A Long-Run Approach to Money, Unemployment, and Equity Prices

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Abstract

We propose a search and matching model that can account for a long-run joint relationship between inflation, unemployment, and equity prices. The model predicts the following long-run joint relationship: (i) a positive relationship between inflation and unemployment; (ii) a negative relationship between unemployment and equity prices; and (iii) a negative relationship between inflation and equity prices. Empirical evidence for this trivariate relationship in the post-WWII US data has been documented in previous studies. We reconfirm this evidence with expanded dataset. Furthermore, our calibration exercises show that the model results driven solely by US monetary policy can account for 62.9% and 29.8% of variations of the long-term trends of US unemployment rate and real equity prices, respectively.

Keywords: Inflation, Unemployment, Equity Prices, Search Models

JEL Classification Numbers: E40, E50, G18

1 Introduction

This paper readdresses one of the seminal questions in macroeconomics: what is the long-run relationship between inflation, unemployment rate, and equity prices? Our approach to answering this question is novel because we concentrate on the *trivariate* relationship between them. Previous research has largely focused on the bivariate relationship between two of the three variables in isolation. We, on the other hand, examine the *trivariate* relationship together in this paper. The merit of our approach seems clear particularly from a monetary policy perspective. Monetary authorities may well have greater confidence simultaneously responding to both equity prices and unemployment if they understand what drives the *trivariate* relationship jointly. This point is imperative because central bankers around the world have recently paid attention not just to unemployment, but also to the relevance of equity prices for macroeconomic stability.

Our focus is also narrowed down to the *long-run* relationship. A vast majority of macroeconomic research is traditionally concerned with how these three variables behave over the business cycle. We do not doubt the importance of this short-run approach. Yet, it is also admittedly true that both empirical and theoretical lessons drawn from this approach tend to be obscure due to factors affecting short-run macroeconomic conditions such as nominal rigidities, asymmetric information, learning, and self-fulfilling prophecies, etc. The long-run approach allows one to abstract away from such factors, and therefore may well be better suited for setting long-term monetary policy goals.

In light of the aforementioned focuses, what we do in this paper is twofold: 1. building a theory of the long-run joint relationship between the three variables, 2. conducting quantitative assessment of the theory. To begin with, we construct a dynamic general equilibrium model that generates a steady-state joint relationship between the three variables. Our model aims to propose a framework allowing for joint analysis of unemployment, money, and equity with parsimony as well as logically consistent microfoundations. To that end, we adopt a model simultaneously incorporating frictional labor and goods market. Others have also worked with microfounded models based on frictions in both labor and goods markets, e.g., Berentsen, Menzio, and Wright (2011) and Williamson (2015). While their models have a single asset, that is, money, our innovation here is to add a second asset, i.e., equity, in a tractable manner so that the joint correlation between money, equity and unemployment can be analyzed.[1]

Our model structure goes as follows. Every agent ought to make an entry choice every period. She could choose to be either a worker, a producer or a consumer. A worker and a producer can jointly produce output that generates no consumption utility for them. In addition, a producer must possess a patent in advance in order to participate in the production activity. Yet, neither a producer

[1] Gomis-Porqueras, Huangfu, and Sun (2020) add capital to Berentsen et al. (2011), but their question and model framework are different from ours. Gabrovski, Geromichalos, Herrenbrueck, Kospentaris, and Lee (2023) add trading frictions in over-the-counter markets in addition to frictional goods and labor markets. Their focus is how inflation-unemployment-asset prices relationship is affected by the degree of liquidity in frictional financial markets.

nor a worker is endowed with any technology to produce the patent. Meanwhile, a consumer can produce a patent and enjoy utility stemming from consuming the output. This gives rise to a trading opportunity between all three of them. For a producer to obtain a patent, we assume she seeks equity financing from consumers and pays a fixed portion of future profits to equity holders. Once the equity issuance becomes successful, a mutually beneficial trade can take place among the three types. However, we impose a crucial assumption that a Walrasian market, where they can frictionlessly trade, does not exist due to anonymity and limited commitment. Thus, they must search for each other to exploit gains from trade. When a successful triplet is formed, a consumer is assumed to pay money to the worker/producer pair in exchange for the output. Following labor and monetary search literature, we assume the three types split the trading surplus through bargaining. Matching is not perfect due to search frictions; hence, equilibrium unemployment naturally arises in the current model as in labor search models.

The key feature of the model is that monetary policy serves as a driving force for the joint relationship between inflation, unemployment and real equity prices. Intuitively, a higher nominal interest rate or inflation reduces real money balances, and therefore generates downward pressure on the amount of output produced within a successfully matched triplet, i.e., an intensive margin effect of inflation. At the same time, a higher inflation distorts the extensive margin of labor supply as well, because a higher cost of holding money leads to a relatively higher portion of workers and producers seeking to be matched with fewer consumers who bear all the money holding costs. As in standard search-based monetary/labor models, both of these effects result in a long-run positive correlation between the inflation rate and unemployment rate.

The novelty here lies in the transmission mechanism of monetary policy on real equity prices. Similar to unemployment, it also works through both intensive and extensive margin effects. A higher inflation lowers real money balances available within each matched triplet whose total trading surplus consequently becomes smaller. This, in turn, depresses real equity prices, i.e., the intensive-margin distortion of a higher inflation on real equity prices. Note that similar channel also exists in other money/labor search-based models. For instance, in the study of Berentsen et al. (2011), a higher nominal interest rate or inflation induces agents to economize on their real balances, which in turn hurts the *total profit* over all firms. However, profit per each identical firm, which is the equivalent of our real equity prices, remains constant because the number of total vacancies falls, which completely offsets the effect of the profit reduction for all firms. That is the extensive margin distortion of firm formation in Berentsen et al. (2011) works exactly opposite way to ours, which in turn causes zero elasticity of real equity prices with respect to inflation.

Thus, it is imperative to emphasize that the negative relationship between real equity prices and inflation (or nominal interest rates) in the steady state is unique to our model. Unlike the study of Berentsen et al. (2011), our triplet-wise matching mechanism induces greater measures of firms to enter the search market in light of a higher nominal interest rate or inflation. This effectively allows our model to have a reduction in aggregate firm profits as well as profit per firm, i.e., equity value, if the central bank tightens the supply of money.

One may concern whether our model predictions are empirically substantive. Our theoretical predictions are summarized by: (i) a positive correlation between unemployment and inflation; (ii) a negative correlation between real stock prices and inflation; and (iii) a negative correlation between unemployment and real stock prices. As a matter of fact, these theoretical predictions have been repeatedly supported by many previous studies as explained in section 1.1. Our online appendix reconfirms these correlations in the context of post-WWII US data. One could also raise potential concerns about whether our proposed mechanism, i.e., the extensive margin fluctuations caused by monetary policy, and its implications for the real equity price are empirically supported. We argue they are indeed empirically substantive. Online appendix documents compelling evidence in the case of the United States that the extensive margin of firm entry, proxied by the US index of net business formation during the postwar period, is positively (negatively) correlated with inflation (real equity price) in the long run, consistent with our model predictions.[2]

The second objective of this paper is to conduct quantitative analysis by calibrating the model and asking to what extent it can account for the observed *trivariate* correlations. We conduct a counterfactual analysis similar to that of Berentsen et al. (2011). Assuming nothing but the monetary policy during the sample period of 1950-2007 was a driving force for unemployment and stock price movements in the US, we ask how well our model can explain the low-frequency or trend movement in unemployment and stock prices. Our results confirm that a considerable fraction of both of the trend movements can be attributed to observed changes in trend inflation and/or interest rates during the sample period. First, except for a few periods of time when non-monetary factors are thought to have played a significant role in driving labor and equity market performances, e.g., relatively higher aggregate productivity growth rates in the 1960s and the late 1990s, our results confirm that monetary policy alone could account for a sizable portion of low-frequency movements in unemployment and real equity prices: The model results driven solely by the US monetary policy can account for 62.9% and 29.8% of variations of the long-term trends of US unemployment rate and real equity prices, respectively. In fact, the model-implied series of unemployment could almost match not just the trend but also the actual levels observed in the data during the early 1980s. As for real equity prices, our counterfactual series could also account for the overall pattern in real equity prices, i.e., a downward trend until 1980 and an upward trend afterward.

1.1 Related Literature

[2] Bergin and Corsetti (2008) also document this positive and unconditional correlation in their figure 1, arguing the feedback effect of monetary policy on business cycle fluctuation as a main reason. On the other hand, our explanation is based on reverse causation, i.e., changes in monetary policy causes business formation.

Many previous studies on the relationships between the three variables exist although most of them focus on the bivariate relationship between two of the three variables in insolation. They also rely on different theoretical models, empirical methodologies, and dataset. First, using post-WWII US data, Beyer and Farmer (2007) show a positive long-run unemployment-inflation relationship based on cointegration analysis. Farmer (2017) goes so far as to argue that the natural rate hypothesis is false by showing that average inflation and unemployment by decade for the US are positively correlated. Moreover, Haug and King (2014) also find the positive long-run unemployment-inflation relationship in the post-WWII US data by applying band-pass filters to the data. Most attempts to rationalize this positive relationship have been based on models explicitly incorporating trading frictions into labor and goods markets, e.g., Berentsen et al. (2011) and Williamson (2015). Central to their ideas is the inflation-distorted extensive and intensive margin of labor supply, which also works in our framework.

A negative correlation between real equity prices and the nominal interest rate in the US data is by no means a new finding either. In fact, this empirical observation has been reported by numerous studies since the late 1970s. For instance, Asness (2000), Sharpe (2002), Lansing (2004), and Bordo and Wheelock (2007) among others documented that the US real equity prices are negatively correlated with US nominal bond yields. This correlation, rather regarded anomalous in the literature, has been dubbed the so-called *Fed Model* among finance practitioners, see Lagos and Zhang (2015) and references therein.[3] While finance literature has traditionally attempted to rationalize this empirical observation, e.g., money illusion story, see Bekaert and Engstrom (2010) and references therein, there is a growing body of search-based literature attempting to investigate the effects of search and matching frictions on asset prices, e.g., see Vayanos (1998), Duffie, Gârleanu, and Pedersen (2005), and Vayanos and Weil (2008). Recent examples include Lagos and Zhang (2015), Geromichalos and Jung (2019), and Geromichalos, Jung, Lee, and Carlos (2021). Common to both studies is that assets exhibit liquidity properties, although the former exploits the model environment where money and assets are complements, whereas the latter assumes the opposite, i.e., money and assets are substitutes.[4] Our model mechanism differs in that we abstract away from liquidity properties of equities, and yet provide a justification for the negative relationship only through extensive and intensive margin effects of labor supply.

[3] The Fed model has been represented in various forms. Some concern the effect of inflation on the price earning ratio, while some others compare nominal interest rate movements with the earnings yield of stocks. We adopt the form taken by Lagos and Zhang (2015), a direct relationship between real equity prices and inflation/nominal interest rate. Moreover, the Fed model may not hold universally across all periods. Gourio and Ngo (2016), among others, point out that the negative relationship between inflation and asset prices might be different during the great recession.

[4] Hu and Rocheteau (2015) also study the relationship between monetary policy and asset prices by taking asset liquidity seriously. Adopting the mechanism design approach, their model justifies a negative correlation between asset prices and inflation as well. For further details on the liquidity based explanation for the relationship between asset prices and interest rates, see Geromichalos, Licari, and Suarez-Lledo (2007), Geromichalos and Herrenbrueck (2016), and references therein.

The long-run negative connection between unemployment and the equity market in the US, has already been shown by many[5]. Much to one's surprise, this observation has turned out to be robust to changes in the periods or the choice of a country. In fact, the positive relationship between share prices (normalized by labor productivity) and the employment rate (one minus the unemployment rate) has been dubbed the *Phelps Curve* in the literature. Phelps (1999) documents that the stock market boom in the 1990s coincided with a fall in the unemployment rate. Moreover, Phelps and Zoega (2001) and Zoega (2009) also document that the *Phelps Curve* prevails in the OECD countries. Farmer (2017) shows strong evidence on the negative link between changes in the unemployment rate and changes in the stock prices in the US. Moreover, Farmer (2017) even argues that the stock market Granger-causes the unemployment rate. Several theories have been proposed to make sense of these negative links, e.g., see Hall (2017) and Rocheteau and Rodriguez-Lopez (2014). By and large, traditional explanations so far have been based on Phelps (1994)'s structuralist model of the natural rate of unemployment: firms' expectations on future profits affect investment and productivity, which in turn have a long-run effect on the labor market.[6] Our explanation differs in that the negative link between equity prices and unemployment is not a result of a causal chain from changes in the equity market to changes in unemployment, but could be potentially interpreted as a consequence of monetary policy effects on both equity prices and unemployment.

The rest of the paper is organized as follows. Sections 2 describes the model. Section 3 solves for the equilibrium. Section 4 conducts the quantitative analysis. Section 5 concludes.

2 The Model

The model builds on Williamson (2015). Time is discrete, and runs forever. Each period is divided into two sub-periods. Time discounting applies only across periods, i.e., $0 < \beta < 1$. There is a unit measure of agents living forever. Each period agent is categorized into three types: workers with a measure of l_t, producers with a measure of f_t, and consumers with a measure of b_t; i.e., $l_t + f_t + b_t = 1$. Moreover, there are two types of perishable goods; a *numeraire* good and a special good. Only consumers are endowed with a technology that turns one unit of labor into one unit of *numeraire* goods. By contrast, the special good by assumption can be produced only by a pair of worker and producer

Two different types of assets exist in this model economy. The first one is a perfectly divisible and non-perishable *fiat* currency or money. The supply of money is stochastically determined by

[5] Similarly, the long-run positive relationship between the labor market tightness and the equity value is also empirically supported by existing studies, e.g., Petrosky-Nadeau, Zhang, and Kuehn (2018).

[6] Recently, Farmer (2012) offers an alternative theory based on a model of multiple equilibria, where the steady-state unemployment rate is not unique, and changes in aggregate demand can permanently shift the equilibrium unemployment rate from one to another.

the monetary authority *via* lump-sum transfers or taxes to agents at the end of each period. To be more precise, the money stock is initially given by $M_0 \in \mathbb{R}_{++}$, and thereafter it grows at a stochastic rate given by γ_t, i.e., $M_{t+1} = \gamma_t M_t$. We assume that γ_t follows a Markov process defined by its transition function $F(\gamma', \gamma) = Pr(\gamma_{t+1} \leq \gamma' | \gamma_t = \gamma)$. Following the standard approach in the literature, we assume that the transition function F has a stationary distribution $\Phi(\cdot)$ and a unique solution to $\Phi(\gamma') = \int F(\gamma', \gamma) d\Phi(\gamma)$, and F has the Feller property. Each period agents can purchase and/or sell any amount of money at the ongoing market price φ_t in the first sub-period. Note that φ_t is a real price of money in units of the time t *numeraire* goods. The lump-sum money transfers are denoted as τ_t; therefore, $\varphi_t(M_t - M_{t-1}) = \tau_t$, $\forall t$.

The second type of asset is equity. In each first sub-period, each producer must acquire one unit of the patent to be able to produce the special good in the second sub-period. The patent is assumed to be perfectly divisible and valid only for one period. However, only consumers have access to technology that allows them to produce a unit of the patent with endogenous labor costs, κ_t units of labor in time t.[7] Plus, only consumers can gain utility from consuming special goods. Given this mutually beneficial trading opportunity for both consumers and producers, we assume that each producer can only issue a unit of equity and sells it at the ongoing market price ψ_t in each first sub-period. Eventually, she must purchase a unit of the patent using the proceeds of an equity sale. This implies that a unit of the patent should be worth of one unit of equity in equilibrium.

Further details on the equity are as follows. Like φ_t, ψ_t represents a real equity price in units of the time t *numeraire* goods. Importantly, this homogeneous equity issued by each producer is a one-period IOU contract promising to pay $(1 - \eta)$ portion of the second sub-period trading surplus at the beginning of the following period $t + 1$. If the producer does not earn any trading surplus because she or he doesn't match with any consumer in the second sub-period, she or he pays nothing to equity holders in the following period. Despite these simplifying assumptions, one could still think of this IOU contract equity, given the one-period living firms. For instance, $(1 - \eta)$ represents the average dividend payout ratio. The fact that only profit generating producers in the model pay out dividends reflects upon the feature of limited liability firms in practice. Lastly, note that $\kappa_t = \psi_t$ in equilibrium. This is because a unit of labor must generate exactly the same real value in terms of *numeraire* goods whether it is used for producing patents, which would create the real value of ψ_t/κ_t, or for producing *numeraire* goods, which would create the real value of 1. Otherwise, the no-arbitrage condition would be violated in equilibrium.

Now, we describe detailed economic activities in each sub-period. In the first sub-period, agents consume the *numeraire* goods. All agents can trade money, equity, and the patent in the Walrasian

[7] One can instead interpret "patent" as "human capital". For instance, one can instead assume that our model requires three inputs factors: labor, technology, and human capital. Only producers possess technology(or knowledge) to combine labor input with human capital, while consumers are assumed to have exclusive rights to produce human capital with labor costs. So, one can essentially think of the term patent in our model as human capital in the endogenous growth model.

or centralized market (CM henceforth) where agents are price takers, and both money and equity markets clear. The real CM price for money and equity at time t are again φ_t and ψ_t respectively. Most importantly, every agent must choose whether to be a worker, a producer, or a consumer in the subsequent second sub-period. In addition, an agent who chooses to be a consumer in the subsequent period carries G_t units of equity.

The second sub-period is characterized by triplet-wise trading in the decentralized market (DM). In this sub-period, three types of agents are simultaneously trying to form a triplet. The total number of such triplets is exogenously determined by a matching function $N(f_t, l_t, b_t)$. This function is assumed to be strictly increasing in all arguments, and exhibit a constant return to scale (CRS) property along with standard properties taken in the literature. Within a successful match worker/producer pair produces y_t units of the special good and hands it over to the consumer in exchange for real money balances Q_t in units of the $t+1$ *numeraire* good. Workers suffer labor disutility cost of $\tau(y_t)$ but receive real money balances W_t in units of the $t+1$ *numeraire* good out of Q_t. Importantly, y_t, Q_t and W_t are determined through a two-stage Kalai (1977) bargaining. α and δ represent exogenously fixed worker's and producer's bargaining power within the negotiation respectively. Note that each must face the same expected payoff to being a worker or a producer or a consumer in equilibrium. Lastly, we adopt a simplifying assumption from Williamson (2015). That is, consumers can share money and equities through contingent contracts such that only matched consumers carry Q_t units of money and G_t units of equity into the DM each period.[8]

The lifetime utility of an agent is given by a linear combination of consumptions and labor disutilities as follows.

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ C_t - h_t - \nu_t + c_t - \tau(c_t) \right\},$$

where C_t denotes consumption of the *numeraire* good, h_t is labor supply for the *numeraire* good production, ν_t stands for total labor supply for the patent production, and c_t and $\tau(c_t)$ are consumption and labor disutility costs in the DM, respectively.

[8] The assumption taken here that the firm-worker pair is one-period-lived and the firm issues the one-period patent may seem at odds with reality. For example, 1-period equity in this model implies a price-dividend ratio of 1, rarely observed in reality even in the long run. However, we chose these simplifying assumptions mainly for two reasons. First, multi-period dividend payout would break the main benefit from this model, tractability. Second and more importantly, our simplifying assumptions are really what differentiates ours from Berentsen et al. (2011). As explained earlier, the extensive margin distortion of firm formation in our model works opposite way to Berentsen et al. (2011), which in turn enables only our model to account for the negative relationship between nominal interest rate and real equity prices. One could instead design a framework in which worker-producer pair and consumer-producer pair are separately formed, which in turn brings about congestion externalities between the two markets, e.g., Veronica, Lorenzoni, Straub, and Werning (2020). This may generate richer implications on the trivariate relationship. We leave this interesting exercise for future research.

3 Equilibrium

Let $\mathbf{s}_t = (\gamma_t, M_t)$ denote the aggregate state of the economy. We solve for a recursive equilibrium in the following way. First, each agent's *ex-ante* expected payoffs among the three alternative activities in the DM must be equalized. This leads to

$$\frac{N(f_t, l_t, b_t)}{b(\mathbf{s}_t)}\left(y(\mathbf{s}_t) - \frac{Q(\mathbf{s}_t)\varphi(\mathbf{s}_t)}{\varphi(\mathbf{s}_{t+1})} + D(\mathbf{s}_t)\right) = \frac{N(f_t, l_t, b_t)}{l(\mathbf{s}_t)}\left(\beta W(\mathbf{s}_t) - \tau(y(\mathbf{s}_t))\right) \tag{1}$$

$$= \frac{N(f_t, l_t, b_t)}{f(\mathbf{s}_t)}\beta\eta\left(Q(\mathbf{s}_t) - W(\mathbf{s}_t)\right), \tag{2}$$

where $D(\mathbf{s}_t) \equiv \psi(\mathbf{s}_t)G(\mathbf{s}_t) - \kappa(\mathbf{s}_t)G(\mathbf{s}_t) = 0$ represents a zero net payoff from holding equities by a consumer in equilibrium. Equation (1) and (2) gives rise to two measures of market tightness defined as follows:

$$\theta(\mathbf{s}_t) = \frac{f(\mathbf{s}_t)}{l(\mathbf{s}_t)} = \frac{\beta\eta[Q(\mathbf{s}_t) - W(\mathbf{s}_t)]}{\beta W(\mathbf{s}_t) - \tau(y(\mathbf{s}_t))}, \tag{3}$$

$$\sigma(\mathbf{s}_t) = \frac{b(\mathbf{s}_t)}{l(\mathbf{s}_t)} = \frac{y(\mathbf{s}_t) - Q(\mathbf{s}_t)[\varphi(\mathbf{s}_t)/\varphi(\mathbf{s}_{t+1})]}{\beta W(\mathbf{s}_t) - \tau(y(\mathbf{s}_t))}, \tag{4}$$

where $\theta(\mathbf{s}_t)$ and $\sigma(\mathbf{s}_t)$ denote labor market and goods market tightness, respectively.

Now, we move on to the bargaining solution in the DM. Following Williamson (2015), we adopt the 2-stage Kalai (1977) bargaining. In the first stage, worker/producer pair decides how to divide $Q(\mathbf{s}_t)$. Applying the Kalai (1977) method, the ratio of two surpluses equals to the ratio of exogenously given bargaining power, given by

$$\frac{\alpha}{\alpha+\delta} = \frac{\beta W(\mathbf{s}_t) - \tau(y(\mathbf{s}_t))}{(\alpha+\delta)[y(\mathbf{s}_t) - \tau(y(\mathbf{s}_t))]} \Rightarrow W(\mathbf{s}_t) = \frac{\alpha y(\mathbf{s}_t) + (1-\alpha)\tau(y(\mathbf{s}_t))}{\beta}. \tag{5}$$

Taking the solution in equation (5) as given, a worker/producer pair negotiates with a consumer regarding the distribution of surpluses. The Kalai (1977) bargaining problem between a consumer and a worker/producer pair should therefore be given by

$$\max_{\{Q(\mathbf{s}_t), y(\mathbf{s}_t)\}} \left\{y(\mathbf{s}_t) - \tau(y(\mathbf{s}_t))\right\}, \text{ st. } y(\mathbf{s}_t) - \beta Q(\mathbf{s}_t) = (1-\alpha-\delta)\left\{y(\mathbf{s}_t) - \tau(y(\mathbf{s}_t))\right\},$$

focusing on the equilibrium where the liquidity constraint is always binding. This gives rise to the following bargaining solution:

$$Q(\mathbf{s}_t) = \frac{(\alpha+\delta)y(\mathbf{s}_t) + (1-\alpha-\delta)\tau(y(\mathbf{s}_t))}{\beta}. \tag{6}$$

Equation (6) can pin down how much real balances consumers should acquire in the CM. Therefore, given equation (6), the maximization problem that a consumer faces in the CM is effectively equivalent to an optimal choice problem for $y(\mathbf{s}_t)$, given as follows:

$$\max_{\{y(\mathbf{s}_t)\}} \left\{ y(\mathbf{s}_t) - \frac{Q(\mathbf{s}_t)\varphi(\mathbf{s}_t)}{\varphi(\mathbf{s}_{t+1})} \right\}, \text{ st. } y(\mathbf{s}_t) - \beta Q(\mathbf{s}_t) = (1-\alpha-\delta)\left\{ y(\mathbf{s}_t) - \tau(y(\mathbf{s}_t)) \right\}.$$

Solving this problem leads to the following first-order condition (FOC):

$$y(\mathbf{s}_t) = \left\{ y(\mathbf{s}_t) : \tau'(y(\mathbf{s}_t)) = \frac{\beta\varphi(\mathbf{s}_{t+1})/\varphi(\mathbf{s}_t) - (\alpha+\delta)}{1-\alpha-\delta} \right\}. \tag{7}$$

Combining equation (5) and (6) together, one could achieve

$$Q(\mathbf{s}_t) - W(\mathbf{s}_t) = \frac{\delta}{\beta}\left[y(\mathbf{s}_t) - \tau(y(\mathbf{s}_t)) \right]. \tag{8}$$

Also, equations (3), (4), (5), (6), and (8) lead to

$$\theta(\mathbf{s}_t) = \frac{f(\mathbf{s}_t)}{l(\mathbf{s}_t)} \equiv \theta = \frac{\eta\delta}{\alpha}, \tag{9}$$

$$\sigma(\mathbf{s}_t) = \frac{b(\mathbf{s}_t)}{l(\mathbf{s}_t)} = \frac{y(\mathbf{s}_t) - \frac{\varphi_t}{\beta\varphi_{t+1}}\tau(y(\mathbf{s}_t))}{\alpha\left[y(\mathbf{s}_t) - \tau(y(\mathbf{s}_t)) \right]} - \frac{\alpha+\delta}{\alpha}\frac{\varphi(\mathbf{s}_t)}{\beta\varphi(\mathbf{s}_{t+1})}. \tag{10}$$

We then apply two market clearing conditions. First, money supply should be equal to money demand in the CM at every state $\mathbf{s}_t$, $\forall t$.

$$\begin{aligned} \varphi(\mathbf{s}_t)M_t &= \frac{Q(\mathbf{s}_t)\varphi(\mathbf{s}_t)}{\varphi(\mathbf{s}_{t+1})} N(l(\mathbf{s}_t), f(\mathbf{s}_t), b(\mathbf{s}_t)) \\ &= \frac{(\alpha+\delta)y(\mathbf{s_t}) + (1-\alpha-\delta)k(y(\mathbf{s_t}))}{\beta} \frac{N(1,\theta,\sigma(\mathbf{s_t}))}{(1+\theta+\sigma(\mathbf{s_t}))} \frac{\varphi(\mathbf{s_t})}{\varphi(\mathbf{s_{t+1}})}, \end{aligned} \tag{11}$$

where the second equality follows from equation (6). Second, the total real value of equity supply should also equal to the total real value of equity demand in the CM at every state $\mathbf{s}_t$, $\forall t$.

$$\psi(\mathbf{s}_t) f(\mathbf{s}_t) = N(f(\mathbf{s}_t), l(\mathbf{s}_t), b(\mathbf{s}_t)) G(\mathbf{s}_t) \frac{N(f(\mathbf{s}_t), l(\mathbf{s}_t), b(\mathbf{s}_t))}{f(\mathbf{s}_t)} \beta(1-\eta)\left[Q(\mathbf{s}_t) - W(\mathbf{s}_t) \right].$$

Note that $N(f(\mathbf{s}_t), l(\mathbf{s}_t), b(\mathbf{s}_t))$ equals the total number of equity holders in time t, i.e., a total number of matched consumers in time t. Thus, $N(f(\mathbf{s}_t), l(\mathbf{s}_t), b(\mathbf{s}_t))G(\mathbf{s}_t)$ represents total units of equities demanded that must be equal to $f(\mathbf{s}_t)$ in equilibrium. $N(f(\mathbf{s}_t), l(\mathbf{s}_t), b(\mathbf{s}_t))/f(\mathbf{s}_t)$ denotes the probability that an equity pays out the dividend. Lastly, $\beta(1-\eta)\left[Q(\mathbf{s}_t) - W(\mathbf{s}_t)\right]$ represents the time t real value of dividends actually paid out to an equity holder at the beginning of time

$t+1$. Combining with equation (8), an equilibrium equity pricing formula is given by

$$\psi(\mathbf{s}_t) = (1-\eta)\delta\{y(\mathbf{s}_t) - \tau(y(\mathbf{s}_t))\}N\left(1, \frac{1}{\theta}, \frac{\sigma(\mathbf{s}_t)}{\theta}\right). \tag{12}$$

Finally, a recursive equilibrium is a list of time-invariant functions of the aggregate state $\theta(\mathbf{s})$, $\sigma(\mathbf{s})$, $\varphi(\mathbf{s})$, and $\psi(\mathbf{s})$ such that equations (9), (10), (11), and (12) are all satisfied. Two market tightness measures along with the real money price in recursive equilibrium, i.e., equations (9), (10), and (11), have essentially similar interpretation as in Williamson (2015). What is novel here is the equilibrium equity pricing formula in eq (12). The effects of dividend payout ratio and bargaining power, i.e., $(1-\eta)$ and δ, on a real equity price are standard. A higher dividend payout ratio should lead to a higher real equity price. A higher total surplus generated within the triplet, i.e., $y(\mathbf{s}_t) - \tau(y(\mathbf{s}_t))$ would also put an upward pressure on the real equity price. Lastly, a higher matching probability for a producer, i.e., $N(1, 1/\theta, \sigma(\mathbf{s}_t)/\theta)$, would increase the real equity value too.

Unemployment and vacancy rates in recursive equilibrium can also be obtained easily as follows.

$$u(\mathbf{s_t}) = 1 - N\left(\theta, 1, \sigma(\mathbf{s_t})\right) \quad v(\mathbf{s_t}) = 1 - N\left(1, \frac{1}{\theta}, \frac{\sigma(\mathbf{s_t})}{\theta}\right).$$

Before we state our final analytical results of the paper, i.e., the effects of inflation or nominal interest rate on unemployment and equity prices in the long run, we first need to characterize inflation and nominal interest rate in recursive equilibrium. The nominal price of a *numeraire* good is then $1/\varphi(\mathbf{s}_t)$. Then, an actual gross inflation rate in terms of the *numeraire* good between t and $t+1$, i.e., π_t, are defined as

$$\pi_t(\mathbf{s_{t+1}} = \mathbf{s}', \mathbf{s_t} = \mathbf{s}) = \frac{\varphi(\mathbf{s})}{\varphi(\mathbf{s}')}.$$

One can also define expected gross inflation as the rate at which the money price of a *numeraire* good, conditional on the information available at $\mathbf{s}_t$, changes. Denote the expected gross inflation rate at state $\mathbf{s}_t$ as $\tilde{\pi}_t$, which can be defined as follows:

$$\tilde{\pi}_t(\mathbf{s_t}) = \int \pi_t(\mathbf{s_{t+1}} = \mathbf{s}', \mathbf{s_t} = \mathbf{s}) d\boldsymbol{F}(\mathbf{s}', \mathbf{s}).$$

where $\Pr(\mathbf{s}_{t+1} \leq \mathbf{s}' \mid \mathbf{s}_t = \mathbf{s}) = \mathbb{I}_{\{\gamma M \leq M'\}} F(\gamma', \gamma) \equiv \boldsymbol{F}(\mathbf{s}', \mathbf{s})$. Also let $\mathbf{\Psi}$ be the associated stationary distribution, i.e., let $\mathbf{\Psi}$ be the unique solution to $\mathbf{\Psi}(\mathbf{s}') = \int \boldsymbol{F}(\mathbf{s}', \mathbf{s}) d\mathbf{\Psi}(\mathbf{s})$.

Lastly, the gross nominal interest rate between t and $t+1$ is defined as R_t. The *Fisher effect* then implies that βR_t must equal $\tilde{\pi}_t(\mathbf{s_t})$ in equilibrium. Then, the following proposition reveals one important equilibrium property regarding the effect of nominal interest rate on unemployment.

Proposition 1 $\partial u(\mathbf{s}_t)/\partial R_t > 0$. *and* $\partial u(\mathbf{s}_t)/\partial \tilde{\pi}_t(\mathbf{s}_t) > 0$.

Proof. The proof is straightforward from equations (7) and (10). ■

This equilibrium property states that a higher expected inflation and/or nominal interest rate leads to an increase in unemployment, which is a common result across a wide class of monetary-search models including that of Berentsen et al. (2011) and Williamson (2015). Intuitively, when anticipated inflation and/or nominal interest rate increases, fewer agents would choose to be a consumer due to higher real costs of carrying money. This, in turn, would lead to the less tighter goods market condition, i.e., $\sigma(\mathbf{s}_t)$ would fall as a result of a rise in R_t. Eventually, a matching probability for a worker would fall, i.e., $N(1, \theta, \sigma(\mathbf{s}_t))$ decreases, leading to a higher equilibrium unemployment rate.

The novelty in this paper is the equilibrium property on the negative effect of anticipated inflation and/or nominal interest rate on equity prices, shown in the following proposition.

Proposition 2 $\partial \psi_t(\mathbf{s}_t)/\partial R_t < 0$. *and* $\partial \psi_t(\mathbf{s}_t)/\partial \tilde{\pi}_t(\mathbf{s}_t) < 0$.

Proof. Proof is straightforward from equations (10) and (12). ■

Proposition 2 works through both extensive and intensive margin channels. First, a higher anticipated inflation and/or nominal interest rate would induce agents to choose to be a consumer to a lesser extent due to higher holding costs of money. As a result, an increase in producer/worker pair measures loosens the goods market tightness, i.e., $\sigma(\mathbf{s}_t)$ falls from equation (10). In turn, it would result in a lower matching success rate for producers, i.e., $N\left(1/\theta, 1, \sigma(\pi(\mathbf{s_t}))/\theta\right)$ falls as seen in equation (12). This effect would eventually lower real equity prices, i.e., the extensive-margin distortion of a higher nominal interest rate on real equity prices. The negative relationship between the nominal interest rate/expected inflation and real equity prices in this model is also driven by the intensive-margin effect. A higher nominal interest rate would lower the amount of real balances each consumer carries over to the DM, i.e., $y(\mathbf{s}_t)$ would fall in response to a higher nominal interest rate. As a result, the total DM trading surplus within each triplet should fall, which would in turn lower the real benefit each matched producer generates. This would consequently cause the real equity price to fall, materializing the intensive-margin distortion effect of a higher nominal interest rate.

4 Quantitative Analysis

Section 3 shows our model framework generates a theoretical joint relationship between inflation and/or interest rate, unemployment rate, and equity prices, consistent with US long-run data found in previous studies. The model is very simple, and works through a standard real money balance and a labor market search friction channel. Now, we show that our model can also account for sizable variations of the long-run trends of US unemployment rate and real equity prices during the post-WWII period quantitatively, assuming that the driving force is only monetary policy as in Berentsen et al. (2011). For this quantitative work, we specify a process for R_t as a mean-reverting stochastic process given by[9]

$$R_{t+1} = \bar{R} + \rho(R_t - \bar{R}) + \varepsilon_t,\ \ \varepsilon_t \sim N(0, \Omega) \tag{13}$$

4.1 Calibration

In the benchmark calibration, we set the period length equal to one quarter. Time preference is described by the discount factor β. We assume a simple quadratic functional form of the labor disutility cost $\tau(y_t) = \gamma y_t^2$, where $\gamma > 0$. The matching function takes a simple constant-return-to-scale (CRS) form $N(f_t, l_t, b_t) = A f_t^{(1-\zeta)/2} l_t^{(1-\zeta)/2} b_t^{\zeta}$, where $0 < \zeta < 1$. A stochastic monetary policy is governed by $(\rho, \bar{R}, \Omega)$. The dividend payout ratio is $(1-\eta)$. α and δ represent a bargaining power of a worker and producer respectively; therefore $\Lambda = \alpha + \delta$ refers to the combined bargaining power of a worker and a producer in a triplet.

β is set such that $1/\beta$ equals the average annual real interest rate where the latter in each year is calculated as the difference between Aaa bonds and realized inflation as in Berentsen et al. (2011). We set η so that $1-\eta$ matches with the average annual dividend payout ratio in the United States, collected from Shiller (2015). $\bar{R}$ is set through the OLS estimation of equation (13). We adopt the same approach as Berentsen et al. (2011) to calibrate Λ. The latter is set such that the markup in DM matches with the retail data used by Faig and Jerez (2005), that is, a target markup of 30 percent. Note that the markup e in the DM is given by

$$\begin{aligned} e =& \frac{\text{Nominal price of a DM good}}{\text{Nominal marginal labor disutility}}, \\ =& \frac{M_t/y_t}{\tau'(y_t)/\varphi_t}, \\ =& \bar{R}\Big[\frac{\Lambda}{\big(1/\bar{R} - \Lambda\big)/1 - \Lambda} + \frac{1-\Lambda}{2}\Big], \end{aligned}$$

where the third equality comes from equations (6) and (7). Thus, we set Λ so that $e = 1.3$. We also set α and δ such that θ, which equals $\eta(\Lambda - \alpha)/\alpha$ from equation (9), matches with the average ratio of vacancies to unemployment between 2001 and 2007, given by *the Federal Reserve Bank of St.Louis*(FRED).

For the DM cost parameter γ, we follow the standard approach taken by the monetary economics

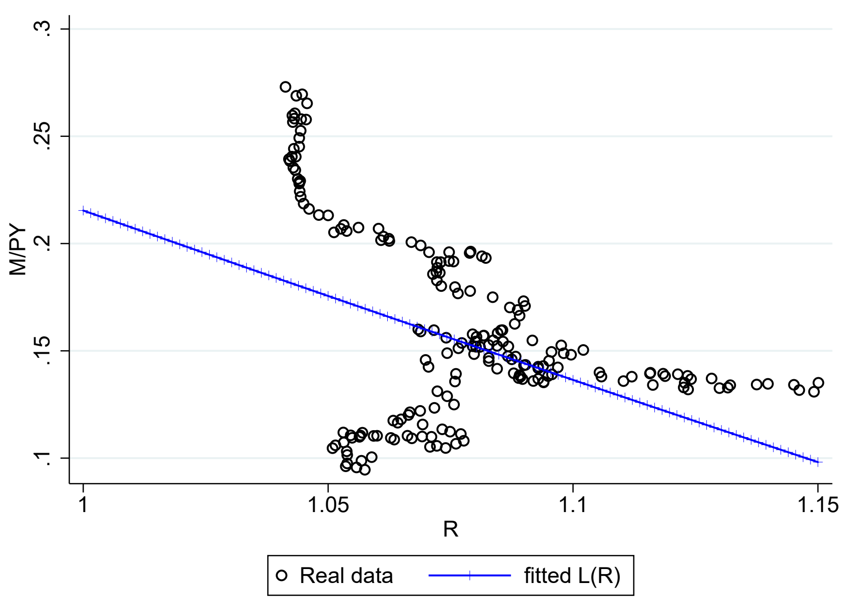
[9] One could instead specify a stochastic process for money growth rate γ_t, and use this for calibration. However, the majority of recent monetary policy literature focuses on the nominal interest rate rule as a major monetary policy instrument. Hence, our calibration exercise only focuses on the interest rate process as in Berentsen et al. (2011).

literature. We set γ so that the relationship between $L(R) \equiv M/PY$ and nominal interest rate i in the model matches the data. The real value of special goods produced at time t equals $Q_t(\varphi_t/\varphi_{t+1})N(f_t, l_t, b_t)$, whereas real patent output in CM at time t equals $\psi_t G_t N(f_t, l_t, b_t)$. Note that real *numeraire* good consumption by consumers is indeterminate due to a linear CM utility assumption. We denote it as B_t. The total real output Y_t is then $Q_t(\varphi_t/\varphi_{t+1})N(f_t, l_t, b_t) + B_t b_t + \psi_t G_t N(f_t, l_t, b_t)$. The model implies that

$$
\begin{aligned}
L(R_t) &= \frac{Q_t(\varphi_t/\varphi_{t+1})N(f_t, l_t, b_t)}{Q_t(\varphi_t/\varphi_{t+1})N(f_t, l_t, b_t) + B_t b_t + \psi_t G_t N(f_t, l_t, b_t)}, \\
&= \frac{Q_t(\varphi_t/\varphi_{t+1})N(f_t, l_t, b_t)}{Q_t(\varphi_t/\varphi_{t+1})N(f_t, l_t, b_t) + B_t b_t + \beta(1-\eta)[\delta/(\alpha+\delta)]Q_t(\varphi_t/\varphi_{t+1})N(f_t, l_t, b_t)}, \\
&= \frac{(\alpha+\delta)y(R_t) + (1-\alpha-\delta)\gamma[y(R_t)]^2}{\bar{B} + (1+\beta(1-\eta)(\delta/(\alpha+\delta)))\left[(\alpha+\delta)y(R_t) + (1-\alpha-\delta)\gamma[y(R_t)]^2\right]},
\end{aligned}
\tag{14}
$$

where $y(R_t) = (1/R_t - \Lambda)/(2\gamma(1-\Lambda))$ and $\bar{B} = B_t b_t(\beta\varphi_{t+1}/(\varphi_t N(f_t, l_t, b_t))$, which is assumed to be a constant for parsimony. Note that the second equality in equation (14) comes from equations (8) and (6), while the third one is based on equation (7). Our approach is to fit equation (14) to match the actual US data on M1 to GDP ratio by choosing $\bar{B}$ and γ. We set the latter to match the average M/PY and empirical elasticity of M/PY with respect to R_t simultaneously. The result is shown in Figure 1, which generates a reasonable fit with our calibrated parameters, $\bar{B} = 7.1588$ and $\gamma = 0.2114$.

Figure 1: The "money demand" data and fitted $L(R)$



The triplet matching function adopted in the current model is not common; hence, empirical estimates on matching efficiency parameter A and matching elasticity with respect to the measure of consumers ζ have been rarely provided in the literature.[10] Our strategy here is to simultaneously set $\{A, \zeta\}$ to satisfy two conditions: (i) the average US unemployment during the post-WWII period; and (ii) the minimum unemployment rate in the model is bounded below by zero, i.e., $u = 0$ when $R = 1$.

Table 1 summarizes the calibration targets and data sources. Calibrated parameters are summarized in Table 2.

Table 1: Calibration targets

Description	Value	Sources
Average real interest rate	0.031	FRED
Average dividend payout ratio	0.4877	Shiller (2015)
Average nominal interest rate	0.0703	FRED
Average money demand M/PY	0.16	FRED
Elasticity of M/PY w.r.t. R_t	-4.434	FRED
Average unemployment rate	0.056	FRED
Average vacancy to unemployment ratio	0.522	FRED
Retail sector markup	0.3	Faig and Jerez (2005)

Table 2: Parameter values

Description		Values
Discount factor	β	0.9695
Dividend payout ratio	η	0.5123
Average nominal interest rate	$\bar{R}$	0.0698
Weight on the matching function	A	1.4187
Matching elasticity	ζ	0.107
Weight on the cost function	γ	0.2114
Consumer bargaining power	1-Λ	0.2212
Worker bargaining power	α	0.3857
Producer bargaining power	δ	0.3930

4.2 Results

Based on the calibrated parameters in Table 2 and the actual time series for R_t between 1955Q1 and 2007Q3 , we compute the model-implied path for unemployment rate, u and real stock prices, φ. Following Berentsen et al. (2011), we focus on the low-frequency movement of unemployment rate and real equity prices by Baxter and King (1999) filtering. Thus, the results show our counterfactual prediction on the trend unemployment rate and real equity prices in the absence of any

[10] Recent studies have started to estimate U.S. matching efficiency in various sample periods, which makes long-run calibration of matching efficiency possible. One could adopt this methodology to improve the quantitative performance of our model. We leave this fruitful exercise for future research.

macroeconomic shocks except for monetary policy over the sample period. Data sources for actual R_t, unemployment and real equity prices used in this section are given in the online appendix.

Figure 2: Counterfactual unemployment

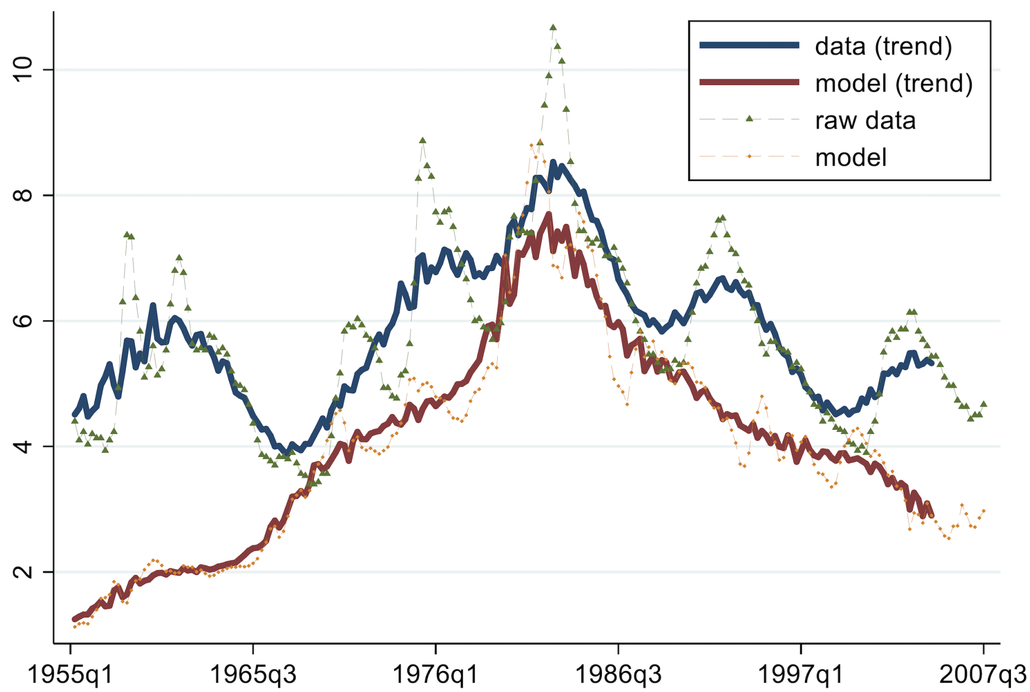


Figure 2 shows the filtered and raw time series of actual and counterfactual movements in u. First, the model implied u appears to be always lower than the actual u in data. This seems natural given many other extra factors such as productivity and unemployment benefits, causing unemployment. Second, although the model implied unemployment does not appear to capture all movements in actual unemployment during the sample period, Figure 2 shows a similar trend between the two series. The long term trend of u predicted by the model accounts for 62.9 percent of variations of the long term trend of the real data unemployment rate. In particular, the upward trend in unemployment rate until the mid-1980s and the downward trend afterward are well captured by the model. Changes in R alone can account for 76.5 percent of variations in u (trend) between 1968q1 and 1982q4. Furthermore, 74.2 percent of variations in u (trend) between 1983q1 and 1992q4 are also captured by changes in R alone according to Figure 2.[11] We also find that the model results driven by changes in R (without the filtering) account for 48.1 percent of variations in u between 1968q1 and 1982q4, and 22.5 percent of variations in u between 1983q1 and 1992q4. Figure 3 shows the scatter plot of actual (blue) and counterfactual (black) R versus u (trend). The correlations generated by the model and the data appear to be very similar as well.

[11] It appears that the actual and counterfactual series move in the opposite direction only in the 1960s. This was also detected in Berentsen et al. (2011), which could be attributed to many other non-monetary factors, see Berentsen et al. (2011) for details.

Figure 3: Interest Rate and Unemployment

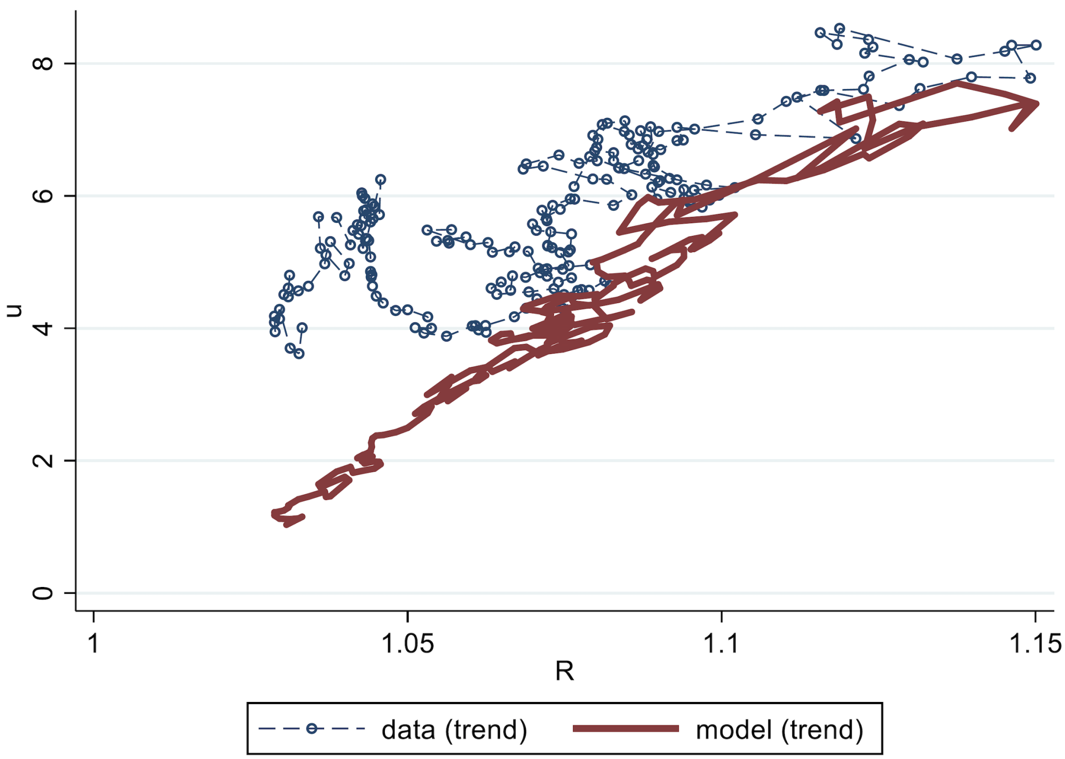


Figure 4: Counterfactual Real Equity Prices

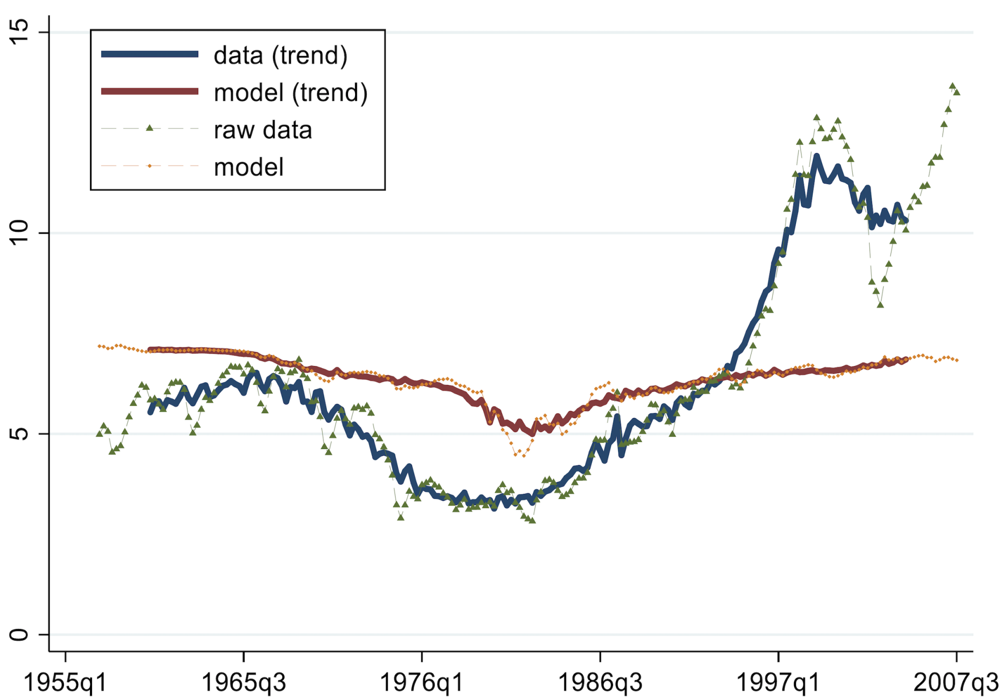
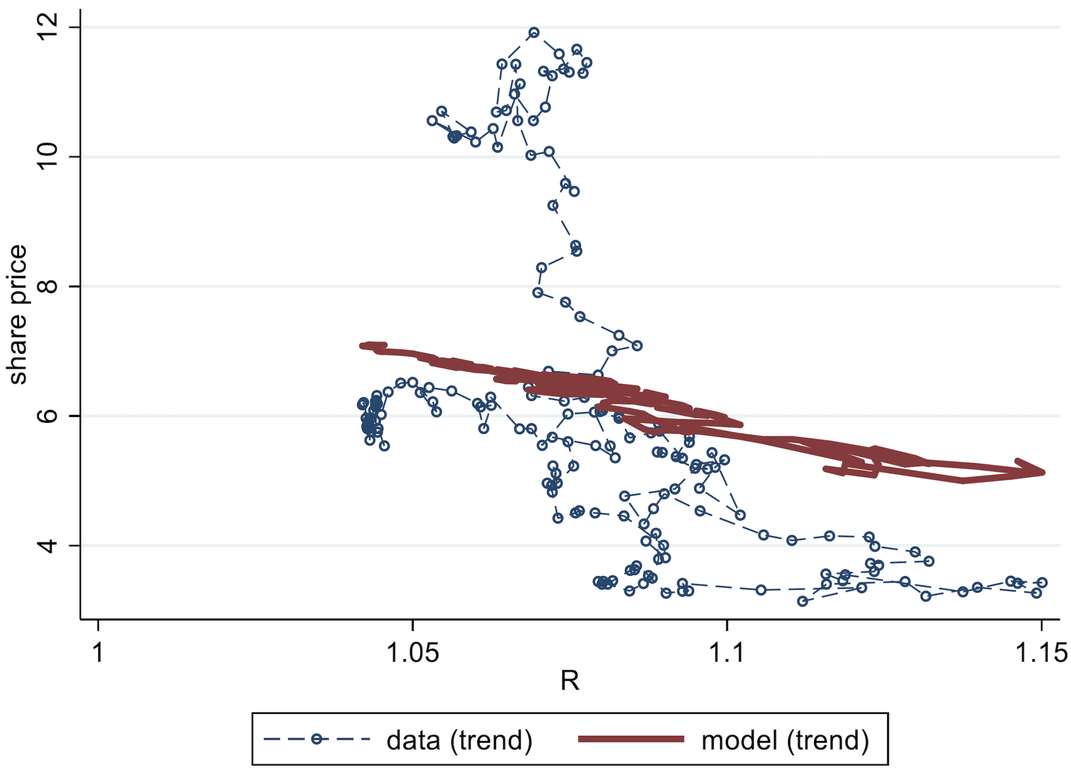


Figure 4 illustrates the results with respect to real equity prices, φ. The blue line plots the actual trend in φ, whereas the red one shows the model implied trend. Units differ between the

Figure 5: Interest Rate and Real Equity Prices



two series; thus, we adjust the mean from the model results to the mean of real stock prices. Similar to the unemployment case, the model implied φ does not fully capture all the movements in actual φ. For example, the two series move in the opposite direction during much in the 1960s; that is, actual data shows an upward trend, while the model shows a downward pattern during the 1960s. We find this pattern interesting, because this divergence also appears for u during the 1960s as witnessed in Figure 2. One factor that could potentially account for both of these two patterns is productivity. The United States might have experienced a strong productivity growth in the 60s, which simultaneously boosted equity prices and lowered unemployment rate. Despite this drawback, our model can capture long-term trends in real equity prices.

The model generates the overall decline in φ between 1955 and the mid-1980s, which coincides with the actual data. A subsequent rise in the actual trend for φ after the mid-1980s is also well captured by our model. Specifically, the long term trend of φ predicted by the model accounts for 29.8 percent of variations of the long term trend of real stock prices. Note that the magnitude of the increase in actual φ appears to be greater than what our model predicts. Yet, we do not think this discrepancy is necessarily a drawback of our model, because our model attempts to explain the long-term trend movement in φ only through changes in R. Obviously, the productivity growth since the 1990s is thought to have played a significant role in boosting equity prices on the same period. Our model does not take into account this real productivity factor. Figure 5 repeats the scatter plot of Figure 3 with real equity prices replacing unemployment rate. The model appears to generate the negative correlation between φ and R similar to the actual correlation between the two variables.

4.3 Robustness

As a robustness check, we also performed a calibration using annual data between 1950 and 2007. Table 3 summarizes the calibration targets and data sources for the annual data. Calibrated parameters are summarized in Table 4. The money demand result based on calibrated parameters using the annual data is shown in Figure 6, which generates a reasonable fit.

Table 3: Calibration targets

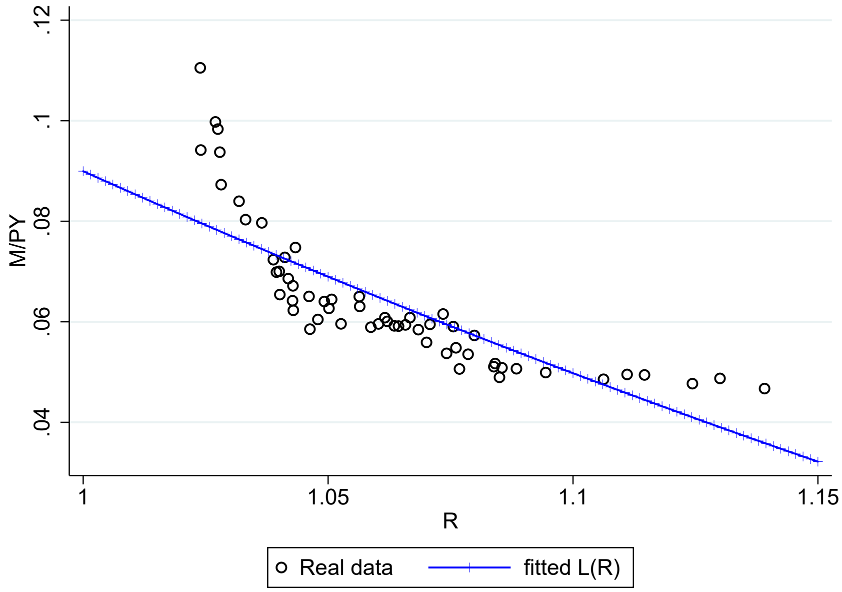
Description of Targets	Value	Sources
Average annual real interest rate	0.02607	Jorda, Schularick, and Taylor (2017)
Average annual dividend payout ratio	0.4934	Shiller (2015)
Average annual nominal interest rate	0.0624	Jorda et al. (2017)
Average annual money demand M/PY	0.06404	Jorda et al. (2017)
Elasticity of M/PY w.r.t. R_t	-6.915	Jorda et al. (2017)
Average annual unemployment rate	0.0562	FRED
Average annual vacancy to unemployment ratio	0.5237	FRED (2001-2007)
retail sector markup	0.3	Faig and Jerez (2005)

Table 4: Parameter Values

Description of Parameters		Values
Discount factor	β	0.9746
Dividend payout ratio	η	0.5066
Average nominal interest rate	$\bar{R}$	1.0624
Weight on the matching function	A	1.4328
Matching elasticity	ζ	0.1080
Weight on the cost function	γ	1.5986
Consumer bargaining power	1-Λ	0.2028
Worker bargaining power	α	0.392
Producer bargaining power	δ	0.4052

Figures 7 and 8 plot the annual data counterparts for figures 2 and 3 respectively. The basic pattern remains similar between the model and data even at an annual frequency. In fact, the model fitness became better. Figure 7 shows that the u predicted by the model becomes almost identical to the actual u in the first half of the 1980s. That is, our model implies that changes in R alone could account for almost the entire movements of u in the data during the early 1980s. The scatter plot of actual (blue) and counterfactual (red) R versus u in Figure 8 also improves at the annual frequency as seen in Figure 3. The counterfactual real asset prices also appear to perform well at the annual frequency (e.g., Figures 9 and 10).

Figure 6: The “money demand” using annual data and fitted $L(R)$



Summing up, our calibration exercises show that money could have been a major driving force behind a significant portion of movements in trend u and φ in the US over the last half century. In this sense, our results reconfirm Berentsen et al. (2011)'s claim that money could be more important in driving labor market conditions than previously thought. We also add a dimension to the claims of Berentsen et al. (2011), that is, the effects of monetary policy on real equity prices. We show that money may have been responsible for low-frequency movements in real equity prices, which so far has been relatively neglected in the literature. Finally, our model also shows that it could capture not only the low-frequency movements between money, unemployment, and real equity prices in the US, but also the actual magnitude of such movements, that is, the model implied u series in the early 1980s.

5 Conclusion

In this paper, we readdress one of the most important questions in macroeconomics. Is there a joint relationship between inflation, unemployment, and real equity prices in the long run? For this end, we develop a dynamic general equilibrium theory that can explain the steady-state joint relationship between the three variables. Our model takes trading frictions both in labor and goods markets seriously. By integrating these search and matching frictions in both markets within an integrated framework, our model shows clear predictions on how the three variables should be correlated in the long run. Through the extensive margin effect of inflation on labor supply, the model predicts a positive correlation between inflation and unemployment rates as in data. With

Figure 7: Counterfactual unemployment

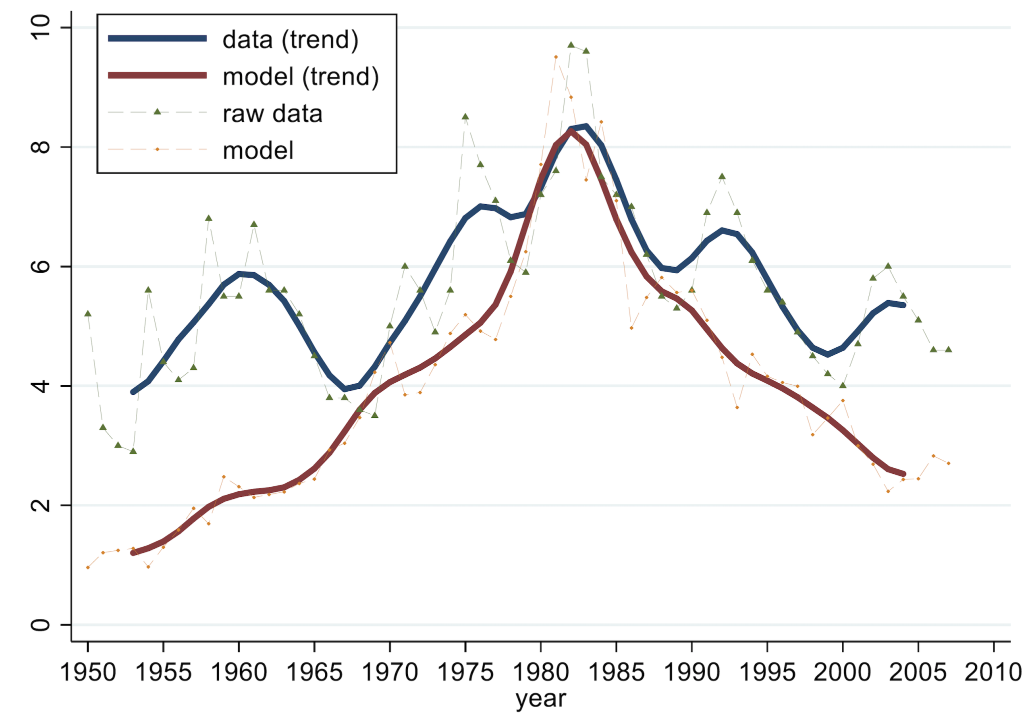


Figure 8: Interest rate and unemployment

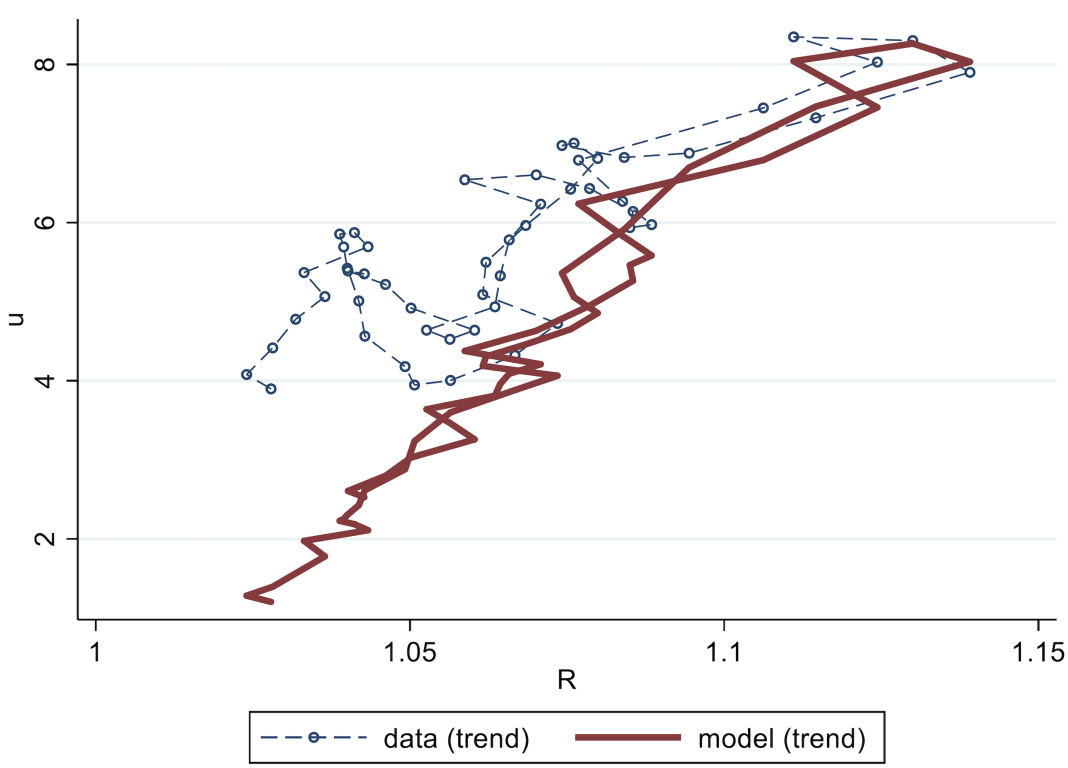


Figure 9: Counterfactual real equity prices

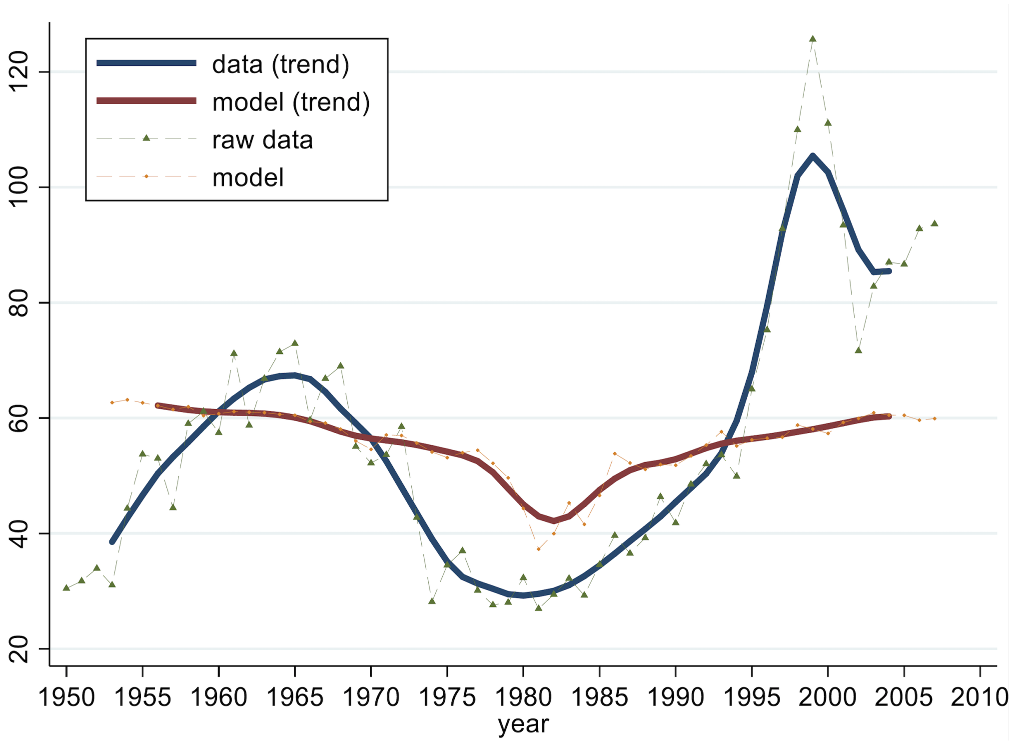
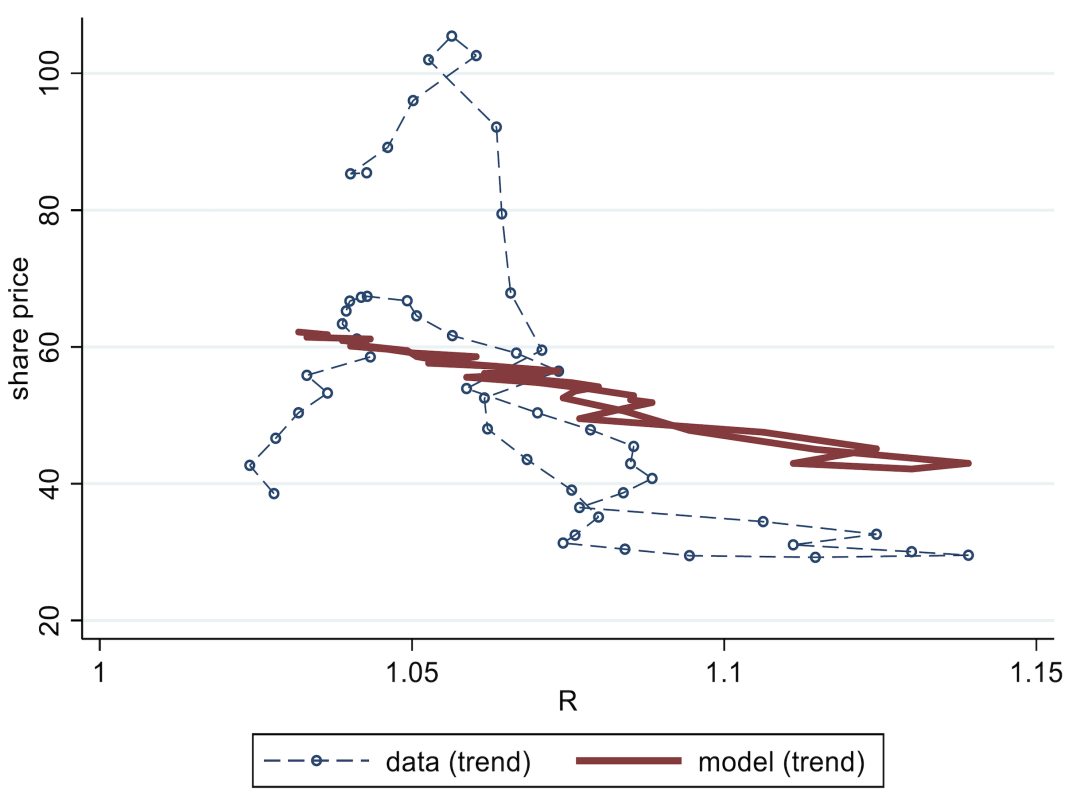


Figure 10: Interest rate and real equity Prices



respect to real equity prices, both intensive as well as extensive margin effects of inflation turn out to negatively affect the real equity prices. These predictions are well supported by US data, already illustrated by many previous studies. We reconfirm these findings with expanded dataset in the online appendix. Lastly, we also examined whether our model can account for the long-run behavior of unemployment and real equity prices in the US, assuming that the sole driving force is monetary policy, that is, changes in nominal interest rates. We found that our model can considerably elucidate the US long-run behavior of unemployment and real equity prices.

Our results shed new light on the long-run relationship between these three essential macro variables even in the current post-pandemic era. During COVID-19, central bank liquidity injections led to the return of global inflation. Now is the journey back to normal. While weathering through short-term negative consequences of monetary policy during this journey matters, long-term monetary policy effects should never be underrated. This is because history shows that sudden shift in monetary policy direction has far reaching consequences on long-term economic performances of real sectors, e.g., the Great depression, the Great disinflation of 70s, and etc. Our analysis could help monetary policy makers achieve a better long-run goal by revealing the role of search and matching distortions in goods/labor markets on the long-run relationship among three variables. Recently, others have started considering the long-run effects of monetary policy through other channels, e.g., Jorda, Singh, and Taylor (2020), Baqaee, Farhi, and Sangani (2021), and Meier and Reinelt (2020). We hope our study can help warrant further research on this topic.

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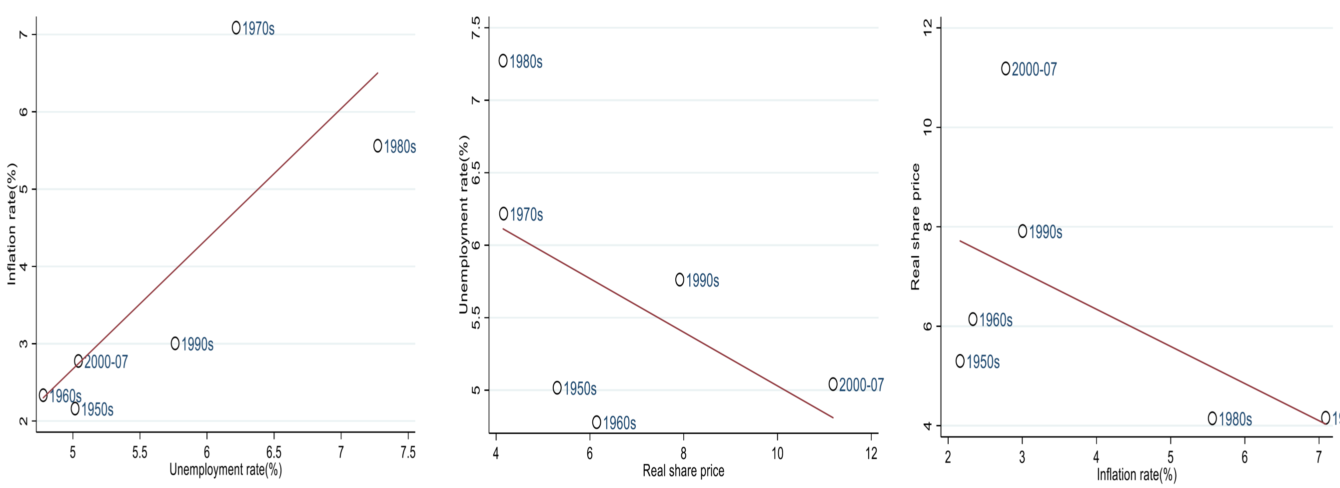
Online Appendix (not for publication)

This appendix illustrates empirical evidence that US inflation, unemployment, and real stock prices are closely related in the long run during the sample period 1950-2007. First, we show our analysis by computing 10-year averages of three variables and describing the correlations among them. Then, we decompose individual variables into the long-term trend and short-term fluctuations, and examine whether their long term trends show any systematic relationships. Finally, we use a cointegration test to check whether a long run relationship exists between variables, and estimate cointegration vectors.

We collect US quarterly inflation rate and unemployment rate from the FRED, a quarterly nominal share price index from the OECD statistics over the post WW-II period (from 1957q1 to 2007q3). We also limit our data by 2007 to avoid the global financial crisis which might have a structural shock on three variables. For the real stock price, we normalize nominal share prices by GDP per workers which is the proxy for labor productivity (e.g., Zoega (2009)). For the robustness of the result, we employ the US annual data collected from Jorda et al. (2017).

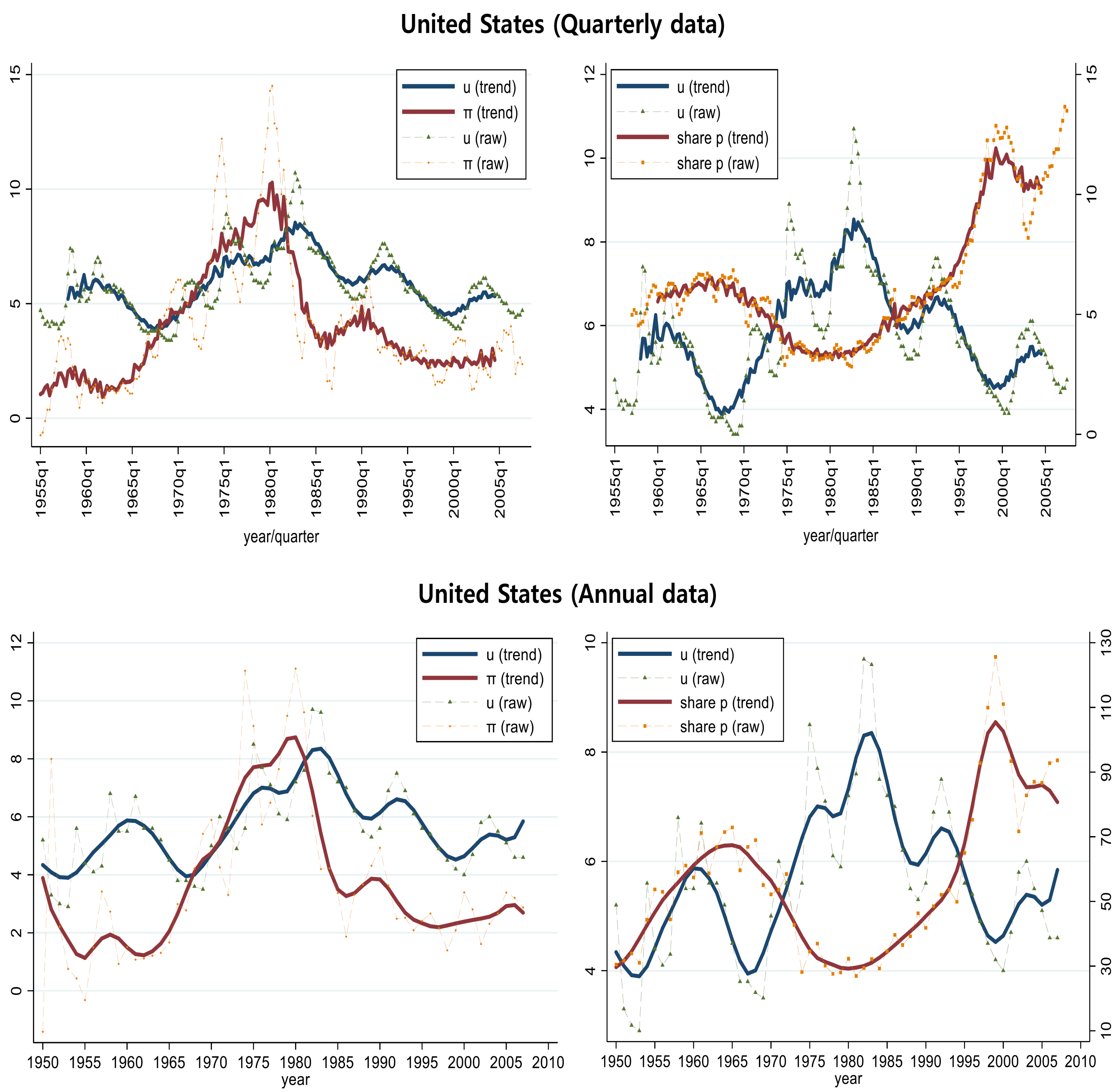
Figure 11: The 10-year average rates of unemployment, inflation, and real stock prices



This figure shows a bilateral relationship of 10-year averages for the inflation rate, unemployment rate, and real stock prices for the period 1950-2007. Red lines indicate the fitted lines.

Figure 11 shows the scatter plots for the 10-year averages of the US inflation rate, unemployment rate, and real stock prices. By calculating the mean of three variables by decade, we focus on the long term changes of three variables and their correlations. Specifically, we put two variables in the separate panels and draw the fitted line for 10-year averages of the two variables. From the first panel to the third panel, it is clear that the scatter plots document a positive relationship between inflation rate and unemployment rate (Beyer and Farmer (2007); Berentsen et al. (2011); Haug and King (2014)), a negative relationship between unemployment and real equity prices (Phelps and Zoega (2001); Zoega (2009)), and a negative relationship between inflation rate and real equity prices.

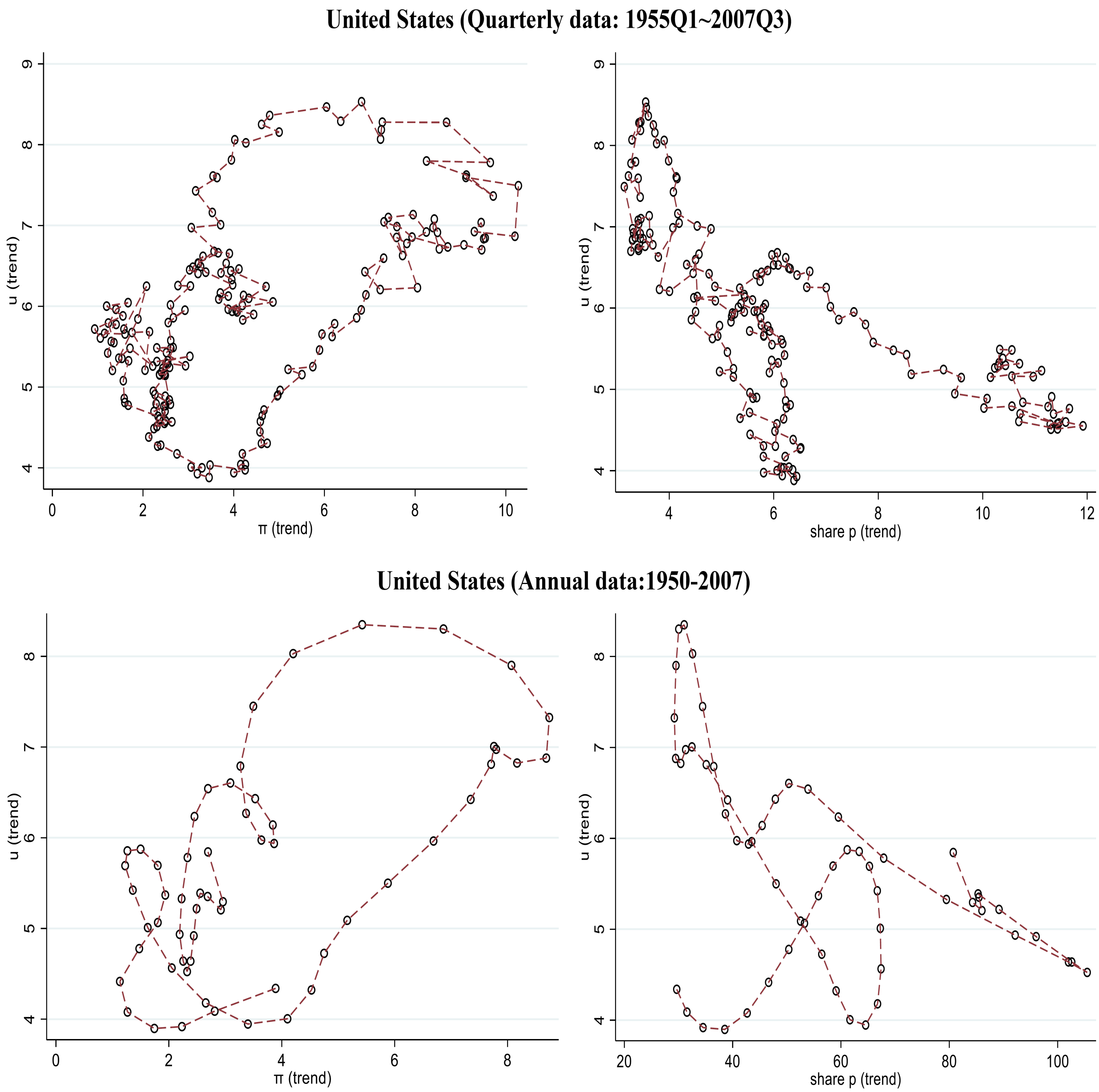
Figure 12: Long term trends using Baxter and King's (1999) filter



This figure shows the long term trends of unemployment rate (u), inflation (π), and real share prices (share p). The band-pass filter of Baxter and King (1999) is used to decompose raw data into a trend and a cyclical component. The upper and lower panels show the US quarterly data and annual data, respectively, and their long term trends.

To provide more concrete evidence on the long term relationship for the three variables, Figure

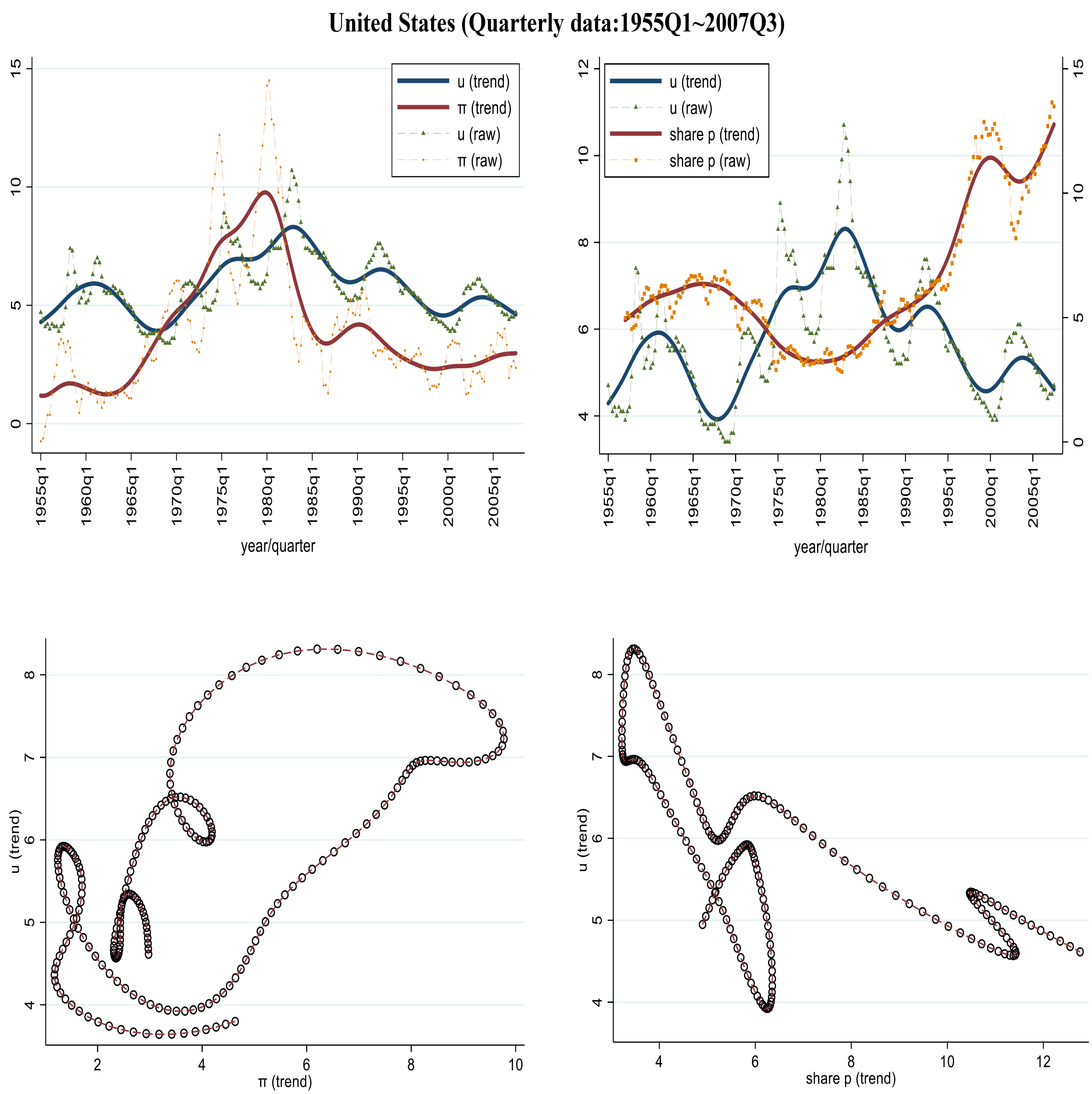
Figure 13: Long term trends using Baxter and King's (1999) filter (Scatter plot)



This figure shows scatter plots of the long term trends of unemployment rate (u), inflation (π), and real share prices (share p). The band-pass filter of Baxter and King (1999) is used to decompose raw data into a trend and a cyclical component. The upper and lower panels show the US quarterly data and annual data, respectively, and their long term trends.

12 removes more of the higher frequency fluctuations from raw data by using the band-pass filter of Baxter and King (1999). We choose the filter weight parameters to pick out specific frequency bands such as cycles that last from 1.5 years to 8 years (business cycles), similarly to Haug and

Figure 14: Long term trends using Hodrick-Prescott filter



This figure shows the long term trends of unemployment rate (u), inflation (π), and real share prices (share p). The Hodrick-Prescott filter is used to decompose raw data into a trend and a cyclical component. The smoothing parameter for quarterly data is 1600. The upper panel shows the US quarterly data and their long term trends, and the lower panel shows the scatter plot of the long term trends of the US quarterly data.

King (2014). After filtering out the higher frequencies, we plot the long term trend components with raw data of two variables each. The left panel shows that the long term trends of inflation and unemployment move in the same direction: the long term trends of inflation and unemployment had increased until early 1980s and decreased afterward. Similar findings for US inflation and unemployment rates are also observed by Berentsen et al. (2011). The right panel displays the long-term trends of unemployment rate and real share prices. A strong negative relationship exists between the long term trends in these variables: The trends of two variables go across each other especially in the 1980s and 2000s. In the lower panel with US annual data, we also observe a positive relationship between the the long term trends of inflation and unemployment. In particular, the long term inflation and unemployment show an increasing trend until the early 1980s but a decreasing trend afterward. In sum, the graphs with annual data support a negative relationship between the long term trends of unemployment and real stock prices, which shows very similar patterns to the results with US quarterly data.

Figure 13 shows the data in Figure 12 differently by displaying the scatter plot for the long term trends of unemployment rate and inflation rate (left panel), and that for the long term trends of unemployment rate and real stock prices (right panel). Dotted lines connect quarter-by-quarter and year-by-year observations. This figure confirms the positive relationship between unemployment rate and inflation rate and the negative relationship between unemployment rate and real share prices in the long term, as shown in Figure 12.

Figure 14 use Hodrick-Prescott (HP) filter (the smoothing parameter for quarterly data =1600), instead of the Baxter and King filter used in Figures 12 and 13. The results with HP filter describe smoother long term trends and support a positive relationship between the long term trends of inflation and unemployment and a negative relationship between the long term trends of unemployment and real stock prices.

Table 5 reports the results of bivariate cointegration tests to check the long term relationship between the three variables more thoroughly. Before implementing the cointegration tests, we conducted unit root tests for three variables. Inflation rate and real stock prices cannot reject the null hypothesis that the variable contains a unit root even by including various lags of the difference of the variable, trend and drift. Unemployment rate itself also cannot reject the null of a unit root.[12] Table 5 also provides the estimated cointegration vectors among the three variables. The first test (in the first row) is to capture the Phillips curve relationship between US inflation and unemployment. The second test examines Phelps and Farmer's hypothesis on the relationship between unemployment and real stock prices. Lastly, we focus on the Fed model implication on the relationship between the inflation rate and real stock prices. Our test statistics indicate that the null hypothesis of no cointegration in all tests at a reasonable statistical significance level would be rejected. More interestingly, the estimated cointegration vectors for the variables are consistent with the aforementioned empirical findings in Figures 11, 12, 13 and 14. For the robustness check, we allow for various assumptions on the existence of the trend, but our results do not alter. Note that we also implement a multivariate cointegration test to check whether cointegrations exist

[12] However, when adding the lagged difference terms, we reject that the unemployment rate contains a unit root.

among three variables simultaneously and find that more than 1 cointegration vector exist among three variables.

Lastly, since the extensive margin of firm entry in response to monetary policy is the distinct feature in our model, we provide micro-evidence on the relationship between the long run trend of the US firm entry and that of the US inflation and real equity prices during the postwar period. For a proxy variable for the extensive margin of firm entry, we use the US index of net business formation collected from the Survey of Current Business published in February 1996.[13] The index is normalized at 100 in the year of 1967, and covers from January 1948 to December 1994. Note that the data are only available up to 1994. Figure 15 plots fluctuation in US net business formation and US inflation rates and real equity prices at both quarterly and yearly frequencies and also draw the long term trends of both variables using the HP filter. One could soon discern that the trend of the index of new business formation moves in the same direction with that of inflation rates during the sample period at both frequencies, while the trend of firm entry is negatively associated with that of real equity prices. This finding suggests that the extensive margin of US firm entry is positively associated with US inflation rate and negatively related to US share prices over the long run, respectively.

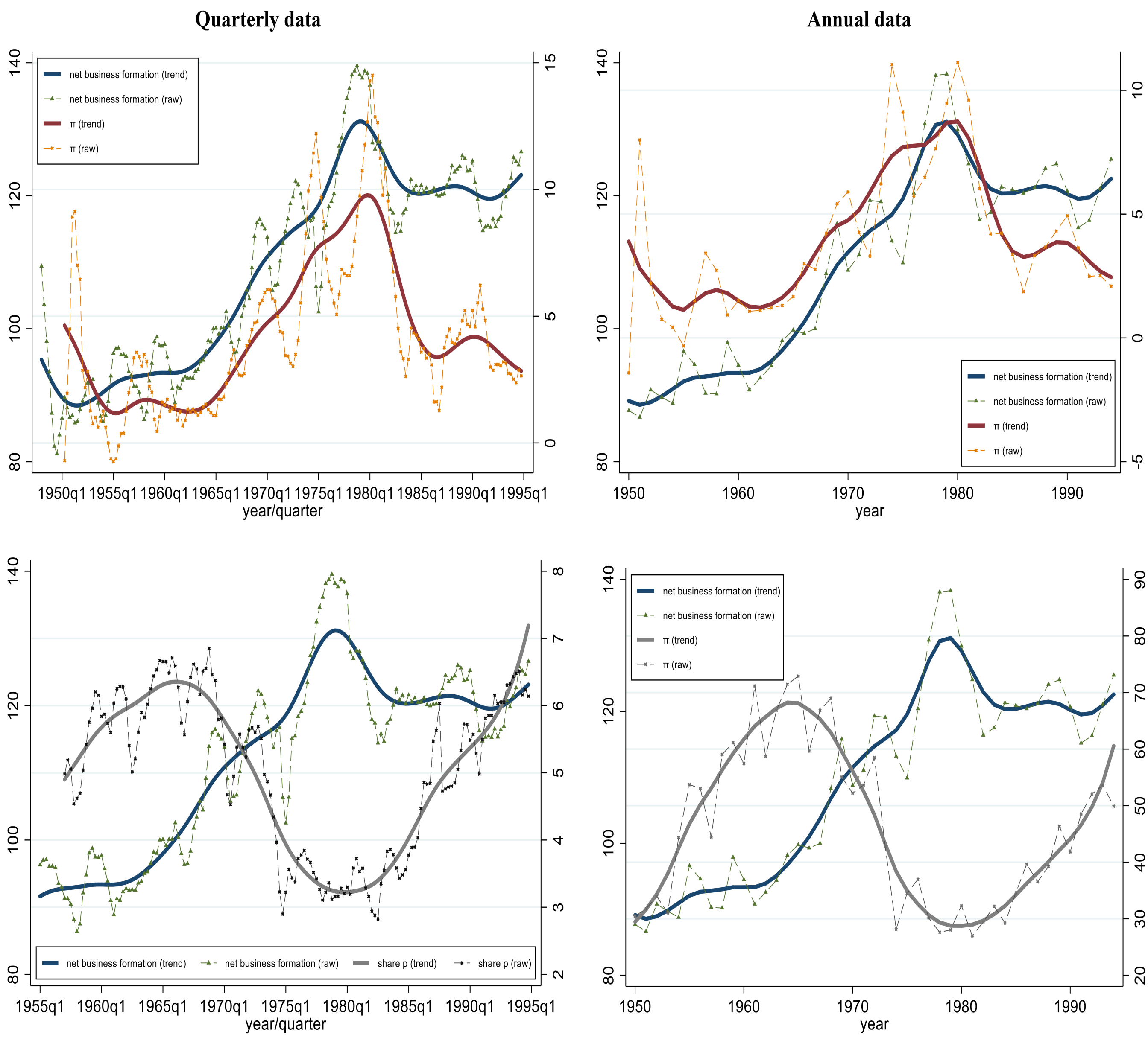
Table 5: Cointegration vectors with the US quarterly data

	Test result: Cointegration?	Cointegration vector	
	H_0: no cointegration	Inflation	Unemployment rate
No trend	Reject at the 1%	1.00	0.633***
Trend	Reject at the 1%	1.00	0.634***
Quadratic trend	Reject at the 5%	1.00	0.046
		Unemployment rate	Share price index
No trend	Reject at the 5%	1.00	-0.282***
Trend	Reject at the 5%	1.00	-0.459***
Quadratic trend	Reject at the 5%	1.00	-0.612***
		Inflation	Share price index
No trend	Reject at the 1%	1.00	-0.519***
Trend	Reject at the 1%	1.00	-0.773***
Quadratic trend	Reject at the 5%	1.00	-0.527**

This table shows bivariate cointegration among the three variables. Cointegration vectors are reported in the last two columns. We allow for three lags. The residual based Engle-Granger test indicates that the null hypothesis of no cointegration at the reasonable significance level is rejected. Critical values of the test statistics for cointegration are not tabulated. However, *** , **, and * indicate test statistics lie inside the 1, 5 and 10 percent critical region respectively.

[13] https://apps.bea.gov/scb/pdf/NATIONAL/BUSCYCLE/1996/0296cpgs.pdf

Figure 15: Comovement of inflation, real equity prices and new business formation in the United States



This figure shows the long term trends of inflation (π), real equity prices and the index of net business formation collected from Survey of Current Business. The Hodrick-Prescott filter is used to decompose raw data into a trend and a cyclical component. The smoothing parameter for quarterly data is 1600 and that for annual data is 6.25. The left panel shows the US quarterly data and their long term trends, and the right panel shows the US annual data.