. However, no segregated data on FDI in terms of trading characteristics was available, so this study had to exclude FDI in the measurement of OTC-channeled foreign investment inflows.

[27] Early literature on capital flight (Cuddington (1986) and Dooley (1987), for example) constitutes an early attempt to empirically justify the explicit use of errors and omissions in the analysis of concealed and unrecorded private capital flows.

(iii): C = The sum of monthly gross portfolio equity inflows from EPFR[28]

A baseline estimation in this study employs the DCF above with an unbalanced panel for 53 emerging market countries from 1997 to 2007. Moreover, this study only focuses on emerging markets with substantial amount of reserve holdings. Sample countries are listed in Table 3.

For robustness check later, three alternative measures for foreign (private) investment inflows are devised. Focusing on short-term speculative capital flows, the first alternative is so-called *"hot money"* measures from the capital flight literature. Following Cuddington (1986)'s standard methodology for the hot money, the first alternative measure is defined as[29]

✓ Hot money inflows (HM) = A + B + C + D + E[30]

(i): A = Net errors and omissions from IFS's BOP statistics

(ii): B = Annual change in portfolio equity liabilities from LMF

(iii): C = Annual change in portfolio debt liabilities from LMF

(iv): D = Annual change in financial derivative liabilities from LMF

(v): E = Other short term capital, other sectors from IFS's BOP statistics

The second alternative measure is simply foreign direct investment (FDI) country inflows collected from LMF. FDI inflows characteristically differ from HM on the ground that FDI is usually locked up under rather long-term contracts. Moreover, aggregate FDI inflows do not necessarily constitute OTC-channeled investment flows. The last alternative one is EPFR portfolio country inflow data (or centralized capital flows, CCF). In direct contrast to DCF, this last alternative is meant to be a proxy variable for foreign investment inflows through a centralized market clearing mechanism.

5.2. Empirical Specification

Throughout the empirical section of the paper, a panel regression analysis is conducted based on the following specific equation.

$$y_{i,t} = \lambda \cdot y_{i,t-1} + X'_{i,t}\beta + \eta_i + \epsilon_{i,t} \tag{14}$$

where $y_{i,t}$ is the ratio of official international reserve to GDP, and $y_{i,t-1}$ is its lagged variable. $X_{i,t}$ represents the set of explanatory variables including the various measures for the foreign (private) investment inflows described in the preceding section. η_i captures unobserved time

[28]The summation is for each year, and gross inflows refer to changes in the portfolio liabilities of residents.

[29]This hot money measure allows us to use more of the dataset covering 1980 to 2007.

[30]While most other hot money measures rely on net flows (i.e. difference between assets and liabilities from the BOP) in order to consider both capital out-flight and in-flight, I only focus here on the liability side so that investment activities by only non-residents are considered.

invariant country-specific effects. This regression equation also includes time dummies to control for the common effect of specific years such as global financial crisis. $\epsilon_{i,t}$ is the error term which is assumed to be serially uncorrelated. The subscripts i and t represent country and time period respectively. The data set has a feature of panel structure consisting of 484 annual observations clustered by 53 countries from 1997 to 2007.

Empirically, it is widely observed that the reserve to GDP ratio has high degree of autocorrelation with its lagged variable for many countries. In the presence of a highly autocorrelated dependent variable, Arellano and Bond (1991) assert that allowing for dynamics (i.e., a lagged dependent variable among regressors) in the panel estimation may be crucial for recovering consistent estimates of other parameters. This provides a crucial reason why the lagged dependent variable $y_{i,t-1}$ is included as one of the regressors in the e.q(14). This dynamic panel approach is relatively new compared to other studies in the literature and also provides some interesting estimation results later.[31] Various estimation methods can then be used for this equation.

First, the pooled OLS is implemented, but the drawback is that it cannot consider country-level heterogeneity (i.e., η_i) that causes a potential bias because $y_{i,t-1}$ is correlated with η_i. Accordingly, the fixed effects (FE henceforth) estimation can be implemented to control for country-level heterogeneity. The FE estimation, in general, draws consistent estimators by controlling for the country-specific unobserved components. However, FE estimators are biased in the case of a lagged dependent variable among regressors. This is because the $y_{i,t-1}$ becomes still correlated to error terms in the differenced regression of the *'within'* model (i.e., Nickell (1981) bias). Therefore the FE and OLS estimators both face an endogeneity problem.

To fix these problems, the Arellano and Bond (1991) type correction is employed next. The first difference of e.q(14) will eliminate the country-specific effects η_i and generate the following equation.

$$y_{i,t} - y_{i,t-1} = \lambda(y_{i,t-1} - y_{i,t-2}) + (X_{i,t} - X_{i,t-1})'\beta + (\epsilon_{i,t} - \epsilon_{i,t-1}) \quad (15)$$

Instruments are needed in order to control for the endogeneity (i.e., correlation between $(y_{i,t-1} - y_{i,t-2})$ and $(\epsilon_{i,t} - \epsilon_{i,t-1})$ in e.q(15)). Arellano and Bond (1991)'s methodology specifies that lags of the dependent variable and the first-differences of other regressors can be used as instruments, $Z'_{i,t} = \left[y_{i,t-2}, y_{i,t-3}, \cdots, \Delta X'_{i,t}\right]$. In addition, the common assumption in dynamic panel estimation that all explanatory variables are strictly exogenous (i.e. they are uncorrelated with the error term ϵ at all leads and lags) needs to be relaxed. To this end, a weak

[31] Obstfeld, Shambaugh, and Taylor (2010) for instance do take the autocorrelation seriously by clustering the standard errors by country to allow for heteroskedasticity across countries and an unstructured serial correlation in the error term within countries. However, their methods are only limited to OLS estimator and do not control possible endogeneity problems discussed in Difference-GMM and System-GMM estimation methods later.

exogeneity assumption for explanatory variables are adopted.[32] This allows for the possibility of simultaneity between the dependent variable and explanatory variables such as the DCF, consistent with the model.

Then, the GMM method can be used to estimate this differenced system based on the moment conditions. It is often referred to as the Difference-GMM. However, Alonso-Borrego and Arellano (1996) and Blundell and Bond (1998) also point that the Difference-GMM estimator cannot take the cross-country variation into account, and the lagged levels of the regressors might be weak instruments for the first-differences if the regressors are persistent over time. Particularly, they show through Monte Carlo experiments that the weakness of the instruments can produce biased coefficients in small samples.

To avoid this potential bias and imprecision, Arellano and Bover (1995) proposed the System-GMM estimator that combines the regression (15) in differences with the regression (14) in levels. In the equation (14), variables in levels are instrumented with suitable lags of their own first-differences. The assumption needed is that these differences are uncorrelated with the unobserved country fixed effects and error terms.[33]

The consistency of the GMM estimator depends on whether lagged values of the explanatory variables are valid instruments. Thus, the over-identification restriction test (i.e., Hansen (1982) test) is conducted. The failure to reject the null hypothesis gives support for the valid instruments. Lastly, for the consistency of the GMM, it is necessary to check that the error term ϵ is not serially correlated. If the original error term $\epsilon_{i,t}$ has no serial correlation, the differenced error terms at order 1, $\Delta\epsilon_{i,t}$ and $\Delta\epsilon_{i,t-1}$, are expected to have serial correlation. As a result, it is expected that the differenced error terms at order 2, $\Delta\epsilon_{i,t}$ and $\Delta\epsilon_{i,t-2}$, have no serial autocorrelation. On this account, test results for the first and second order autocorrelation in the differenced error terms are also reported.

5.3. Basic Results

This section starts with estimating the e.q(14) with DCF as a measure for the "foreign private investment inflows". Five different estimation methods (i.e. OLS, FE, FE with lagged dependent variables, Difference-GMM, and System GMM estimators) are implemented. The results

[32] When the set of explanatory variables (X) are weakly exogenous, the following moment conditions hold

$$\begin{aligned}\mathbb{E}\big[y_{i,t-k}(\epsilon_{i,t}-\epsilon_{i,t-1})\big] &= 0 \quad \textit{for} \quad k\geq 2, t=3,\cdots,T\\ \mathbb{E}\big[X_{i,t-k}(\epsilon_{i,t}-\epsilon_{i,t-1})\big] &= 0 \quad \textit{for} \quad k\geq 2, t=3,\cdots,T\end{aligned}$$

[33] The moment conditions of (14) are therefore

$$\begin{aligned}\mathbb{E}\big[(y_{i,t-1}-y_{i,t-2})(\eta_i+\epsilon_{i,t})\big] &= 0\\ \mathbb{E}\big[(X_{i,t-1}-X_{i,t-2})(\eta_i+\epsilon_{i,t})\big] &= 0\end{aligned}$$

are reported in columns (1) to (5) in Table 4.

Basic results from OLS are broadly consistent with previous studies. Countries that trade more tend to hold more reserves. Also the size of financial market measured by M2/GDP turns out to be a significant predictor for the level of a country's reserves as is the case in Obstfeld, Shambaugh, and Taylor (2010). Interestingly, financial openness measures do not appear to matter. This is particularly believed to stem from a time trend in Chinn and Ito (2008)'s financial openness index. Since the latter is highly sensitive to regime switching capital control policy, the time trend of such index rather shows a stepwise increase. Thus, the weak linkage between financial openness and reserve levels found in the results is hardly surprising and does not necessarily contradict the model.

Switching to the FE estimation method, variables such as Trade/GDP and M2/GDP now turn out to be statistically insignificant, indicating a potential omitted variable bias caused by country fixed effects.[34] Moreover, as explained earlier, pooled OLS and fixed effect estimation may be biased due to a possible correlation between the lagged dependent variable and error terms. This, therefore, leads one to focus on the results with the Difference-GMM and System-GMM estimators.

The Difference-GMM in column (4) of Table 4 corrects possible errors of estimating dynamic fixed effects model. As expected, the lagged reserve to GDP is significantly positive at the 1 % critical level. Table 4 also reports specification tests of first and second order autoregression (AR) of first differences in error terms. The null hypothesis of no first-order AR in differences in the error terms is rejected, while the second-order AR hypothesis is not. It means that the error terms in the specification are independent as assumed earlier. Table 4 displays not only the Hansen test but also the number of instruments in line with Roodman (2009). Too large instrument collection in both GMM methods overfits endogenous variables and weakens the Hansen test of the instruments' joint validity.[35] So the collaps method of Roodman (2009) is applied and the lag range of instruments in the Hansen test is restricted to avoid the problem of instrument proliferations. The Hansen test in the Table 4 also implies that the null hypothesis that instruments are valid can not be rejected.

The Difference-GMM results show some interesting implications on the conventional determinants of foreign-exchange reserves. First, a significantly positive coefficient on the DCF suggests that the liquidity-based model of this paper should be a good predictor of emerging economy's reserve hoarding behavior. As a matter of fact, figure 7 also illustrates how easily the positive link between the DCF and the reserve levels can be detected even with naked eyes

[34] Obstfeld, Shambaugh, and Taylor (2010) also report FE results and found Trade/GDP and M2/GDP still remain significant. However, their sample included all the advanced countries whereas the country sample in this study only includes a selective group of emerging economies. This may signal the weakness of their financial stability model in the context of emerging economies.

[35] Bowsher (2002) argues that instrument proliferation vitiates the Hansen (1982)'s test of over-identification. The Hansen test implausibly returns a perfect p-value of 1.

for major reserves holding countries.[36] Furthermore, it is shown that controlling for potential endogeneity problems discredits conventional reserve determinants such as Trade/GDP and M2/GDP ratio. In addition, the income and population size of a country turn out to be negatively correlated with reserve to GDP, which is somewhat at odds with conventional wisdom. However it might be the case that these results are caused by weak instrument problems related to the highly persistent regressors over time (i.e. Trade/GDP, population, and GDP per capita etc.). In order to fix these potential drawbacks, one can finally move on to the System-GMM estimator.

The System-GMM results are reported in column (5). As is the case in the Difference-GMM, AR specification tests as well as Hansen (1982)'s over identification test are performed to confirm the appropriateness of instruments and model specification. Test results in column (5) support the validity of the System-GMM estimates. As conjectured, population, GDP per capita, and trade to GDP ratio all turn into statistically insignificant regressors after controlling for the weak instrument problems. In fact, only two independent variables remain valid. These are the DCF and exchange rate volatility which both turn out to be positively correlated with reserve holdings. A positive impact of the DCF on reserve accumulation is not surprising as it is already detected even with the Difference-GMM. A positive influence that exchange rate volatility has on reserves also makes sense since emerging economies normally vulnerable to external shocks would voluntarily accumulate reserves as a primary measure to lean against a high degree of exchange rate fluctuation.

Lastly, for comparison purpose, the DCF is omitted from the specification. The results are shown in column (6) and (7) for the two different GMM estimation methods. From the value of R-squared in the System-GMM, the message is clear. A model based on liquidity motives should be a better predictor of reserve holdings than one ignoring those factors. To sum up, the basic results here reinforces the model's claim that the liquidity property of official reserves especially in OTC international investment markets should be treated as a key driving force behind the recent upsurge of global imbalances.

5.4. Robustness Check

This section subjects the estimation to various alternative measures for foreign investment inflows to check robustness of the DCF measure. To that end, the equation 15 is estimated, employing the three different measures-HM, FDI, and CCF from the data section-for foreign investment inflows. Table 5, 6, and 7 report the results with only Difference-GMM and System-GMM estimators.

First and most important thing to notice is that coefficients on the FDI and the CCF be-

[36] Figure 7 actually compares the reserves/GDP to the equity inflows through OTC trading which is a major part of the DCF variable (i.e., the (B-C) part of the DCF defined earlier).

come insignificant as shown in Table 6 and 7. This result again provides strong empirical support to the liquidity mechanism of the model. Unlike the FDI and CCF, the HM appears to have a positive correlation with the reserve to GDP ratio as shown in Table 5 although the coefficient under the System-GMM becomes statistically insignificant. Several interpretations could follow from this observation.

It is clear that not all hot money investments penetrate into emerging economies through decentralized trade channels. Definition of hot money measures in the data section reveals this clearly. In fact, only the *"errors and omissions"* part intersects with what constitutes the DCF. Thus, it may well be the case that the positive correlation between the HM and the reserves largely emanates from that intersection part. The reduced significance level also raises doubts about the short-term foreign investment inflows as a genuine bellwether of reserve accumulation. To finally dismiss the effect of the HM, a horse race between all the different measures for the foreign investment inflows is performed. This exercise would allow one to see the magnitude of the contribution of each relative to the other. Table 8 does indeed confirm the DCF as a sole and statistically significant driver for the country's reserve hoarding behavior.

Lastly, coefficients on the Debt/GDP deserve some discussion. Conventional wisdom in the literature is that emerging economies with larger debt may hold larger amount of reserves as a precautionary measure against possible currency attacks. In fact, the causality might as well run reversely. A country with a larger amount of reserves may opt for the issuance of sterilization bonds to fight inflation. In any case, the positive effect of the debt to GDP ratio on the level of reserves are expected. On the contrary, this paper insists neutrality of domestic asset supply simply because the home asset (i.e., emerging market debt) does not serve any liquidity role in the FIM. Empirical results at first seem to provide counter evidence on the model (i.e., Table 5 and 6). Yet, controlling for the DCF, the positive link between the debt level and reserve hoarding decisions breaks (i.e., Table 4).[37] Thus further scrutiny would be also necessary particularly on this issue in future research.

5.5. Out-of-Sample Prediction

This section focuses on model fit and quantitative significance associated with the out-of-sample prediction. Two models-the baseline System-GMM model with the DCF (column (5) of the Table 4), and a financial stability model proposed by Obstfeld, Shambaugh, and Taylor (2010)-are estimated on data from 1997 to 2001. The estimates are then used to make fore-

[37] Obstfeld, Shambaugh, and Taylor (2010) also find more foreign currency liabilities do not necessarily lead a country to hold more reserves and therefore provide another example of ambiguous relationship between public debt and reserves.

casts of reserves to GDP ratio in subsequent years up to 2007.[38] The actual reserve to GDP is graphed against the out-of-sample prediction results of the two models.

Figure 8 compares the out-of-sample prediction results of the two models for a selection of eight most important reserve holders in the sample. Several intriguing points are worth noting. First off, reserve accumulation by these countries over the last decade seems in no way inexplicable at least in the System-GMM specification with the DCF. For instance, actual reserves to GDP ratio for countries like Korea, India and Brazil follow very close to what the System-GMM specification would have predicted. Interestingly, Hong-Kong and Singapore who had been usually identified as extremely excessive reserve holders by many studies (e.g., Jeanne and Ranciere (2011) and Obstfeld, Shambaugh, and Taylor (2010)) even turn out to be undersavers according to this model's specification. Chinese and Russian reserve accumulation still seem excessive even from this model's specification standpoint. Yet, as Obstfeld, Shambaugh, and Taylor (2010) suggest, there might be several other reasons for reserve demand in the case of world's largest reserve holder, China.[39]

This model's specification seems to outperform the financial stability model in terms of the out-of-sample forecasting. This better prediction is not just about the predicted magnitude of the reserves to GDP ratio at 2007. In fact, even with naked eye one could see that the degree to which this model's prediction co-moves together with the actual ratio is higher than its counterpart.[40]

6. Concluding Remarks

Markets from which developing countries need to acquire foreign capital have been increasingly characterized by OTC features. This paper demonstrates that this trend is a key to understanding emerging economies' extraordinary reserve accumulation over the last decade. Since these decentralized markets lack perfect credit and commitment, a facilitator of trade (i.e., a liquid asset) is required. Typically, U.S. T-bills have served this role, either through collateral or buffer stocks against the repatriation of foreign capital. Declining search frictions in these markets thus enhance assets' liquidity property, which induces developing counties in need of sustained foreign investment to require U.S. T-bills relatively more. As a result, the amount of emerging economies' U.S. T-bill holdings is on the rise in equilibrium. Fur-

[38]Explanatory variables used in Obstfeld, Shambaugh, and Taylor (2010)'s out-of-sample forecasting include advanced country dummies. To facilitate a comparison between the Obstfeld, Shambaugh, and Taylor (2010)'s and this model's, the advanced country dummies are replaced with individual country fixed effects for the financial stability model's out-of-sample prediction.

[39]China may need a lot more international reserves to recapitalize domestic banks. Furthermore, the high level of non-performing loans by Chinese banks may require large holdings of reserves.

[40]One can actually conduct a regression of actual reserves/GDP on the predicted ratio and compare the R-squared for the two models. It would be surprising to see the lower R-squared for this model specification.

thermore, the sustained increase in U.S. T-bill's liquidity attribute leads to a higher liquidity premium on these assets, thereby causing low real U.S. interest rates.

Indeed, the empirical results of this paper lend quite strong support for this liquidity-based story. Interestingly, some important policy implications can be drawn from these empirical results. In line with some recent studies, such as Obstfeld, Shambaugh, and Taylor (2010), this study yields a similar conclusion with respect to major emerging economies' current reserve adequacy. Their current reserve holdings appear to be neither excessive nor inexplicable based on the empirical estimates of this study. In short, the findings in this study call for a more prudent liquidity-based approach to the reserve adequacy criteria that are carefully gauged against the amount of bilateral foreign investment transactions through OTC markets.

Before concluding, one final remark needs to be emphasized. While this paper chooses to explain the basic mechanism underlying the recent upsurge in emerging economies' foreign exchange reserve holdings from a simple liquidity point of view, it should be apparent from many preceding studies that there are multiple channels through which massive demand for reserve assets could come about. In fact, as Lagos (2011) argues, shocks to risk premiums on the U.S. T-bills could easily translate into an aggregate liquidity premium to the extent that these assets serve as a source of liquidity. Thus, while they may appear to be different on the surface, conventional explanations and new liquidity-based theory might be often equivalent. In this regard, one should view this paper's alternative explanation as complementary to existing studies rather than competing. I hope that the liquidity mechanism proposed here can enrich the dimension of research in this field. In particular, it would be interesting to extend this study towards an analysis of the future of global imbalances, especially in light of the changing status of the U.S. dollar as a world reserve currency.

References

AIZENMAN, J., AND J. LEE (2007): "International Reserves: Precautionary versus Mercantilist Views, Theory and Evidence," *Open Economics Review*, 18(2), 191–214.

AIZENMAN, J., AND N. MARION (2002): "International Reserve Holdings with Sovereign Risk and Costly Tax Collection," *NBER Working Paper Series*, Working Paper 9154.

ALONSO-BORREGO, C., AND M. ARELLANO (1996): "Symmetrically Normalised Instrumental Variable Estimation Using Panel Data," *Journal of Business and Economic Statistics*, 17, 36–49.

ARELLANO, M., AND S. BOND (1991): "Some Tests of Specification for Panel Data: Monte Carlo Evidence and an Application to Employment Equations," *Review of Economic Studies*, 58, 277–297.

ARELLANO, M., AND A. BOVER (1995): "Another Look at the Instrumental Variable Estimation of Error Components Model," *Journal of Econometrics*, 68, 29–51.

BERNANKE, B. (2005): "The Global Saving Glut and the U.S Current Account Deficit.," *Speech at the Sandridge Lecture, Virginia Association of Economists*.

BLUNDELL, R., AND S. BOND (1998): "Initial Conditions and Moment Restrictions in Dynamic Panel Data Models," *Journal of Econometrics*, 87, 115–143.

BOWSHER, C. (2002): "On Testing Overidentifying Restrictions in Dynamic Panel Data Models," *Economic Letters*, 77, 211–220.

CABALLERO, R. J., E. FARHI, AND P.-O. GOURINCHAS (2008): "An Equilibrium Model of Global Imbalances and Low Interest Rates," *American Economic Review*, 98(1), 358–393.

CHINN, M., AND H. ITO (2008): "A New Measure of Financial Openness," *Journal of Comparative Policy Analysis*, 10(3), 309–322.

CHIU, J., AND M. MOLICO (2010): "Liquidity, Redistribution, and the Welfare Cost of Inflation," *Journal of Monetary Economics*, 57, 428–438.

CUDDINGTON, J. T. (1986): "Capital Flight: Estimates, Issues, and Explanations," *Princeton Studies in International Finance*, (58).

DEVEREUX, M. B., AND S. SHI (2013): "Vehicle Currency," *International Economic Review*, 54, 97–133.

DOOLEY, M. (1987): "Capital Flight, A Response to Differences in Financial Risks," *IMF Staff Papers*, (34), 422–436.

DOOLEY, M. P., D. FOLKERTS-LANDAU, AND P. GARBER (2004): "Direct Investment, Rising Real Wages and the Absorption of Excess Labor in the Periphery," Working Paper Series 10626, NBER.

DOOLEY, M. P., D. FOLKERTS-LANDAU, AND G. PETER (2005): "Living with Bretton Woods II," Working paper, Deutsche Bank Global Markets Research.

DUFFIE, D., N. GÂRLEANU, AND L. H. PEDERSEN (2005): "Over-the-Counter Markets," *Econometrica*, 73(6), 1815–1847.

DURDUE, C., E. MENDOZA, AND M. TERRONES (2007): "Precautionary Demand for Foreign Assets in Sudden Stop Economics: an Assessment of the New Merchantilism," *NBER Working Paper*, (13123).

GEROMICHALOS, A., AND L. HERRENBRUECK (2013): "Monetary Policy, Asset Prices, and Liquidity in Over-the-Counter Markets," *Working Paper Series, UC Davis Department of Economics*.

GEROMICHALOS, A., AND I. SIMONOVSKA (2011): "Asset Liquidity and International Portfolio Choice," *NBER Working Paper*, (17331).

GHIRONI, F., J. LEE, AND A. REBUCCI (2007): "The Valuation Channel of External Adjustment," *NBER Working Paper*, (12937).

GOURINCHAS, P., AND H. REY (2007): "From World Banker to World Venture Capitalists: US External Adjustment and the Exorbitant Privilege," *In Clarida, R. H. (Ed), G7 Current Account Imbalances: Sustainability and Adjustment*, pp. 11–66.

HANSEN, L. P. (1982): "Large Sample Properties of Generalised Method of Moments Estimators," *Econometrica*, 50, 1029–1054.

JEANNE, O., AND R. RANCIERE (2011): "The Optimal Level of International Reserves For Emerging Market Countries: a New Formula and Some Applications," *Economic Journal*, 121(555), 905–930.

KEHOE, T. J., K. J. RUHL, AND J. B. STEINBERG (2013): "Global Imbalances and Structural Change in the United States," *NBER Working Paper Series*, 19339.

KRISHNAMURTHY, A., AND A. VISSING-JORGENSEN (2012): "The Aggregate Demand for Treasury Debt," *Journal of Political Economy*, 120(2), 233–267.

LAEVEN, L., AND F. VALENCIA (2012): "Systemic Banking Crises Database: An Update," Working Paper 12/163, IMF.

LAGOS, R. (2010): "Asset prices and liquidity in an exchange economy," *Journal of Monetary Economics*, 57(8), 913–930.

——— (2011): "Asset Prices, Liquidity, and Monetary Policy in an Exchange Economy," *Journal of Money, Credit and Banking*, 43, 521–552.

LAGOS, R., AND G. ROCHETEAU (2008): "Money and Capital as Competing Media of Exchange," *Journal of Economic Theory*, 142(1), 247–258.

LAGOS, R., AND R. WRIGHT (2005): "A Unified Framework for Monetary Theory and Policy Analysis," *Journal of Political Economy*, 113(3), 463–484.

LANE, P. R., AND G. MILESI-FERRETTI (2007a): "A Global Perspective on External Position," *G7 Current Account Imbalances: Sustainability and Adjustment*, pp. 67–98.

LANE, P. R., AND G. M. MILESI-FERRETTI (2007b): "The External Wealth of Nations Mark II: Revised and Extended Estimates of Foreign Assets and Liabilities, 1970-2004," *Journal of International Economics*, 73, 223–250.

LESTER, B., A. POSTLEWAITE, AND R. WRIGHT (2012): "Information, Liquidity, Asset Prices, and Monetary Policy," *Review of Economic Studies*, 79(3), 1209–1238.

LUCAS, ROBERT E, J. (1978): "Asset Prices in an Exchange Economy," *Econometrica*, 46(6), 1429–45.

MCGRATTAN, E., AND E. PRESCOTT (2007): "Technology Capital and the U.S Current Account," Discussion Paper 406, Federal Reserve Bank of Minneapolis, Staff Report.

MENDOZA, E. G., V. QUADRINI, AND J.-V. RIOS-RULL (2009): "Financial Integration, Financial Development, and Global Imbalances," *Journal of Political Economy*, 117(3).

NICKELL, S. (1981): "Biases in Dynamic Models with Fixed Effects," *Econometrica*, 49, 1417–1426.

OBSTFELD, M., J. C. SHAMBAUGH, AND A. M. TAYLOR (2010): "Financial Stability, The Trilemma, and International Reserves," *American Economic Journal: Macroeconomics*, 2, 57–94.

ROCHETEAU, G. (2011): "Payments and Liquidity under Adverse Selection," *Journal of Monetary Economics*.

ROCHETEAU, G., AND R. WRIGHT (2005): "Money in Search Equilibrium, in Competitive Equilibrium, and in Competitive Search Equilibrium," *Econometrica*, 73(1), 175–202.

ROODMAN, D. (2009): "A Note on the Theme of Too Many Instruments," *Oxford Bulletin of Economic and Statistics*, 71, 135–158.

SHAMBAUGH, J. C. (2004): "The Effects of Fixed Exchange Rates on Monetary Policy," *Quarterly Journal of Economics*, 119(1), 301–352.

SUMMERS, L. (2006): "Reflections on Global Account Imbalances and Emerging Markets Reserves Accumulation," L.k. jha memorial lecture, Reserve Bank of India.

VENKATESWARAN, V., AND R. WRIGHT (2013): "Pledgability and Liquidity: A New Monetarist Model of Financial and Macroecomic Activity," Working paper, University of Wisconsin-Madison.

WARNOCK, F. E., AND V. C. WARNOCK (2009): "International Capital Flows and U.S. Interest Rates," *Journal of International Money and Finance*, 28(6), 903–919.

Appendix

Proof of Lemma 2.

Unlike the usual bargaining solutions in the literature, the participation constraint for the F during this bargaining also depends on how much foreign assets in reference to the first best choice of R in the subsequent DM (i.e., $\breve{a}$) that F has brought up. I consider three possible scenarios regarding the amount of F's foreign asset holdings.

Scenario 1: $a_f^* \geq \breve{a}$

In this scenario, the bargaining problem simplifies to:

$$\max_{\{t,b^*\}} \Big\{ u(t) - \beta b^* \Big\}$$

$$s.t. \quad 1.\ c(t) \leq \beta b^*$$

$$2.\ b^* \leq a_h^*$$

Solution for this problem is standard and straightforward.

$$\textit{If} \quad a_h^* \geq \frac{c(t^{op})}{\beta} \quad \textit{then} \quad \begin{cases} t = t^{op} \\ b^* = \frac{c(t^{op})}{\beta} \end{cases}$$

$$\textit{If} \quad a_h^* \leq \frac{c(t^{op})}{\beta} \quad \textit{then} \quad \begin{cases} t = \big\{t : \beta a_h^* = c(t)\big\} \\ b^* = a_h^* \end{cases}$$

So given the assumption of $a_f^* \geq \breve{a}$, the above two solutions correspond to the region 1 and 2.

Scenario 2: $\breve{a} - b^* \leq a_f^* \leq \breve{a}$

In this scenario, the F would get the first best liquidity choice for the subsequent DM (i.e., $\breve{a}$) only after the bargaining. Hence, the bargaining problem is described by:

$$\max_{\{t,b^*\}} \Big\{ u(t) - \beta b^* \Big\}$$

$$s.t. \quad 1.\ c(t) \leq \beta b^* + \sigma\big[u(\tilde{q}) - \tilde{q}\big] - \sigma\big[u(q(a_f^*)) - \beta n(a_f^*)\big]$$

$$2.\ b^* \leq a_h^*$$

First order conditions for this problem follows as:

$$t : u'(t) = \lambda_1 c'(t) \tag{16}$$

$$b^* : -\beta + \lambda_1 \beta - \lambda_2 = 0 \tag{17}$$

where λ_1 and λ_2 are the associated lagrange multipliers for the above two constraints. Let us consider two possible cases.

When $\lambda_2 = 0$

If we let $\lambda_2 = 0$ then $b^* < a_h^*$ must hold. Since $a_f^* + b^* \geq \check{a}$ by assumption in this scenario, the e.q.(16) ensures $\lambda_1 = 1 \Rightarrow t = t^{op}$. Moreover, the first constraint in the bargaining problem (i.e., the participation constraint) also becomes binding due to $\lambda_1 = 1$. Hence,

$$c(t^{op}) = \beta b^* + \sigma\big[u(\tilde{q}) - \tilde{q}\big] - \sigma\big[u(q(a_f^*)) - \beta n(a_f^*)\big] \tag{18}$$

It is understood that the maximum value of b^* must equal to $\frac{c(t^{op})}{\beta}$ since e.q.(18) implies $b^* \propto a_f^*$ and $max\{a_f^*\} = \check{a}$ by the assumption. We also need to make sure that these solutions satisfy conditions imposed in this scenario. First, in order to ensure $b^* < a_b^*$ stemming from $\lambda_2 = 0$, one needs the following condition based on e.q.(18).

$$c(t^{op}) - \sigma\big[u(\tilde{q}) - \tilde{q}\big] + \sigma\big[u(q(a_f^*)) - \beta n(a_f^*)\big] < \beta a_h^* \tag{19}$$

On top of that, one would also need to verify $b^* \geq \check{a} - a_f^*$ imposed by the scenario 2 assumption which gives out:

$$c(t^{op}) - \sigma\big[u(\tilde{q}) - \tilde{q}\big] + \sigma\big[u(q(a_f^*)) - \beta n(a_f^*)\big] \geq \beta(\check{a} - a_f^*) \tag{20}$$

Equations (19) and (20) hold if $a_h^* + a_f^* > \check{a}$ which therefore needs to be imposed. Next, a_f^* must be bounded from below due to the following reason. Combining the e.q.(19) and (20) generates;

$$\sigma\big[u(\beta a_f^*) - \beta a_f^*\big] + \beta a_f^* \geq \beta\check{a} + \sigma\big[u(\check{q}) - \check{q}\big] - c(t^{op}) \tag{21}$$

This e.q.(21) confirms that $\exists \bar{a}_f^*$ such that

$$\sigma\big[u(\beta a_f^*) - \beta a_f^*\big] + \beta a_f^* = \beta\check{a} + \sigma\big[u(\check{q}) - \check{q}\big] - c(t^{op}) \tag{22}$$

Note here the condition $a_h^* + a_f^* > \check{a}$ imposed above is redundant as long as e.q.(19) holds and $a_f^* > \bar{a}_f^*$ since the LHS of inequality e.q.(19) is increasing in a_f^* while $\check{a} - a_f^*$ falls with a_f^*. It is also important to notice that the relative size of cost of producing *capital goods* during the FIM critically determines the sign of $\bar{a}_f^*$. If $c(t^{op})$ turns out to be greater than $\beta\check{a} + \sigma\big[u(\check{q}) - \check{q}\big] - c(t^{op})$ then it is obvious that $\bar{a}_f^*$ must be a negative value otherwise $\bar{a}_f^*$ becomes positive. Hence, the bargaining solution depends on the set of parameter space for $\big\{c(t^{op}), u(\tilde{q}), \textit{and}, \check{a}\big\}$ For now, let us restrict ourselves to the case, $c(t^{op}) < \beta\check{a} + \sigma\big[u(\tilde{q}) - \tilde{q}\big]$ and consider the other case later.

Finally, in order for $\bar{a}_f^*$ to be less than $\breve{a}$, one would have to bound $c(t^{op})$ from below such that

$$\begin{aligned}\sigma\big[u(\beta\breve{a})-\beta\breve{a}\big]+\beta\breve{a} &> \beta\breve{a}+\sigma\big[u(\breve{q})-\breve{q}\big]-c(t^{op}) \\ c(t^{op}) &> \sigma\big[u(\breve{q})-\breve{q}\big]-\sigma\big[u(\beta\breve{a})-\beta\breve{a}\big] \\ &> 0\end{aligned} \tag{23}$$

To sum up, for the solution to be $t=t^{op}$ and $b^*<a_h^*$ such that e.q.(18) holds;

1. $\bar{a}_f^* \leq a_f^* \leq \breve{a}$
2. $a_h^* \geq \frac{c(t^{op})-\sigma\left[u(\tilde{q})-u(\beta a_f^*)+\tilde{q}-\beta a_f^*\right]}{\beta}$

which corresponds to the region 3 solution.

<u>*When* $\lambda_2>0$</u>

If we let $\lambda_2>0$ then $b^*=a_h^*$ must hold. Since $a_f^*+b^*\geq\breve{a}$ by assumption in this scenario, $a_f^*+a_h^*\geq\breve{a}$ and the e.q.(16) ensures $\lambda_1>1\Rightarrow t<t^{op}$. Moreover, the first constraint in the bargaining problem (i.e., the participation constraint) also becomes binding due to $\lambda_1>1$. Hence the solution for t must satisfy:

$$\begin{aligned}c(t) &= \beta a_h^*+\sigma\big[u(\tilde{q})-\tilde{q}\big]-\sigma\big[u(q(a_f^*))-q(a_f^*)\big] \\ c(t^{op}) &> \beta a_h^*+\sigma\big[u(\tilde{q})-\tilde{q}\big]-\sigma\big[u(q(a_f^*))-q(a_f^*)\big] \\ a_h^* &\leq \frac{c(t^{op})-\sigma\big[u(\tilde{q})-u(\beta a_f^*)+\tilde{q}-\beta a_f^*\big]}{\beta}\end{aligned} \tag{24}$$

Furthermore, combining e.q.(24) with $a_h^*\geq\breve{a}-a_f^*$ yields:

$$\begin{aligned}c(t)-\sigma\big[u(\tilde{q})-\tilde{q}\big]+\sigma\big[u(q(a_f^*))-q(a_f^*)\big] &\geq \beta(\breve{a}-a_f^*) \\ \sigma\big[u(\beta a_f^*)-\beta a_f^*\big]+\beta a_f^* &\geq \beta\breve{a}+\sigma\big[u(\breve{q})-\breve{q}\big]-c(t)\end{aligned} \tag{25}$$

From e.q.(22) and (25), it is understood that $a_f^*>\bar{a}_f^*$ in this case 2 as well. Thus for the solution to be $b^*=a_h^*$ and $t<t^{op}$ such that e.q.(24) holds;

1. $\bar{a}_f^* \leq a_f^* \leq \breve{a}$
2. $a_h^*+a_f^*\geq\breve{a}$
3. $a_h^* \leq \frac{c(t^{op})-\sigma\left[u(\tilde{q})-u(\beta a_f^*)+\tilde{q}-\beta a_f^*\right]}{\beta}$

which corresponds to the region 4.

Scenario 3: $a_f^*+b^*\leq\breve{a}$

In this scenario, the F would never get the first best liquidity choice for the subsequent DM (i.e., $\breve{a}$) even after the bargaining. Hence, the bargaining problem is described by:

$$\max_{\{t,b^*\}} \left\{ u(t) - \beta b^* \right\}$$

$$s.t. \quad 1.\ c(t) \leq \beta b^* + \sigma\big[u(q(a_f^* + b^*)) - \beta n(a_f^* + b^*)\big] - \sigma\big[u(q(a_f^*)) - \beta n(a_f^*)\big]$$
$$2.\ b^* \leq a_h^*$$

First order conditions for this problem follows as:

$$t : u'(t) = \lambda_1 c'(t) \tag{26}$$
$$b^* : -\beta + \lambda_1\big[\beta + \sigma u'(q(a_f^* + b^*))\beta - \sigma\beta\big] - \lambda_2 = 0 \tag{27}$$

where λ_1 and λ_2 are associated Lagrange multipliers for the above two constraints. Let us consider two possible cases.

<u>*When $\lambda_2 = 0$*</u>

If we let $\lambda_2 = 0$ then the second constraint becomes slack and therefore $b^* < a_h^*$. Also from e.q.(27) it is obvious that $\lambda_1 < 1 \Rightarrow t > t^{op}$. Again the first constraint binds due to positive value of λ_1 and therefore the following must hold.

$$c(t) = \beta b^* + \sigma\big[u(q(a_f^* + b^*)) - \beta n(a_f^* + b^*)\big] - \sigma\big[u(q(a_f^*)) - \beta n(a_f^*)\big] > c(t^{op}) \tag{28}$$

As before, we again need to make sure that these solutions satisfy conditions imposed in this scenario. First, in order to ensure $b^* < a_b^*$ stemming from $\lambda_2 = 0$, one needs the following condition based on e.q.(28).

$$c(t) - \sigma\big[u(q(a_f^* + b^*)) - \beta n(a_f^* + b^*)\big] + \sigma\big[u(q(a_f^*)) - \beta n(a_f^*)\big] < \beta a_h^* \tag{29}$$
$$c(t) - \sigma\big[u(q(a_f^* + b^*)) - u(q(a_f^*))\big] + \sigma\beta b^* < \beta a_h^*$$

On top of that, one would also need to verify $b^* \leq \breve{a} - a_f^*$ imposed by the scenario 3 assumption which gives out:

$$c(t) - \sigma\big[u(q(a_f^* + b^*)) - \beta n(a_f^* + b^*)\big] + \sigma\big[u(q(a_f^*)) - \beta n(a_f^*)\big] < \beta(\breve{a} - a_f^*) \tag{30}$$
$$\sigma u(\beta a_f^*) + (1-\sigma)\beta a_f^* < \beta\breve{a} + \sigma\big[u(q(a_f^* + b^*)) - q(a_f^* + b^*)\big] - c(t) \tag{31}$$

Now the question is whether $a_h^* + a_f^* < \breve{a}$ or not. From e.q.(28), it is easy to see that

$$\sigma u(q(a_f^*)) = (1-\sigma)\beta b^* + \sigma u(q(a_f^* + b^*)) - c(t) \tag{32}$$

which confirms that $\{b^*, t\}$ is not uniquely determined and yet positively related (i.e., $b^* \propto t$). This in turn ensures that a_h^* must be bounded from below for the following reason. Due to

$b^* \propto t$, $c(t) > c(t^{op})$ and e.q.(32), minimum value for b^*, b^*_m is such that

$$c(t^{op}) = \sigma\big[u(q(a^*_f + b^*_m)) - u(q(a^*_f))\big] + (1-\sigma)\beta b^*_m \tag{33}$$

By the implicit function theorem, e.q.(33) confirms $\frac{\partial b^*_m}{\partial a^*_f} > 0$

$$\frac{\partial b^*_m}{\partial a^*_f} = -\frac{\frac{\partial G}{\partial a^*_f} \to \ominus}{\frac{\partial G}{\partial b^*_m} \to \oplus} > 0$$

where $G(b^*_m, a^*_f) = \sigma\big[u(q(a^*_f + b^*_m)) - u(q(a^*_f))\big] + (1-\sigma)\beta b^*_m - c(t^{op})$ and

$$\frac{\partial G}{\partial a^*_f} = \sigma\beta\big[u'(q(a^*_f + b^*_m)) - u'(q(a^*_f))\big] < 0 \quad \text{due to the concavity assumption of } u(\cdot)$$
$$\frac{\partial G}{\partial b^*_m} = \sigma\beta\big[u'(q(a^*_f + b^*_m)) - 1\big] + \beta b^* > 0 \quad \text{due to } a^*_f + b^* < \check{a}$$

Thus $min\{a^*_h\}$ (i.e., $a^*_{h,Min}$) must be an increasing function of a^*_f. In addition, when $a^*_f = 0$ the $a^*_{h,Min}$ must satisfy the following.

$$c(t^{op}) = \sigma u(q(a^*_{h,Min})) + (1-\sigma)\beta b^* \tag{34}$$

Since we earlier restricted the parameter space into *Case 1* such that $c(t^{op}) < \sigma\big[u(\tilde{q}) - \tilde{q}\big] + \beta\check{a}$, one can easily verify that $a^*_{h,Min} \equiv \tilde{a}^*_n < \check{a}$. Lastly we need to verify that when $a^*_f = \bar{a}^*_f$, $a^*_{h,Min}$ is such that $a^*_{h,Min} + a^*_f = \check{a}$ so that the feasible domain for a^*_f in this scenario must be bounded from above (i.e., $\bar{a}^*_f$). This can be done easily by comparing e.q.(33) and (18). Considering the knife-edge case between region 3 and this region, e.q.(18) and (33) respectively gives out

$$c(t^{op}) - \sigma u(\tilde{q}) + \sigma u(\beta\bar{a}^*_f) + \sigma\beta a^*_h = \beta a^*_h \tag{35}$$
$$c(t^{op}) - \sigma u(q(\bar{a}^*_f + b^*_m)) + \sigma u(\beta\bar{a}^*_f) + \sigma\beta b^*_m = \beta b^*_m \tag{36}$$

These two equations become identical when $b^*_m = a^*_h$ and therefore $a^*_{h,Min} + a^*_f = \check{a}$ must hold at this knife-edge case of $a^*_f = \bar{a}^*_f$. To sum up, for the indeterminate combination of $(t, b^*) = \big\{(t,b^*) : t > t^{op}, b^* < a^*_h, b^* < \check{a} - a^*_f, c(t) = (1-\sigma)\beta b^* + \sigma\big[u(\beta(a^*_f + b^*) - u(\beta a^*_f)\big]\big\}$ to be the solution, the following restrictions on a^*_h and a^*_f must hold true.

1. $a^*_h \geq a^*_{h,Min}$
2. $a^*_f \leq \bar{a}^*_f$

which corresponds to region 5.

<u>*When* $\lambda_2 > 0$</u>

If we let $\lambda_2 > 0$ then $b^* = a^*_h$ must hold. Moreover, the first constraint in the bargaining problem (i.e., the participation constraint) also becomes binding due to $\lambda_1 > 0$. Hence the solution for t must satisfy:

$$c(t) = (1-\sigma)\beta a_h^* + \sigma\big[u(q(a_f^* + a_h^*)) - u(q(a_f^*))\big] \tag{37}$$

Now the question is whether t here is bigger or less than t^{op}. As a matter of fact, it can be easily shown that $c(t) < c(t^{op})$ in this case. First, the comparison between e.q.(37) and (33) confirm that $c(t) \le c(t^{op})$ if $a_f^* \le \bar{a}_f^*$. Second, if $a_f^* \ge \bar{a}_f^*$ and $a_f^* + a_h^* < \breve{a}$ then e.q.(37) tells us that the max of $c(t)$ (i.e., $c(t^{max})$) occurs at the point where $a_f^* = \bar{a}_f^*$ and $a_h^* = \breve{a} - \bar{a}_f^*$ due to again the concavity assumption on $u(\cdot)$. Thus plugging $a_f^* = \bar{a}_f^*$ and $a_h^* = \breve{a} - \bar{a}_f^*$ into e.q.(37) would yield the condition for $c(t^{max})$ as:

$$\begin{aligned} c(t^{max}) &= (1-\sigma)\beta(\breve{a} - \bar{a}_f^*) + \sigma\big[u(\tilde{q}) - u(q(\bar{a}_f^*))\big] \\ &= (1-\sigma)\beta a_h^* + \sigma\big[u(\tilde{a}) - u(q(\bar{a}_f^*))\big] \end{aligned} \tag{38}$$

which is same as e.q.(35). This completes the proof that $c(t)$ regardless of a_f^* domain becomes bounded from above, $c(t^{op})$ in this case. To sum up, for the $b^* = a_h^*$ and t such that e.q.(37) holds to be the solution, the following condition should be met.

1. $a_h^* \le a_{h,Min}^*$
2. $a_f^* \le \breve{a} - a_h^*$

which corresponds to region 6.

Lastly, let us consider the parameter space such that $\beta\breve{a} + \sigma\big[u(\tilde{q}) - \tilde{q}\big] \le c(t^{op})$. When $c(t^{op}) < \beta\breve{a} + \sigma\big[u(\tilde{q}) - \tilde{q}\big]$, $\bar{a}_f^*$ becomes negative. This essentially eliminates the indeterminate solution region 5. The reason for this disappearance is quite intuitive. Recall the incentive compatibility condition for the F in the FIM bargaining problem. The F's liquidity evaluation of foreign assets (i.e., $\sigma\big[u(q(a_f^* + b^*)) - \beta n(a_f^* + b^*)\big] - \sigma\big[u(q(a_f^*)) - \beta n(a_f^*)\big]$) even when the her initial foreign asset holdings are zero should reach an upper bound. Once the disutility of producing t^{op} (i.e., $c(t^{op})$) exceeds this boundary, the F would never be willing to produce more than t^{op} and the terms of trade would never settle at the point where $t > t^{op}$ even if a_f^* falls into zero as shown in figure 6. All the other specifics regarding the remaining regions stay same. The following summarizes and graphically illustrates the bargaining solution under this new parameter space.

$$\textit{If} \quad \begin{cases} a_h^* \ge \frac{c(t^{op})}{\beta} \\ a_f^* \ge \breve{a} \end{cases} \quad \textit{then} \quad \begin{cases} t = t^{op} \\ b^* = \frac{c(t^{op})}{\beta} \end{cases} \quad \Rightarrow \textit{Region 1}$$

$$\textit{If} \quad \begin{cases} a_h^* \le \frac{c(t^{op})}{\beta} \\ a_f^* \ge \breve{a} \end{cases} \quad \textit{then} \quad \begin{cases} t = \big\{t : \beta a_h^* = c(t)\big\} \\ b^* = a_h^* \end{cases} \quad \Rightarrow \textit{Region 2}$$

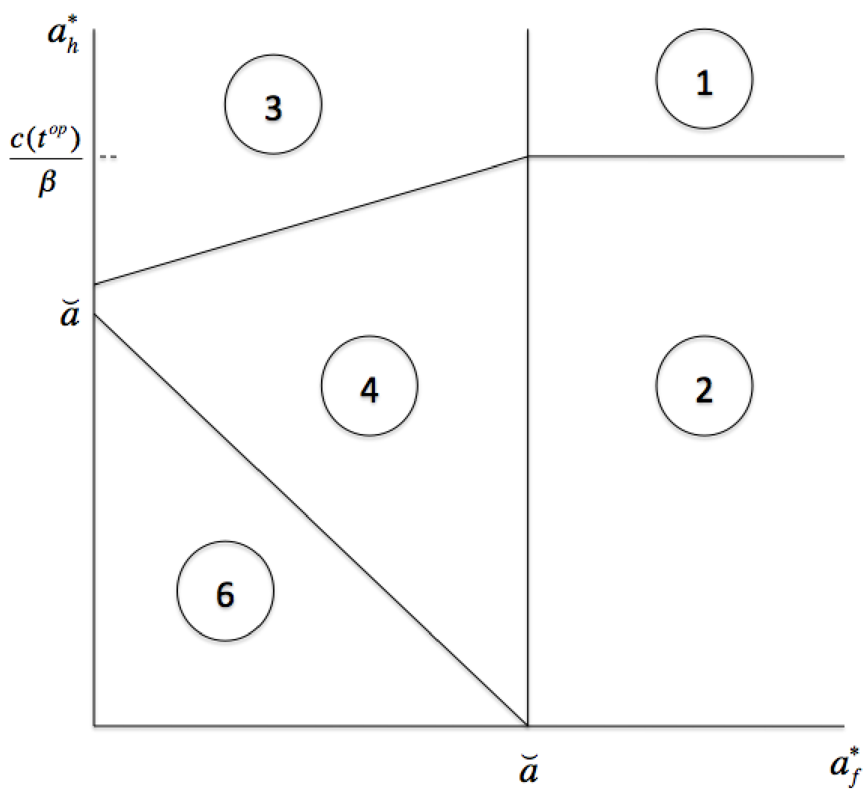
$$\textit{If} \quad \begin{cases} a_h^* \geq \frac{c(t^{op})-\sigma\left[u(\tilde{q})-u(\beta a_f^*)+\tilde{q}-\beta a_f^*\right]}{\beta} \\ a_f^* \leq \breve{a} \end{cases} \quad \textit{then} \quad \begin{cases} t = t^{op} \\ b^* = \frac{c(t^{op})-\sigma\left[u(\tilde{q})-u(\beta a_f^*)+\tilde{q}-\beta a_f^*\right]}{\beta} \end{cases} \quad \Rightarrow \textit{Region 3}$$

$$\textit{If} \quad \begin{cases} a_h^* \leq \frac{c(t^{op})-\sigma\left[u(\tilde{q})-u(\beta a_f^*)+\tilde{q}-\beta a_f^*\right]}{\beta} \\ \breve{a} - a_h^* \leq a_f^* \leq \breve{a} \end{cases} \quad \textit{then} \quad \begin{cases} t = \{t : c(t) = \beta a_h^* \\ \quad +\sigma\left[u(\tilde{q}) - u(\beta a_f^*) - \tilde{q} + \beta a_f^*\right]\} \\ b^* = a_h^* \end{cases} \quad \Rightarrow \textit{Region 4}$$

$$\textit{If} \quad a_f^* + a_h^* \leq \breve{a} \quad \textit{then} \quad \begin{cases} t = \{t : c(t) = \beta a_h^* + \sigma\left[u(\beta(a_f^* + a_h^*)) - u(\beta a_f^*)\right] \\ \quad -\sigma\left[\beta(a_f^* + a_h^*) - \beta a_f^*\right]\} \\ b^* = a_h^* \end{cases} \quad \Rightarrow \textit{Region 6}$$

This completes the proof. *Q.E.D*

Figure 6: Regions of the FIM Bargaining Solution $\left(\beta\breve{a} + \sigma\left[u(\tilde{q}) - \tilde{q}\right] \leq c(t^{op})\right)$



Proof of Lemma 3.

From the *Case 1* in lemma 2, it is easy to check that the terms of trade in the FIM (i.e. t and b^*) in regions 1,3, and 5 have nothing to do with $\widehat{a}_h^*$. Thus the third line e.q.(11) basically becomes a constant term. This makes the first derivative of J^H with respect to $\widehat{a}_h^*$ simply equal to $-\psi^* + \beta$. On the contrary, the H would experience the liquidity shortage in region 2,4, and 6. Therefore she would have to give up all of her foreign asset holdings in the bargaining. This would in turn bring about the $\widehat{a}_h^*$ dependent bargaining solution for the t as well. Thus the partial derivatives in these regions should take a form as:

$$\frac{\partial J_i^H(\widehat{a}_h, \widehat{a}_h^*)}{\partial \widehat{a}_h^*} = -\psi^* + \beta + \chi_h \beta \left\{ u'(t(\cdot)) \frac{\partial t(\cdot)}{\partial \widehat{a}_h^*} - \beta \frac{\partial b^*(\cdot)}{\partial \widehat{a}_h^*} \right\} \quad i = 2, 4, 6$$

$\frac{\partial b^*(\cdot)}{\partial \widehat{a}_h^*} = 1$ since $b^* = a_h^*$ for all regions of 2,4, and 6 while applying the Implicit Function Theorem to the FIM bargaining protocol described in *Case 1* of lemma 2 indicates:

$$\frac{\partial t(\cdot)}{\partial \widehat{a}_h^*} = \begin{cases} -\frac{-\beta}{c'(t)} & \text{if} \quad i = 2, 4 \\ -\frac{-\beta - \sigma u'(\beta(a_f^* + a_h^*))\beta + \sigma \beta}{c'(t)} & \text{if} \quad i = 6 \end{cases}$$

This completes the proof. *Q.E.D*

Proof of Lemma 4.

The budget constraint of the centralized market implies that the H can exploit more labor units in period t by dL_t^h and get either $\psi_{t+1} dB_{t+1}^h$ units of home bonds or $\psi_{t+1}^* dR_{t+1}^h$ units of foreign reserves. In the next period the H can therefore decrease the amount labor units exploited by $dL_{t+1}^h = dB_{t+1}^h$ or dR_{t+1}^h. The net utility gain of doing this strategy is

$$\begin{aligned} d\mathcal{U}_t^h = -dL_t^h + \beta dL_{t+1}^h &= -\, dB_{t+1}^h \big[\psi_{t+1} - \beta\big] \\ &= -\, dR_{t+1}^h \big[\psi_{t+1}^* - \beta\big] \end{aligned}$$

So, either of $\beta > \psi_{t+1}$ or $\beta > \psi_{t+1}^*$ implies that $d\mathcal{U}_t > 0$ which would in turn cause for infinite labor demand every period. Therefore in any any equilibrium $\beta \leq \psi$ and $\beta \leq \psi^*$. This can be applied to the F exactly in the same way. By the similar budget constraint of the F in the centralized market, F can also exploit more labor units in period t by dL_t^f and get dR_{t+1}^f units of foreign assets. In the next period the F can therefore decrease the amount labor units exploited by $dL_{t+1}^f = dR_{t+1}^f$. The net utility gain of doing this strategy is

$$d\mathcal{U}_t^f = -dL_t^f + \beta dL_{t+1}^f = -dR_{t+1}^f \big[\psi_{t+1}^* - \beta\big]$$

Again, this confirms that in any any equilibrium $\beta \leq \psi^*$. *Q.E.D*

Proof of Lemma 5.

Consider the case where $\widehat{a}_f^* > \breve{a}$. It becomes obvious from lemma (1) that the terms of trade in the F's local DM are fixed regardless of the amount of foreign assets F chooses to hold. Hence by taking the first order condition of e.q.(13) with respect to $\widehat{a}^*$, we obtain

$$J_{\widehat{a}^*}^F(\widehat{a}^*) = -\psi^* + \beta \leq 0$$

where the weak inequality sign comes from lemma (4). From this one can easily verify that the optimal choice of foreign asset holdings for the F can neither be same as or exceed $\breve{a}$ unless

$\psi^* = \beta$. We now consider the second case where $\widehat{a}^*_f \le \check{a}$. Again following from the bargaining solution in lemma (1) we have a FOC as:

$$J^F_{\widehat{a}^*}(\widehat{a}^*) = -\psi^* + \beta + \sigma\beta\big\{u'(q(a^*_f)) - 1\big\}$$

This justifies the optimality condition in lemma 5. For the uniqueness of $\widetilde{a^*_f}$, we need following observations. Given the strict concavity assumption of agent's utility function, it is easy to understand the second derivative of the F's objective function with respect to $\widehat{a}^*$ is strictly negative (i.e. $J^F_{\widehat{a}^*\widehat{a}^*}(\widehat{a}^*) < 0$ for all $\widehat{a}^* \in (0, \check{a}]$). Furthermore, one can also easily show that the following two conditions must hold in the limit.

$$\lim_{\widehat{a}^* \to 0} J^F_{\widehat{a}^*}(\widehat{a}^*) > 0$$

$$\lim_{\widehat{a}^* \to \check{a}^-} J^F_{\widehat{a}^*}(\widehat{a}^*) \le 0$$

Combining all these results above, we can finally conclude that the optimal choice of $\widetilde{a}^*_f$ is unique and it satisfies $\widetilde{a^*_f} \in (0, \check{a})$ when $\psi^* < \beta$. On the other hand, if ψ^* happens to be same as β then the F's optimal foreign asset holdings could be either same as $\check{a}$ or anything bigger than that. *Q.E.D*

Proof of Lemma 6.
With regard to the optimal home asset holdings (i.e. $\widetilde{a}$), one can refer to the proof of lemma 5 since the exactly same line of reasoning applies. From lemma 2 and 3, one can infer the parameter space of (ψ^*, a^*_f) that is consistent with the optimal choice of $\widetilde{a^*_h}$ in each of the six regions.

<u>Region 1:</u> First, from lemma 3, the optimality (i.e., *FOC*) requires that $\psi^* = \beta$. Second, lemma 2 restricts $\widetilde{a^*_h}$ to be greater than or equal to $\frac{c(t^{op})}{\beta}$.

<u>Region 2:</u> The optimality condition based on lemma 3 asks $\psi^* - \beta = \chi_h \beta \Big\{\frac{u'(t)}{c'(t)} - 1\Big\}$. Since the lemma 2 implies $t < t^{op}$ in this region, the optimality should be consistent with $\psi^* > \beta$.

<u>Region 3:</u> The optimal condition based on lemma 3 implies $\psi^* = \beta$. At the same time, lemma 2 pins down the $\widetilde{a}^*_h$ such that $\widetilde{a^*_h} = \mathbb{R}_{++} \ge \frac{c(t^{op})}{\beta} - \frac{\sigma\left[u(\tilde{q}) - u(\beta a^*_f) + \tilde{q} - \beta a^*_f\right]}{\beta}$.

<u>Region 4:</u> Lemma 2 restricts $\widetilde{a}^*_h$ to be less than $\frac{c(t^{op})}{\beta}$ and hence implies $t < t^{op}$. At the same time, the optimality from lemma 3 requires $\psi^* - \beta = \chi_h \beta \Big\{\frac{u'(t)}{c'(t)} - 1\Big\}$. Combining the two results, it is obvious that the optimality should be consistent with $\psi^* > \beta$. Nevertheless, the upper bound of ψ^* (i.e., $\overline{\psi^*}$) that is consistent with the optimality should exist. This condition is attributed to the fact that the lemma 2 also bounds $\widetilde{a^*_h}$ from below (i.e., $\check{a} - a^*_f$). If ψ^* grows too big, the optimal amount of $\widetilde{a^*_h}$ defined in lemma 2 may fall below $\check{a} - a^*_f$. In order to prevent this, $\overline{\psi^*}$ should be such that it satisfies the optimality (i.e., $\overline{\psi^*} - \beta = \chi_h \beta \big\{\frac{u'(t)}{c'(t)} - 1\big\}$) given t that guarantees the minimum value of $\widetilde{a^*_h}$ (i.e., $c(t) = \sigma\big[u(\tilde{q}) - \tilde{q}\big] - \sigma\big[u(q(a^*_f)) - q(a^*_f)\big] + \beta(\hat{a} - a^*_f)$)

<u>To the right side of $\bar{a}^*_f$ in Region 6:</u> Similar to the region 4 case, lemma 2 restricts $\widetilde{a}^*_h$ such that

$t < t^{op}$. Given the optimal condition of $\psi^* - \beta = \chi_h \beta \{\frac{u'(t)}{c'(t)}\{(1-\sigma) + \sigma u'(\beta a_f^*)\} - 1\}$ from lemma 3, the optimality should imply $\psi^* > \beta$. However, the region 4 case showed that the foreign asset price range of $\beta < \psi^* < \overline{\psi^*}$ should lead to the $\widetilde{a}_h^*$, which dominates the one implied by the optimality in this region. For this reason, only $\psi^* > \overline{\psi^*}$ is compatible with the optimal choice in this region.

Region 5: From lemma 3, the optimality requires that $\psi^* = \beta$. Moreover, lemma 2 restricts $\widetilde{a_h^*}$ to be greater than or equal to $a_{h,Min}^*$.

To the left side of $\bar{a}_f^*$ in Region 6: The optimality condition based on lemma 3 asks $\psi^* - \beta = \chi_h \beta \{\frac{u'(t)}{c'(t)}\{(1-\sigma) + \sigma u'(\beta a_f^*)\} - 1\}$. Since the lemma 2 implies $t < t^{op}$ in this region, the optimality should be consistent with $\psi^* > \beta$.

Rearranging the results above should suffice to explain the H's optimal choice of foreign asset holdings. This completes the proof. *Q.E.D*

Proof of Lemma 7.

When T is plentiful: $T \geq \check{a} + \frac{c(t^{op})}{\beta}$

Figure 2 confirms that the region 1, 2, 3, and 5 could be all potentially possible equilibrium region. It is obvious from lemma 6 that $\psi = \beta$ if the equilibrium happens to occur in either of these regions.

(i): Now suppose the equilibrium $\{a_h^*, a_f^*\}$ lies in the region 2. Then the lemma 5 tells $a_f^* > \check{a}$ must be consistent with $\psi^* = \beta$. Yet, $\frac{\partial J_2^H(\widehat{a}_h, \widehat{a}_h^*)}{\partial \widehat{a}_h^*}$ from the lemma 3 implies that $t = t^{op}$ which is a contradiction to lemma 2. Thus the region 2 can not be the equilibrium region.

(ii): Suppose the equilibrium lies in either of the region 3 or 5. Then the lemma 5 tells that $\psi^* > \beta$ but again from $\frac{\partial J_3^H(\widehat{a}_h, \widehat{a}_h^*)}{\partial \widehat{a}_h^*}$ or $\frac{\partial J_3^H(\widehat{a}_h, \widehat{a}_h^*)}{\partial \widehat{a}_h^*}$ from the lemma 3 indicates that only $\psi^* = \beta$ must be the H's optimality consistent price. Hence, the region 3 can not be the equilibrium region either.

(iii): Suppose the equilibrium lies in the region 1. Then the lemma 5 implies $\psi^* = \beta$. At the same time, $\frac{\partial J_1^H(\widehat{a}_h, \widehat{a}_h^*)}{\partial \widehat{a}_h^*}$ from the lemma 3 also confirms that $\psi^* = \beta$ is consistent with the H's optimality. Hence, the equilibrium can be achieved in this regions subject to:

1. $a_h^* \geq \frac{c(t^{op})}{\beta}$
2. $a_f^* \geq \check{a}$
3. $a_h^* + a_f^* = T$

As long as any combination of $\{a_h^*, a_f^*\}$ meets the above three conditions, the equilibrium can be achieved and therefore the indeterminacy arises in this case.

When T lies within a moderate range: $\check{a} \leq T < \check{a} + \frac{c(t^{op})}{\beta}$

It is understood that the region 2, 3, and 5 can not be the equilibrium region for the same reason in the case of $T \geq \breve{a} + \frac{c(t^{op})}{\beta}$. This only leaves us with the region 4 as the only feasible equilibrium region. Indeed the lemma 5 and 6 restrict the foreign asset price to be greater than the fundamental value in this region. Specifically, the two optimal conditions at the market clearing situation are:

$$\psi^* - \beta = \sigma\beta\big[u'(\beta a_f^*) - 1\big] \tag{39}$$

$$\psi^* - \beta = \chi_h \beta\left[\frac{u'(t)}{c'(t)} - 1\right] \tag{40}$$

$$where \quad c(t) = \beta(T - a_f^*) + \sigma\big[u(\tilde{q}) - u(\beta a_f^*) - \tilde{q} + \beta a_f^*\big]$$

From e.q(39) and (40) the following must be satisfied in equilibrium as well.

$$\frac{\chi_h}{\sigma} = \frac{u'(\beta a_f^*) - 1}{\frac{u'(t)}{c'(t)} - 1} \tag{41}$$

Finally let us prove if $\exists! a_f^* \in (\bar{a}_f^*, \breve{a})$. By rearranging e.q(41), one can define $G(a_f^*)$ as:

$$G(a_f^*) \equiv \sigma\big\{u'(\beta a_f^*) - 1\big\} - \chi_h\left\{\frac{u'(t)}{c'(t)} - 1\right\} = 0 \tag{42}$$

$$where \quad c(t) = \beta(T - a_f^*) + \sigma\big[u(\tilde{q}) - u(\beta a_f^*) - \tilde{q} + \beta a_f^*\big] \tag{43}$$

First, by taking the $G(a_f^*)$ to the limit the following must hold.

$$\lim_{a_f^* \to \bar{a}_f^{*+}} G(a_f^*) = \sigma\big\{u'(\beta \bar{a}_f^*) - 1\big\} - \chi_h\left\{\frac{u'(t^{op})}{c'(t^{op})} - 1\right\} \tag{44}$$

$$= \oplus - 0 > 0$$

$$\lim_{a_f^* \to \breve{a}^-} G(a_f^*) = \sigma\big\{u'(\beta \breve{a}) - 1\big\} - \chi_h\left\{\frac{u'(t)}{c'(t)} - 1\right\} \tag{45}$$

$$= 0 - \oplus < 0$$

where the second term in e.q(45) becomes a negative value since the t in the region 4 happens to be less than t^{op} according to the lemma 2.

$$G'(a_f^*) = \sigma\beta \underbrace{u''(\beta a_f^*)}_{\ominus} \tag{46}$$

$$- \chi_h\left\{\underbrace{u''(t)}_{\ominus}\underbrace{\frac{\partial t}{\partial a_f^*}}_{\ominus} c'(t)^{-1} - u' c'(t)^{-2}\underbrace{c''(t)}_{\oplus}\underbrace{\frac{\partial t}{\partial a_f^*}}_{\ominus}\right\} < 0$$

Finally, all is left to guarantee the uniqueness of equilibrium a_f^* is to show that $G'(a_f^*) < 0$ as shown in e.q. 46 where $\frac{\partial t}{\partial a_f^*} = -\frac{(1-\sigma)\beta + \sigma u'(\beta a_f^*)}{c'(t)} < 0$ from the e.q(43). This proves $G'(a_f^*) < 0$ and therefore the uniqueness of the equilibrium in the region 4 is established.

When T is scarce: $T \leq \breve{a}$
Potentially the equilibrium a_h^*, a_f^* can be in either region 5 and 6. Again It is obvious that the region 5 can not be the equilibrium region for the same reason in the previous two cases. This only leaves us with the region 6 as the only feasible equilibrium region. Indeed the lemma 5 and 6 restrict the foreign asset price to be greater than the fundamental value in this region. Specifically, the two optimal conditions at the market clearing situation are:

$$\psi^* - \beta = \sigma\beta\big[u'(\beta a_f^*) - 1\big] \tag{47}$$

$$\psi^* - \beta = \chi_h\beta\left[\frac{u'(t)}{c'(t)}\big\{(1-\sigma) + \sigma u'(\beta T)\big\} - 1\right] \tag{48}$$

$$where \quad c(t) = \beta(T - a_f^*) + \sigma\big[u(\beta T) - u(\beta a_f^*) + \beta(T - a_f^*)\big]$$

From e.q(47) and (48) the following must be satisfied in equilibrium as well.

$$\frac{\chi_h}{\sigma} = \frac{u'(\beta a_f^*) - 1}{\frac{u'(t)}{c'(t)}\big\{(1-\sigma) + \sigma u'(\beta T)\big\} - 1} \tag{49}$$

Finally let us prove if $\exists! a_f^* \in (0, \breve{a})$. By rearranging e.q(49), one can define $Z(a_f^*)$ as:

$$Z(a_f^*) \equiv \sigma\big\{u'(\beta a_f^*) - 1\big\} - \chi_h\left\{\frac{u'(t)}{c'(t)}\big\{(1-\sigma) + \sigma u'(\beta T)\big\} - 1\right\} = 0 \tag{50}$$

$$where \quad c(t) = \beta(T - a_f^*) + \sigma\big[u(\beta T) - u(\beta a_f^*) + \beta(T - a_f^*)\big] \tag{51}$$

First, by taking the $Z(a_f^*)$ to the limit the following must hold.

$$\lim_{a_f^* \to 0^+} Z(a_f^*) = \sigma\big\{u'(\beta 0) - 1\big\} - \chi_h\left\{\frac{u'(t)}{c'(t)}\big\{(1-\sigma) + \sigma u'(\beta T)\big\} - 1\right\} \tag{52}$$

$$= \infty - \oplus > 0$$

$$\lim_{a_f^* \to \breve{a}^-} Z(a_f^*) = \sigma\big\{u'(\tilde{q}) - 1\big\} - \chi_h\left\{\frac{u'(t)}{c'(t)}\big\{(1-\sigma) + \sigma u'(\beta T)\big\} - 1\right\} \tag{53}$$

$$= 0 - \oplus < 0$$

where the second term in e.q(53) becomes a negative value since the t in the region 6 happens to be less than t^{op} according to the lemma 2. Finally all is left to guarantee the uniqueness of equilibrium a_f^* is to show that $Z'(a_f^*) < 0$. By taking the first derivative of $Z(a_f^*)$ function with respect to a_f^*, the following equation must hold true.

$$\begin{aligned} Z'(a_f^*) =& \sigma\beta \underbrace{u''(\beta a_f^*)}_{\ominus} \\ & - \chi_h \underbrace{\{(1-\sigma)+\sigma u'(\beta T)\}}_{\oplus} \left\{ \underbrace{u''(t)}_{\ominus} \underbrace{\frac{\partial t}{\partial a_f^*}}_{\ominus} c'(t)^{-1} - u'c'(t)^{-2} \underbrace{c''(t)}_{\oplus} \underbrace{\frac{\partial t}{\partial a_f^*}}_{\ominus} \right\} < 0 \end{aligned} \tag{54}$$

where $u'(\beta T) \leq u'(\tilde{q}) = 1$ and $\frac{\partial t}{\partial a_f^*} = -\frac{(1-\sigma)\beta + \sigma u'(\beta a_f^*)}{c'(t)} < 0$ from the e.q(51). This proves $Z'(a_f^*) < 0$ and therefore the uniqueness of the equilibrium in the region 4 is established. *Q.E.D.*

Proof of Proposition 1.

When $\breve{a} < T \leq \breve{a} + \frac{c(t^{op})}{\beta}$
It is obvious that the home country would on aggregate hold on to home assets exceeding the first best amount $\breve{a}$. Then from the lemma 6, $\psi = \beta$ must hold in equilibrium. Since the proof for the lemma 7 confirms $\psi^* > \beta$ for the case $\breve{a} < T \leq \breve{a} + \frac{c(t^{op})}{\beta}$, it should be easy to see $\psi^* > \psi$. For the comparative statics, now recall the e.q(39) and (40). Instead of performing the total differentiation to the optimal conditions, one can simply conduct a thought experiment using the e.q.(39) and (40). Starting with an equilibrium situation, suppose T all of sudden increases. Then by e.q.(39), ψ^* must remain unchanged and also by e.q(40) t must remain same. But by the $c(t)$ function in e.q(40), t must also go up which is a contradiction. Now let us suppose a_f^* falls and a_h^* rises. Then by e.q(39) ψ^* must increase which in turn imply a fall in t by the e.q(40). Yet the $c(t)$ function in e.q(40) again forces t to increase, which contradicts the fall in t by the e.q(40). Therefore these thought experiments leaves us with nothing but $\frac{\partial a_f^*}{\partial T} > 0$ and $\frac{\partial \psi^*}{\partial T} < 0$. Having the effects of ΔT on ψ^* and a_f^* established, one can further pursue the same experiments with the a_h^* and t. Since $\frac{\partial \psi^*}{\partial T} < 0$ for sure, the e.q(40) also makes t rise in response to the increase in T. Consequently, the increase in t and the $c(t)$ function in the e.q(40) also forces a_h^* to go up in equilibrium as T goes up. To sum up, it must be also true that in equilibrium $\frac{\partial a_h^*}{\partial T} > 0$ and $\frac{\partial t}{\partial T} > 0$.

When $T < \breve{a}$
it is easily understood why both ψ^* and ψ exceed the β in equilibrium. For the $\psi^* > \psi$ in equilibrium, one could simply recall the e.q(47) and the optimal condition for the home asset holdings by the home agent in the lemma 6 as:

$$\psi - \beta = \sigma\beta\{u'(\beta T) - 1\}$$
$$\psi^* - \beta = \sigma\{u'(\beta(T - a_h^*)) - 1\}$$

Since $\beta T > \beta(T - a_h^*)$ when $a_h^* \in (0, T)$, $\psi^* > \psi$ must hold in equilibrium. For the comparative statics, the exactly same kind of experiments in the preceding case could be conducted. Recall the e.q(47) and (48). Let us imagine a situation where T rises from the initial steady state.

Suppose $\Delta a_f^* = 0$ and a_h^* goes up in response. The by the e.q(47), $\frac{\partial \psi^*}{\partial T} = 0$ ad by the e.q(48), $\frac{\partial t}{\partial T} < 0$. But these would mean in accordance with the $c(t)$ function in the e.q(48) that a_h^* must fall which is a contradiction. Now suppose a_f^* goes up and a_h^* decreases instead. Then by the same cost function, t must increase and at the same time ψ^* should fall. However by the e.q(47) the ψ^* must rise so again the contradiction arises. Hence $\frac{\partial a_f^*}{\partial T} > 0$ and $\frac{\partial \psi^*}{\partial T} < 0$ must be true just like the preceding example. Nevertheless the effects of ΔT on the t and a_h^* are this time ambiguous. The e.q(48) reveals that given the increase in T the rise in a_f^* and the fall in ψ^* can not guarantee the signs of $\frac{\partial a_h^*}{\partial T}$ and $\frac{\partial t}{\partial T}$. Basically this has been caused by the effect of the terms inside the square bracket in the RHS of e.q(48) (i.e., $(1-\sigma) + \sigma u'(\beta T)$) which generates additional downward pressure for the expected surplus from carrying the asset in the case of rising T. Therefore without this term, this thought experiment would have resulted in exactly same results as the preceding case especially the $\frac{\partial a_h^*}{\partial T} > 0$ and $\frac{\partial t}{\partial T} > 0$. But since this term is present the upward pressure for the a_h^* would be somewhat mitigated. Depending on the parameter values, the precise effect would vary. At least though it is obvious that the positive effect of T changes on the a_h^* in this scarce T case is smaller than the one in the less scarce case of $\check{a} < T \leq \check{a} + \frac{c(t^{op})}{\beta}$. *Q.E.D*

Proof of Proposition 2.

An easy proof could be done by a similar thought experiment as in proposition 1. Recall the e.q(47) and (48). Suppose χ_h increases and as a result ψ^* remains same and t goes up. Then by the cost function in e.q(48), it must be true that a_h^* rises while a_f^* falls. But then since a_f^* falls the e.q(47) implies an increase in ψ^* which is a contradiction. Now suppose t remains same while ψ^* increases but this generates an immediate contradiction from the e.q(48). Next suppose the t falls down and ψ^* increases instead. But again from the cost function, a_h^* must decrease which would automatically imply an increase in the level of a_f^* by the market clearing condition. This combined with the e.q(47) would simply mean a fall in the equilibrium level of ψ^* which is again a contradiction. Lastly now suppose χ_h increases along with t and ψ^*. Then since t goes up it is easily understood that a_h^* increases while a_f^* falls from the cost function. Again since a_f^* falls the e.q(47) implies an increase in ψ^* which is consistent with the assumption. This completes the proof. *Q.E.D*

Table 1: **Definitions and Sources for the Conventional Regressors**

Regressors	**Definitions**	**Sources**
(log) Population	the log of population	World Development Indicator (WDI)
(log) GDP per person	the log of GDP per capita at PPP exchange rates in current international dollar	WDI
Trade/GDP	(Export+Import)/GDP	WDI
Exchange rate volatility	annual standard deviation of monthly exchange rate changes	International Financial Statistics (IFS), IMF
(log) Terms of Trade	the log of the terms of trade in goods and services	World Economic Outlook (WEO), IMF
Financial Openness	Chinn-Ito index	Chinn and Ito (2008)
M2/GDP	M2 to GDP ratio	WDI and the OECD economic outlook
Peg	a pegged exchange rate dummy	Shambaugh (2004) and Obstfeld *et al.* (2010)
Soft peg	a soft-peg exchange rate dummy	Shambaugh (2004) and Obstfeld *et al.* (2010)
Currency crisis	a lagged currency crisis dummy	Laeven and Valencia (2012)

Table 2: **Summary Statistics**

Variables	Observations	Mean	Std. Dev
Lagged reserve to GDP	485	0.196	0.165
DCF	485	0.012	0.072
(log) Population	485	16.884	1.688
(log) GDP per capita	485	8.871	0.851
Trade/GDP	484	0.957	0.689
Exchange rate volatility	485	0.02	0.035
(log) Terms of trade	485	4.6204	0.143
Financial openness	485	0.582	1.418
Peg	485	0.330	0.471
Soft peg	485	0.247	0.432
M2/GDP	485	0.562	0.404
Debt/GDP	485	0.483	0.309
Currency crisis	485	0.0289	0.1676

Table 3: **Countries Included in the Sample**

East and Central Asia	
Bangladesh	Korea
China	Malaysia
Sri Lanka	Pakistan
Hong Kong	Philippines
India	Singapore
Indonesia	Thailand
Kazakhstan	Vietnam
Oil-producing countries	
Bahrain	
Jordan	
Lebanon	
Oman	
Saudi Arabia	
Egypt	
Latin America	
Argentina	Jamaica
Bolivia	Mexico
Brazil	Nicaragua
Chile	Panama
Colombia	Peru
Costa Rica	Trinidad and Tobago
Dominican Republic	Uruguay
El Salvador	Venezuela
Guatemala	
East Europe	
Bulgaria	Hungary
Russia	Lithuania
Ukraine	Croatia
Czech Republic	Slovenia
Slovak Republic	Bosnia and Herzegovina
Estonia	Poland
Latvia	Romania
Macedonia	Moldova

Table 4: **Foreign Investment Inflows through Decentralized Markets and Reserves/GDP**

Dependent Variable: Reserve to GDP

	Estimation Methods						
	(1) OLS	(2) FE	(3) FE w/lagged variable	(4) Difference-GMM	(5) System-GMM	(6) Difference-GMM	(7) System-GMM
Lagged Reserve to GDP			0.6923***	0.5525***	0.8885***	0.2326	0.8290***
			(0.0466)	(0.1324)	(0.0840)	(0.1658)	(0.1080)
DCF	**0.068**	**0.0051**	**0.0742***	**0.0803****	**0.0987****		
	(0.158)	**(0.071)**	**(0.0420)**	**(0.0381)**	**(0.0490)**		
(log) Population	0.009	0.05	0.0173	-0.3676**	-0.0049	-0.1830*	0.0000
	(0.009)	(0.153)	(0.0718)	(0.1625)	(0.0043)	(0.0957)	(0.0041)
(log) GDP per Capita	0.034**	0.201**	0.0633	-0.1842**	0.0084	-0.0903	0.0056
	(0.015)	(0.076)	(0.0407)	(0.0909)	(0.0128)	(0.0716)	(0.0106)
Trade/GDP	0.144***	0.048	0.0075	-0.0912*	-0.0226	0.0046	0.0158
	(0.04)	(0.034)	(0.0253)	(0.0478)	(0.0277)	(0.0707)	(0.0257)
FOREX Volatility	-0.241	-0.021	0.1331*	0.1648	0.2064**	0.0872	0.1455*
	(0.145)	(0.077)	(0.0669)	(0.1149)	(0.0991)	(0.0657)	(0.0747)
Terms of Trade	0.032	–0.021	0.0031	0.0014	0.0155	-0.0078	0.0023
	(0.045)	(0.038)	(0.0217)	(0.0314)	(0.0201)	(0.0303)	(0.0168)
Financial Openness	-0.001	-0.009	-0.0051*	0.0002	-0.0032*	0.0011	-0.0025
	(0.007)	(0.006)	(0.0029)	(0.0067)	(0.0018)	(0.0040)	(0.0019)
M2/GDP	0.122***	0.067	0.0414	0.1065	0.0486	0.1073	0.0359
	(0.039)	(0.058)	(0.0381)	(0.0907)	(0.0349)	(0.0928)	(0.0329)
Debt/GDP	0.083**	0.007	-0.0027	-0.0076	0.0032	-0.0217	0.0081
	(0.039)	(0.044)	(0.0252)	(0.0413)	(0.0195)	(0.0346)	(0.0155)
Peg	-0.03	0.017	0.0053	0.0192*	0.0068	0.0213**	0.0022
	(0.021)	(0.022)	(0.0077)	(0.0112)	(0.0064)	(0.0098)	(0.0056)
Soft Peg	-0.0001	0.012	0.0056	0.0088*	0.0086*	0.0061	0.0046
	(0.009)	(0.009)	(0.0059)	(0.0046)	(0.0050)	(0.0047)	(0.0048)
Currency Crisis $(t-1)$	-0.033*	0.014	0.0128	0.0030	0.0065	0.0057	0.0035
	(0.019)	(0.013)	(0.0126)	(0.0154)	(0.0161)	(0.0097)	(0.0127)
Test for AR(1) in first differences (p-value)	...	...	...	0.005	0.000	0.149	0.000
Test for AR(2) in first differences (p-value)	...	...	...	0.750	0.906	0.551	0.422
Hasen Test (p-value)	...	...	...	0.350	0.784	0.212	0.361
Year Dummy	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	484	484	484	417	484	472	484
Number of Countries	53	53	53	53	53	53	53
R-squared or $Corr(y,y)^2$	0.787	0.943	0.968	0.160	0.941	0.119	0.893

Robust standard errors are in parentheses. Year dummies are included but not reported
* significant at 10%; ** significant at 5%; ***significant at 1%

Table 5: **Robustness Check: Hot Money Measures (Cuddington Method)**

Dependent Variable: Reserve to GDP

	Estimation Methods	
	Difference-GMM	System-GMM
	(4)	(5)
Lagged Reserve to GDP	0.726***	0.775***
	(0.227)	(0.092)
Hot Money Inflows	**0.084***	**0.048**
	(0.049)	**(0.035)**
(log) Population	-0.155**	0.001
	(0.088)	(0.005)
(log) GDP per Capita	-0.152**	0.013
	(0.058)	(0.012)
Trade/GDP	-0.057	0.005
	(0.047)	(0.038)
Exchange Rate Volatility	-0.015**	-0.015*
	(0.007)	(0.009)
(log) Terms of Trade	0.038*	0.019*
	(0.021)	(0.009)
Financial Openness (Chinn-Ito)	-0.003	0.001
	(0.003)	(0.003)
M2/GDP	0.048	0.053
	(0.070)	(0.043)
Debt/GDP	0.016***	0.013***
	(0.002)	(0.001)
Peg	0.004	-0.002
	(0.005)	(0.005)
Soft Peg	-0.003	-0.001
	(0.004)	(0.004)
Currency Crisis	0.0026	0.0045
	(0.0135)	(0.0162)
Test for AR(1) in first differences (p-value)	0.003	0.000
Test for AR(2) in first differences (p-value)	0.614	0.219
Number of instruments	48	64
Hansen Test (p-value)	0.348	0.849
Year Dummy	Yes	Yes
Observations	951	1027
Number of Countries	53	53
R-squared or $Corr(y, \hat{y})^2$	0.166	0.827

Robust standard errors are in parenthesis. Year dummies are included but not reported.
* significant at 10%; ** significant at 5%; ***significant at 1%

Table 6: **Robustness Check: FDI Inflows**

Dependent Variable: Reserve to GDP

	Estimation Methods	
	Difference-GMM	System-GMM
	(4)	(5)
Lagged Reserve to GDP	0.524***	0.821***
	(0.181)	(0.055)
FDI Inflows	**0.069**	**0.002**
	(0.083)	**(0.021)**
(log) Population	-0.138**	0.003
	(0.078)	(0.003)
(log) GDP per Capita	-0.084**	0.007
	(0.043)	(0.006)
Trade/GDP	-0.103***	0.027
	(0.028)	(0.027)
Exchange Rate Volatility	-0.011	-0.017*
	(0.007)	(0.010)
(log) Terms of Trade	0.020	0.016* *
	(0.015)	(0.007)
Financial Openness (Chinn-Ito)	0.01	-0.002
	(0.003)	(0.002)
M2/GDP	0.0425	0.025
	(0.065)	(0.022)
Debt/GDP	0.014***	0.012***
	(0.001)	(0.001)
Peg	0.003	-0.002
	(0.006)	(0.003)
Soft Peg	-0.003	-0.002
	(0.003)	(0.004)
Currency Crisis	0.004	0.004
	(0.007)	(0.007)
Test for AR(1) in first differences (p-value)	0.005	0.000
Test for AR(2) in first differences (p-value)	0.156	0.121
Number of instruments	57	64
Hansen Test (p-value)	0.401	0.952
Year Dummy	Yes	Yes
Observations	969	1044
Number of Countries	53	53
R-squared or $Corr(y, \hat{y})^2$	0.19	0.93

Robust standard errors are in parenthesis. Year dummies are included but not reported.
* significant at 10%; ** significant at 5%; ***significant at 1%

Table 7: **Robustness Check: Foreign Investment Inflows Through Centralized Markets**

Dependent Variable: Reserve to GDP

	Estimation Methods	
	Difference-GMM	System-GMM
	(4)	(5)
Lagged Reserve to GDP	0.385*	0.871***
	(0.197)	(0.078)
CCF	**-0.488**	**0.393**
	(1.742)	**(1.059)**
(log) Population	-0.265	0.002
	(0.174)	(0.004)
(log) GDP per Capita	-0.141	0.001
	(0.097)	(0.006)
Trade/GDP	0.066	0.023
	(0.063)	(0.018)
Exchange Rate Volatility	0.118	0.161*
	(0.093)	(0.088)
(log) Terms of Trade	-0.004	0.0004
	(0.028)	(0.016)
Financial Openness (Chinn-Ito)	-0.0002	-0.002
	(0.004)	(0.002)
M2/GDP	0.068	0.017
	(0.089)	(0.016)
Debt/GDP	-0.032	0.004
	(0.037)	(0.011)
Peg	0.020**	0.003
	(0.009)	(0.005)
Soft Peg	0.007	0.004
	(0.004)	(0.004)
Currency Crisis	0.004	0.005
	(0.014)	(0.016)
Test for AR(1) in first differences (p-value)	0.065	0.000
Test for AR(2) in first differences (p-value)	0.484	0.418
Number of instruments	48	62
Hansen Test (p-value)	0.378	0.7
Year Dummy	Yes	Yes
Observations	424	490
Number of Countries	57	53
R-squared or $Corr(y, \hat{y})^2$	0.219	0.957

Robust standard errors are in parenthesis. Year dummies are included but not reported.
* significant at 10%; ** significant at 5%; ***significant at 1%

Table 8: **Robustness Check: A Horse Race between the DCF, HM, FDI, and CCF**

Dependent Variable: Reserve to GDP

	Estimation Methods	
	Difference-GMM	System-GMM
	(4)	(5)
Lagged Reserve to GDP	0.3253*	0.8907***
	(0.1765)	(0.0816)
DCF	**0.1512****	**0.1103****
	(0.0616)	**(0.0541)**
Hot Money Inflows	**-0.0196**	**0.0153**
	(0.0367)	**(0.0313)**
FDI Inflows	**-0.0105**	**-0.0390**
	(0.0484)	**(0.0261)**
CCF	**0.2840**	**-1.0036**
	(1.5693)	**(1.2055)**
(log) Population	-0.2420	0.0006
	(0.1534)	(0.0032)
(log) GDP per Capita	-0.1242	0.0010
	(0.0811)	(0.0062)
Trade/GDP	-0.0031	0.0189
	(0.0530)	(0.0192)
Exchange Rate Volatility	0.1057	0.1775**
	(0.0789)	(0.0862)
(log) Terms of Trade	-0.0041	0.0080
	(0.0283)	(0.0171)
Financial Openness (Chinn-Ito)	-0.0001	-0.0027*
	(0.0052)	(0.0015)
M2/GDP	0.0515	0.0174
	(0.0813)	(0.0212)
Debt/GDP	0.0005	0.0025
	(0.0433)	(0.0104)
Peg	0.0210**	0.0045
	(0.0103)	(0.0049)
Soft Peg	0.0066	0.0039
	(0.0044)	(0.0035)
Currency Crisis	0.0026	0.0045
	(0.0135)	(0.0162)
Test for AR(1) in first differences (p-value)	0.026	0.000
Test for AR(2) in first differences (p-value)	0.401	0.973
Number of instruments	51	58
Hansen Test (p-value)	0.481	0.396
Year Dummy	Yes	Yes
Observations	417	484
Number of Countries	53	53
R-squared or $Corr(y, \hat{y})^2$	0.183	0.961

Robust standard errors are in parenthesis. Year dummies are included but not reported.
* significant at 10%; ** significant at 5%; ***significant at 1%

Figure 7: Reserves/GDP versus Foreign Equity Inflows Through Decentralized Trading

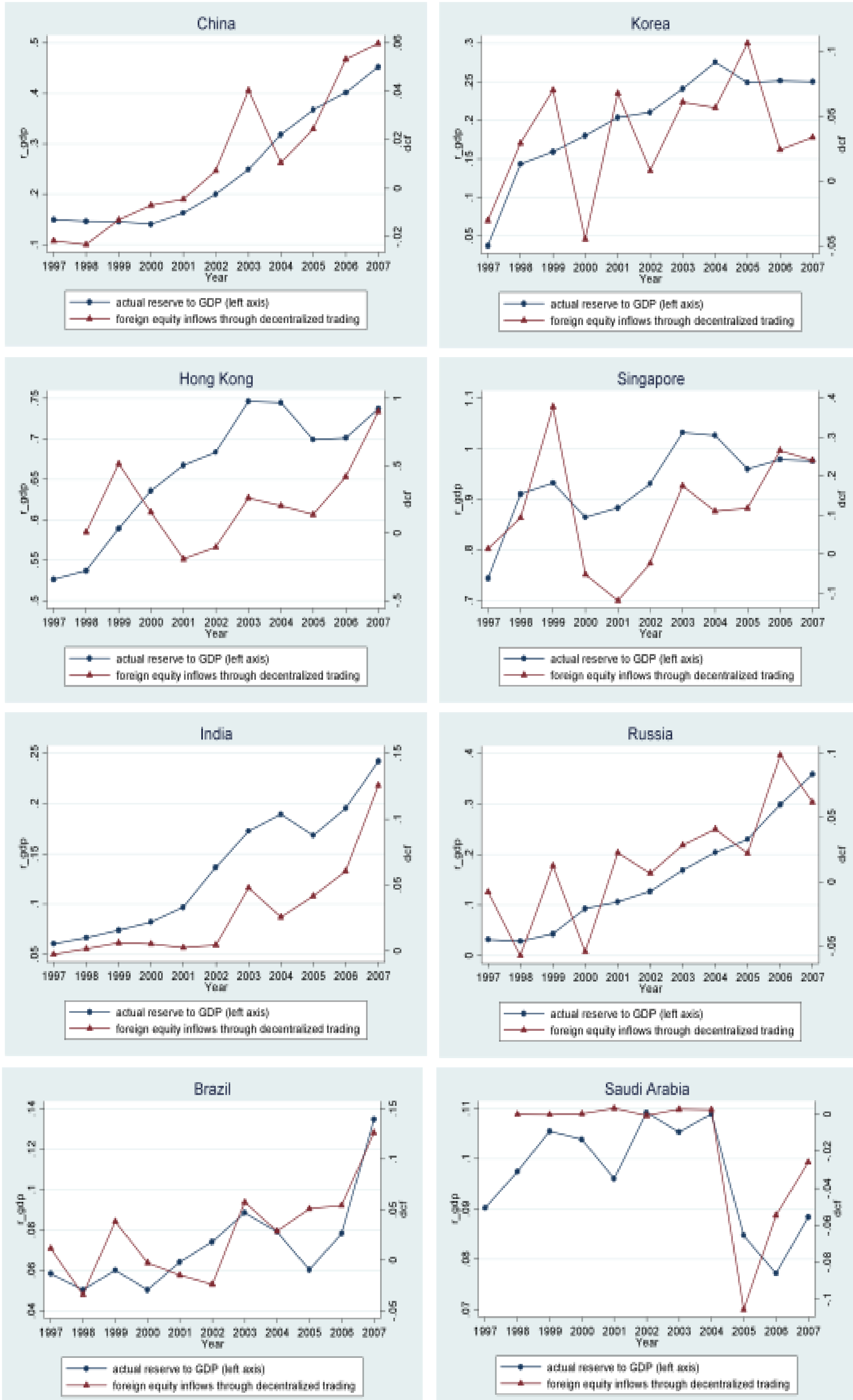


Figure 8: Out-of-Sample Prediction, 2001-2007