Liquidity Risk and Time-Varying Correlation Between Equity and Currency Returns

Kuk Mo Jung*

School of Economics
Henan University, China

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Abstract

Using data on nine major OECD countries over time, this paper documents a new evidence on equity and currency prices: higher returns in the home equity market relative to foreign counterparts are generally associated with home currency depreciation, especially at a monthly frequency, but this negative correlation breaks down or even reverses during non-tranquil time periods. This paper proposes that a long-run risk model, combined with time-varying liquidity risks, can account for the newly found correlation structure between equity and currency returns. With recursive preference for the early resolution of uncertainty, the model demonstrates that severe market liquidity volatility disintegrates the positive link between the intertemporal marginal rate of substitution and asset value. This mechanism becomes a key driving force behind the time-varying correlation structure.

Keywords: foreign exchange rates, long run risks models, liquidity risks

JEL Classification Numbers: E43, F31, G12, G15

*Address: Ming Lun Street, Kaifeng, Henan, 475001, China, Phone Number: +86-378-3885099, e-mail: kmjung@ucdavis.edu. I am grateful to seminar participants at Henan University for helpful comments.
1 Introduction

Since Meese and Rogoff (1983)'s study, a long-standing challenge in international economics has been the difficulty of tying floating exchange rates to macroeconomic fundamentals such as money supplies, outputs, and interest rates. While numerous studies have subsequently claimed to find success for various versions of fundamentals-based exchange rate determination models (sometimes at longer horizons and over different time periods), the success of these models has not proven to be robust. The uncovered interest rate parity puzzle, for instance, is a stark example of this difficulty. For this reason, recent nominal exchange rate determination theory has advanced mostly outside the scope of traditional fundamental-based models. One important strand of this new literature views the link between equity and foreign exchange (FX) markets as a potential solution to the puzzle. Traditionally, international equity markets have been largely overlooked in the exchange-rate determination literature. However, a rapidly growing portion and size of the equity flows reported in Figure 1 as well as a technical development in solving DSGE models involving portfolio choice have recently started requiring for a new exchange rate theory in which exchange rates and equity-market returns are determined jointly.

Figure 1: International Equity Transaction Trend

As for previous literature, the empirical relationship between the exchange rates and the stock markets has been studied for a couple of decades. The results, however, are inconclusive. Most cointegration and standard granger causality tests have found no long-run
association between stock prices and exchange rates.\textsuperscript{1} On the other hand, when it comes to the relationship between relative equity returns and exchange rates, research has revealed a negative relationship at the short to medium frequency was found (see 2000 BIS Quarterly Review, Brooks, Edison, Kumar, and Slock (2001), Cappiello and Santis (2007) and Hau and Rey (2006)). In other words, previous studies have essentially shown that for a pair of countries, one country’s currency appreciation tends to be associated with a fall in relative equity returns to the other country at the short to medium frequency.

This paper finds similar empirical results basically in line with previous findings. However, the critical and novel aspect of this paper’s empirical results—which uses updated data—is that the generally observed negative correlation between FX and equity returns tends to disappear or even turn into a positive one for an extended period of time, especially during an asset market liquidity crisis. Hence, these new empirical results support a time-varying correlation between equity and FX returns, a correlation that no previous study have been able to provide theoretical explanations for. This is exactly where the objective of this paper comes in. My model, which is based on Bansal and Shaliastovich (2013)’s long run risks model, suggests a potential explanation for this newly observed evidence.

The model intuition revolves around some key factors of crisis leading up to the correlation sign-switching behavior. For this purpose, it is worth noting that two major global financial crises in late 1990s and 2000s were essentially triggered by a sudden liquidity shortfall in the financial asset market. For instance, the Russian default and LTCM collapse in late 1990s dried up global financial asset liquidity causing a worldwide financial turmoil. The current financial crisis was triggered by a liquidity shortfall in the U.S. banking system in 2008. All these historical evidence points to asset market liquidity as a potential driving force behind the time-varying correlation structure. My model, therefore, explicitly utilizes this concept of liquidity volatility, combined with the long run risk framework by Bansal and Yaron (2004), to find a solution to the time-varying correlation structure. The model intuition is as follows.

First, the model assumes Epstein-Zin (EZ) preference of agents as in Bansal and Shaliastovich (2013). Introducing EZ preference with a risk aversion and intertemporal elasticity of substitution both greater than 1 would imply that agents prefer early resolution of uncertainty, which in turn implies that a relative rise in today’s level of home consumption volatility would lower today’s level of home asset prices relative to foreign asset prices. Eventually relatively higher returns on home assets would be expected.\textsuperscript{2} In the mean time,  

\textsuperscript{1}See Granger, Huang, and Yang (2000) for details  
\textsuperscript{2}Standard CRRA type preferences could not simply generate consumption volatility-induced asset price changes. This is due to the fact that the pricing kernel under the CRRA preference only depends on the relative size of current and expected future consumption levels and the consumption volatility itself has
a relatively higher domestic consumption volatility would lower the home pricing kernel relative to the foreign counterpart (i.e., home agents value tomorrow’s consumption relatively less than foreign agents). This also indicates that the relative home currency value would have to depreciate tomorrow in order to match the relatively lower consumption demand by home agents.³ This is what my model suggests as a main mechanism behind the negative correlation between currency and equity values.

However, my model also offers a liquidity volatility channel through which the relationship between currency and equity values could be reversed. First, aggregate liquidity volatility has no impact on the pricing kernel and hence FX rates since liquidity volatility is assumed to have no impact on the consumption process at all. Second, if the aggregate liquidity volatility makes aggregate dividend growth more volatile but not consumption growth, then the agents would certainly require a lower risk premium on equities since aggregate dividends would comove with aggregate consumption to a lesser extent. Therefore, a relative rise in the home liquidity volatility today would boost home asset prices today, leading to relatively lower expected returns on home assets tomorrow. This may seem counterintuitive at first given the empirical evidence that high market liquidity volatility tends to be associated with asset market crashes like the 2008-9 financial crisis in the US. However, it is very important to notice that this liquidity effect on asset prices is a per se effect. What is critical is that empirical evidence points to a positive connection between the liquidity volatility and consumption volatility.⁴ In consequence, although the partial or ceteris paribus impact of the liquidity volatility on equity prices is positive, the impact of the liquidity crisis on prices could be very negative since high consumption volatility would bring down the prices at the same time. Therefore, my model explicitly assumes positively related consumption and liquidity volatility through spillover effects.

Finally, in light of huge home liquidity volatility (a situation referred to as a liquidity crisis), the liquidity volatility channel would dominate the consumption volatility channel and would thus cause a relative rise in home asset values today and a relative fall in expected home asset returns tomorrow.⁵ At the same time, a home consumption volatility would more likely to be higher than the foreign counterpart since consumption and liquidity volatility are positively connected. This would in turn imply lower home pricing kernel today and an expected depreciation of home currency tomorrow. All in all, this explains how a positive

³ This intuition is in line with the fact that a relative currency value is positively related to the relative pricing kernel under the complete financial market assumption.
⁴ See Amihud and Mendelson (1989) and Brunnermeier and Pedersen (2008)
⁵ As mentioned above, absolute home asset values could plummet today due to the high consumption volatility at home
link between equity and currency values or the time-varying correlation between FX and equity returns could emerge once a severe liquidity volatility state of the world arrives.

Lastly, it is worth noting what previous literature has provided as theoretical explanations for the jointly determined relationship between equity and FX returns. There are two important portfolio-balance models that theoretically endogenize the dynamics of equity prices and exchange rates. Using a standard international asset-pricing model with demand and supply shock, Pavlova and Rigobon (2007) examined the behavior of the real exchange rate and the stock markets. The model has implications on how equity and foreign exchange markets co-move in response to shocks, which are transmitted internationally across financial markets via the terms of trade. For example, a positive supply shock at home would have a positive effect on the relative domestic stock values. In line with the comparative advantages (Ricardian) theory, the domestic terms of trade would deteriorate and therefore raise the relative prices of foreign goods (i.e., domestic currency depreciation). Hence, the supply shock would generate a negative comovement between FX and equity returns.

However, the dynamics induced by demand shocks are completely different. For instance, a positive demand shock at home would improve the country’s terms of trade due to home bias assumptions of domestic goods. Domestic currency would appreciate as a result, which in turn boost domestic stock values relative to foreign stock prices. Unlike supply shock, a positive correlation between FX and equity returns would be implied by demand shocks. In short, their model predicts that the relationship between FX returns and equity returns critically hinges upon the dominance between demand and supply shocks. However, one important drawbacks deserves attention. To account for the positive comovement between equity and FX returns, Pavlova and Rigobon (2007) have to heavily rely on the demand-shock story, especially during the liquidity crises period, which was both theoretically and empirically hard to prove.

Hau and Rey (2006)’s study is also important and deserves attention. Hau and Rey (2006) developed a theoretical model in which exchange rates, equity market returns and capital flows are jointly determined. They argued that excess equity returns over another country and currency value have a perfect negative correlation due to incomplete FX risk trading. Their key arguments are as follows. When foreign equities outperform domestic equities in terms of rate of return, the relative exposure of domestic investors to exchange-rate risk increases due to incomplete FX risk hedging. To diminish the FX risks exposure, home investors should rebalance their portfolio, decreasing foreign equity holdings. This would

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6 Heathcote and Perri (2013) provided numerical impulse responses for excess equity returns and real exchange rates to supply and demand shocks within their theoretical model. However, the numerical results are basically the same as Pavlova and Rigobon (2007) and Hau and Rey (2006)’s analytical predictions.
in turn generate capital inflows into the domestic country and would therefore result in a
home currency appreciation. In the end, the perfect negative correlation between equity and
currency values would hold. However, Hau and Rey (2006) also admit some flaws in both
theory and the matching of empirical evidence. Most importantly, the perfectly negative
correlation between excess domestic equity returns over foreign equities and domestic currency values implied by their theoretical model can not account for my new empirical results showing a time-varying correlation between equity and FX returns. Again, the main contribution of this paper is to provide a plausible explanation for this newly observed evidence.

The paper is organized as follows. Section 2 documents newly found empirical evidence with updated data. Section 3 and 4 present the model in a rigorous manner. Section 5 discusses the model predictions on FX and equity returns. Section 6 directly tests whether the model can quantitatively replicate the empirical evidence with a calibration exercise. Section 7 concludes.

2 New Evidence

My empirical work focuses on eight OECD countries (i.e., Australia, Germany, the United Kingdom, Switzerland, Norway, New Zealand, Sweden and Korea) relative to the U.S. The nominal FX rate in home currency per unit of foreign currency (e.g., $/£) data and three-month bond rates were obtained from the Federal Reserve Bank of New York and IFS. Data for the major stock market index such as S&P 500 and FTSE 100 were obtained from Yahoo Finance. All data range starts from 1991/1 and ends 2010/10. Since this model explicitly concerns short to medium frequencies, monthly changes in excess stock and FX returns were chosen to analyze the correlation structure.

Table 1 provides empirical estimates on the correlation between foreign currency values relative to the U.S $ and excess foreign stock index returns over S&P 500 returns. It shows the standard OLS regression results of a monthly log change in FX (i.e.,, $/£) on monthly foreign stock index return (in foreign currency) relative to the S&P 500 index return (in dollars) (i.e.,, [R_{ft} - R_{ht}]).

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7 In terms of theoretical flaws in the model, restraints on foreign bond short selling as a way to introduce the market incompleteness are absurd. Because typical foreign equity investors hold equities and bonds as a bundle, no short-selling assumption on the foreign bonds seems to be far too unrealistic. Second and more importantly, the risk-rebalancing channel in their model entirely depends on expected excess returns. However, real-world investors do care about the risk associated with assets. Without risk aspects of excess returns taken into account, their theoretical prediction on the link between FX and equity returns should be treated with much caution.
Table 1: Monthly correlations of foreign currency returns and excess foreign equity returns

\[ \Delta \xi_t = \alpha + \beta [\Delta R_{ft}^* - \Delta R_{ht}^*] + \epsilon_t \]

<table>
<thead>
<tr>
<th>Countries</th>
<th>( \hat{\beta} )</th>
<th>( \hat{\beta} )</th>
<th>( \hat{\beta} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>-0.00386</td>
<td>0.00633</td>
<td>0.20098**</td>
</tr>
<tr>
<td>Germany</td>
<td>-0.06607*</td>
<td>0.025</td>
<td>0.13397***</td>
</tr>
<tr>
<td>UK</td>
<td>-0.11887***</td>
<td>0.07754</td>
<td>-0.03466</td>
</tr>
<tr>
<td>Swiss</td>
<td>-0.06037***</td>
<td>0.05549</td>
<td>-0.02409</td>
</tr>
<tr>
<td>Norway</td>
<td>-0.03831**</td>
<td>-0.00502</td>
<td>0.163405***</td>
</tr>
<tr>
<td>New Zealand</td>
<td>-0.1149***</td>
<td>N/A</td>
<td>0.04889</td>
</tr>
<tr>
<td>South Korea</td>
<td>-0.03883***</td>
<td>N/A</td>
<td>0.18843***</td>
</tr>
<tr>
<td>Sweden</td>
<td>-0.04519*</td>
<td>N/A</td>
<td>0.04887</td>
</tr>
</tbody>
</table>

Notes: *, ** and *** indicate that the coefficient is significant at 30%, 20% and 10% level respectively. Tranquil periods usually cover the whole periods from 1991/01 to 2010/06 except two sub-periods (i.e., Internet bubble periods and Current financial crisis periods defined above). Due to the lack of stock index data, New Zealand, South Korea and Sweden only have data starting from 2000.

First, the OLS estimates on \( \beta \) during the whole time period turn out to be negative for all eight countries and statistically significant except for Australia. This regression evidence in favor of a negative correlation is in fact closely associated with what Hau and Rey (2006) and others already found in data. However, what Hau and Rey (2006) missed is the evidence outlined in the second and third rows of Table 1. As seen in Table 1, the regression evidence for the correlation during the late 1990s liquidity crisis periods and current financial periods tells an entirely different story. In fact, the estimated \( \beta \) or the correlation becomes statistically insignificant and the sign even turns significantly positive in four cases. This evidence contradicts Hau and Rey (2006)’s model prediction that such correlation must be perfectly negative. Another evidence of the non-perfect negative correlation is illustrated in Figure 2.
Figure 2: Correlation between FX and equity returns with a rolling window of 2-year
Figure 2 plots the two-year rolling correlations between \((\Delta \xi_t)\) and \([R^f_t - R^h_t]\). The gray area in each sub-figure indicates the financial crisis periods. It is actually quite surprising to observe the frequently fluctuating correlations. This graph has many interesting implications. First, the correlation structure between the excess foreign stock return over the U.S counterpart and relative foreign currency values must be time-varying. Second, no or positive correlations during the liquidity crisis periods (late 1990s) and the current financial crisis (2007 through 2009) call for a new model that is able to account for the sign-switching correlation behavior associated with financial market crisis.

3 The Model

3.1 Epstein-Zin Recursive Utility

The representative investor preference over the uncertain aggregate consumption stream \(C_t\) are assumed to have a functional form of the EZ utility function.

\[
U_t = \left[(1 - \beta) C_t^{1-\gamma} + \beta (E_t U_{t+1})^{1-\gamma} \right]^{\frac{\theta}{1-\gamma}},
\]

where \(\beta\), \(\phi\) and \(\gamma\) are the time discount factor, the intertemporal elasticity of substitution (IES) and the risk aversion parameter respectively. Parameter \(\theta\) is defined as \(\theta = \frac{1-\gamma}{1-\frac{\phi}{\gamma}}\). As pointed out in Bansal and Shaliastovich (2013), the logarithm of the intertemporal marginal rate of substitution (IMRS) for these preferences is given by

\[
m_{t+1} = \theta \log \beta - \frac{\theta}{\phi} \Delta c_{t+1} + (\theta - 1) r_{c,t+1},
\]

where \(\Delta c_{t+1} = \log(\frac{C_{t+1}}{C_t})\) is the growth rate of aggregate consumption and \(r_{c,t+1}\) is the log of the return on an imaginary asset which delivers aggregate consumption as its dividends each time period. Of course, this return is not observed in data.

3.2 Aggregate Consumption Process

I adopt the exact same consumption process in Bansal and Shaliastovich (2013) where home and foreign countries differ only in consumption volatility (i.e., \(\sigma_{gl}, \sigma_{gl}^*\)) and consumption growth innovations (i.e., \(\eta_{t+1}, \eta_{t+1}^*\)). From now on, foreign country variables are indexed
by a superscript \(^{\ast}\). The consumption dynamics for home are the following:

\[
\begin{align*}
\Delta c_{t+1} &= \mu_g + x_t + \sigma_{gt}\eta_{t+1} \\
x_{t+1} &= \rho x_t + \sigma_{xt}\epsilon_{t+1} \\
\sigma_{g,t+1}^2 &= v_g\sigma_{gt}^2 + \omega_{g,t+1} \\
\sigma_{x,t+1}^2 &= v_x\sigma_{xt}^2 + \omega_{x,t+1},
\end{align*}
\]

(3)

where \(x_t\) is a persistent long-run expected growth component. The fact that home and foreign countries share the same long-run component reflects the almost similar long-run growth prospects across the countries in historical data. Notice here that this long run consumption shock \((\epsilon_t)\) is persistently transmitted into the future consumption process whereas the short run consumption shock \((\eta_t)\) is not. The consumption growth differences between the two countries in this model are captured by the differences in short-run consumption shocks and volatilities. For tractability, \(\eta_{t+1}\) and \(\epsilon_{t+1}\) are assumed to follow the standard normal distribution and the innovations in volatility processes \(\omega_{g,t+1}\) and \(\omega_{x,t+1}\) are assumed to follow a gamma distribution with no contemporaneous correlation. Finally, empirical justifications for the time-varying volatilities of the consumption and long-run components are presented in Bansal and Shaliastovich (2013).

### 3.3 Aggregate Dividend Process

In this model economy, the value of the aggregate dividend is composed of two components: intrinsic factors and liquidity factors (as in Acharya and Pedersen (2005)). The intrinsic component has the process of the form.

\[
\begin{align*}
\Delta id_{t+1} &= \mu_g + \phi x_t + \phi_d\sigma_{gt}\eta_{dt+1},
\end{align*}
\]

(4)

where the average dividend growth rate is equal to the rate for aggregate consumption (i.e., \(\mu_g\)) and the volatility of dividend growth is simply \(\phi_d\) times greater than the consumption counterpart. For tractability, the independence between consumption and dividend growth shocks is assumed.

The liquidity factor process is as follows, where \(\sigma_{l,t}\) is the volatility of the liquidity factor.

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8 Basically what they showed is that in an economy where the aggregate dividend, \(D_t\) and the liquidity cost or liquidity shocks (i.e., the transaction cost of buying and selling equity), \(C_t\) both follow an AR(1) process, the equilibrium equity prices should be same as those of an imagined economy where the aggregate dividend simply equals to \(D_t - C_t\) with no liquidity cost or shocks. They then solved a liquidity-adjusted CAPM. Thus, they justified using the liquidity shock-adjusted aggregate dividend process to bring about the liquidity factor-adjusted equity prices.
and $\zeta_{t+1}$ is the liquidity shock.

$$\Delta l\sigma_{t+1} = \tau x_t + \sigma_{l,t} \zeta_{t+1}. \quad (5)$$

The easiest interpretation of the liquidity factor could be viewing it as a transaction cost (e.g., broker fees and bid-ask spread) or even broadly as a search cost. Strictly speaking, the liquidity factor in my model can be understood as a negative of all the transaction and search costs associated with selling or buying equities. Of course, I allow for a time-varying liquidity factor (i.e., transaction and search costs) in my model. Importantly, $\zeta_t$, a liquidity shock is a main driving force behind the time variation for the liquidity factor. For an individual stock, this liquidity shock could be, for example, highly increased selling pressure resulting from an accounting scandal within a company, which would in turn increase the bid-ask spread for this particular stock. For aggregate dividend, the liquidity shock could include macroeconomic events like sub-prime mortgage or an LTCM crisis that suddenly dries up the entire equity market liquidity. Finally, it is important to emphasize that my model is simply based on exogenous liquidity shocks. In other words, there is no endogenous mechanism whereby the liquidity shocks become more volatile during any recession period for the economy. However, this exogenous shock is enough to show that the time-varying correlation between equity and FX returns is triggered by the magnitude of liquidity volatility in my model. I leave the task of endogenizing the liquidity shock for future research.

Figure 3: Standardized innovations in US equity market liquidity extracted from Acharaya and Pedersen (2005)
Empirically, asset market liquidity is persistent, pro-cyclical and time varying. Acharya and Pedersen (2005) reported highly persistent US equity market liquidity with an autocorrelation of around 0.9 at a monthly frequency. Brunnermeier and Pedersen (2008) demonstrated the pro-cyclical nature of the asset market liquidity provision and offered a theoretical explanation based on the funding-liquidity constrained investment problem. Hence, this evidence justifies the liquidity factor process in eq.(5) containing a persistent long-run component (i.e., pro-cyclicality). Time-varying volatility of equity market liquidity (i.e., $\sigma_{lt}$) was also empirically observed (Acharya and Pedersen (2005), Amihud (2002), Pastor and Stambaugh (2003)).

To sum up, the home and foreign aggregate dividend processes combining intrinsic and liquidity factors take following forms respectively.

$$
\Delta d_{t+1} = \mu_g + (\phi + \tau) x_t + \varphi d \sigma_{gt} \eta_{d,t+1} + \sigma_{lt} \zeta_{t+1}, \\
\Delta d^*_t = \mu_g + (\phi + \tau) x_t + \varphi d \sigma^*_{gt} \eta^*_{d,t+1} + \sigma^*_{lt} \zeta^*_{t+1}.
$$

(6)

In short, the liquidity itself does not affect the intrinsic dividend process at all. However, I incorporated the liquidity factor into the dividend process to bring about the liquidity factor-adjusted equilibrium equity price as in Acharya and Pedersen (2005). For tractability, the intrinsic dividend growth shocks (i.e., $\eta_{d,t+1}$ and $\eta^*_{d,t+1}$) and liquidity shocks (i.e., $\zeta_{t+1}$ and $\zeta^*_{t+1}$) are assumed to be an independent, identically distributed normal process with no covariance. The volatility process of liquidity shock is the following.

$$
\sigma^2_{lt+1} = v_l \sigma^2_{lt} + v_m \sigma^2_{gt} + \omega_{l,t+1}.
$$

(7)

Positive $v_m$ allows for positive spillover effects of consumption volatility on liquidity volatility such that intrinsic dividend shock’s volatility ($\varphi d \sigma_{gt}$) and liquidity shock’s volatility ($\sigma_{lt}$) are positively correlated. This assumption is consistent with the empirical findings of Amihud and Mendelson (1989) and Brunnermeier and Pedersen (2008), for which market liquidity fluctuations intensify as fundamental volatility increases.\(^9\)

\(^9\)Amihud and Mendelson (1989) and Brunnermeier and Pedersen (2008) showed that the asset market liquidity fluctuation intensifies as the volatility of intrinsic or fundamental factors of aggregate dividends increases in the economy. In my model economy, the volatility of intrinsic factors of the aggregate dividend is basically the same as aggregate consumption volatility scaled up by a constant number, which is empirically supported by Bansal and Yaron (2004). Therefore, the consumption and liquidity volatility are positively correlated in this model.
3.4 Inflation Process

Since my empirical evidence deals with nominal terms (i.e., nominal FX and equity returns), an inflation process should be added into the model to derive implications for nominal returns. This model adopts the exogenous inflation dynamics in Bansal and Shaliastovich (2013).

\[
\begin{align*}
\pi_{t+1} &= \mu + \sigma_{\pi g} \eta_{t+1} + \sigma_{\pi x} \xi_{t+1}, \\
\pi_{t+1} &= \mu + \sigma_{\pi g} \eta_{t+1} + \sigma_{\pi x} \xi_{t+1},
\end{align*}
\]

where \( z_t \) is the expected inflation state variable and \( \varphi_{\pi g}, \varphi_{\pi x} \) measure the sensitivity of realized and expected inflation innovations to short and long-run consumption news. As noted in Bansal and Shaliastovich (2013), the shocks to the consumption process also affects the inflation process in this model economy. Finally, the expected inflation component differs across two countries in line with the usual real world data (i.e., \( z_t \neq z^*_t \)).

4 Asset Markets

4.1 Stochastic Discount Factor

First, Bansal and Shaliastovich (2013) showed that the log-linearized return on the imaginary asset that pays out the aggregate dividend every period is given by the following processes, which is linear in state variables.\(^{10}\)

\[
\begin{align*}
rc_{t+1} &= \kappa_0 + \kappa_1 p_c t + \Delta c_{t+1}, \\
p_c t &= A_0 + A_x x_t + A_{gs} \sigma_{gt}^2 + A_{gs} \sigma_{xt}^2,
\end{align*}
\]

where \( p_c t \) is the log wealth or price to consumption ratio and the solution coefficients \( A_x \) and \( A_{gs} \) are as follows:\(^{11}\)

\[
\begin{align*}
A_x &= \frac{1 - \frac{1}{\psi}}{1 - \kappa_1 \rho}, \\
A_{gs} &= \frac{(1 - \gamma)(1 - \frac{1}{\psi})}{2(1 - \kappa_1 \psi_g)}.
\end{align*}
\]

An important thing to notice here is that \( A_x \) and \( A_{gs} \) would have been negative and positive values, respectively, under the CRRA preference with risk aversion greater than 1. Under such conditions, higher consumption volatility (\( \sigma_{gt}^2 \)) would have raised asset prices.

\(^{10}\) Bansal and Shaliastovich (2013) showed that the log-linearized solution to the model is very close to the solution of the model based on numerical methods.

\(^{11}\) The solutions coefficients for \( \kappa_0, \kappa_1, A_0, A_x \) are shown in the appendix of Bansal and Shaliastovich (2013). These coefficients are not important for the later analysis.
today which is certainly counterintuitive. In contrast, if the intertemporal elasticity of substitution and risk aversion are larger than one, $A_{gs}$ becomes negative, resulting in a negative relationship between contemporaneous consumption volatility and asset prices. This is precisely what Bansal and Yaron (2004) argued as a theoretical explanation for the relationship between consumption volatility and asset prices.

Combining the Euler condition in eq. (2) and the equilibrium price-consumption ratio in eq. (9), an analytical expression for the intertemporal marginal rate of substitution (IMRS) can be obtained as follows.\footnote{The computation of IMRS is greatly facilitated through the fact that the expectations of the exponential of the state variables is exponentially linear in the current states. See Bansal and Shaliastovich (2013) for details}

$$m_{t+1} = m_0 + m_x x_t + m_{gs} \sigma^2_{gt} + m_{xs} \sigma^2_{xt}$$
$$- \lambda_\eta \sigma_{gt} \eta_{t+1} - \lambda_e \sigma_{xt} e_{t+1} - \lambda_{gw} \omega_{g,t+1} - \lambda_{xw} \omega_{x,t+1},$$

where $\lambda_\eta$ and $\lambda_e$ are the market prices of short-run and long-run risks, and $\lambda_{gw}$ and $\lambda_{xw}$ are the market prices of short-run and long-run volatility risks. $m_{gs}$ and $\lambda_\eta$ are the two most important solution coefficients since the expected difference between home and foreign stochastic discount factor (SDF) only comes from different consumption volatility levels (i.e., $\sigma^2_{gt}$ and $\sigma^2_{gt}$) which will turn out to be critical in the FX movement.

$$m_{gs} = -\frac{1}{2} (\gamma - \frac{1}{\psi}) (\gamma - 1),$$
$$\lambda_\eta = \gamma.$$

With IES and risk aversion both being large than one, IMRS’s sensitivity to current consumption volatility (i.e., $m_{gs}$) becomes negative. In other words, the pricing kernel’s negative sensitivity to consumption volatility is entirely consistent with eq (9) where asset prices fall when consumption volatility increases. This is the unique feature of the EZ preference since the typical CRRA utility would have implied no impact of consumption volatility on the pricing kernel (i.e., $m_{gs} = 0$).

Finally, the home SDF used to price nominal payoffs is given by the following:

$$m^S_{t+1} = m_{t+1} - \pi_{t+1},$$

Combination of eq (10) and (11) would give linear nominal SDF in state variables, as shown...
below:
\[
m_{t+1}^{NM} = m_0 - \mu + m_x x_t - z_t + m_g s \sigma_g^2 + m_x s \sigma_x^2 \\
- (\lambda \eta + \phi \pi_g) \sigma_g \eta_{t+1} - (\lambda_e + \phi \pi_x) \sigma_x \epsilon_t \xi_{t+1} - \sigma_x \xi_{t+1} \\
- \lambda g_w \omega_{g,t+1} - \lambda x w \omega_{x,t+1}.
\]  
(12)

With this analytical expression for the pricing kernel, equilibrium currency and equity prices as well as expected returns depending on the state variable can also be obtained, which is discussed in the following section.

4.2 Equilibrium Foreign Exchange Rate

Backus, Allan, and Chris (2001) showed that in the case of complete markets, investing in foreign currency amounts to shorting a claim that pays off home SDF and going long in a claim that pays off the foreign SDF. In other words, the following nominal condition holds

\[
s_{NM,t}^{NM} - s_{NM,t} = m_{t+1}^{NM} - m_{t+1}^{NM},
\]  
(13)

where \( s_{t}^{NM} \) is the nominal FX rate in home currency per unit of foreign currency and \( m_{t+1}^{NM} \) and \( m_{t+1}^{NM} \) are the pricing kernel for foreign and home countries respectively. Intuitively, higher foreign SDF is equivalent foreign consumers valuing tomorrow’s consumption goods more than home consumers. This would in turn mean higher relative price for UK goods tomorrow (i.e., £ appreciation).

Under complete markets assumption, the eq.(12) and (13) give the equilibrium nominal FX process in this economy as below.

\[
s_{t+1}^{NM} = s_{t}^{NM} - (z_t^* - z_t) + m_g (\sigma_{gt}^2 - \sigma_{gt}^2) \\
- (\lambda \eta + \phi \pi_g)(\sigma^*_g \eta_{t+1} - \sigma_{gt} \eta_{t+1}) - \sigma_x (\xi_{t+1}^* - \xi_{t+1}) \\
- \lambda g_w (\omega_{g,t+1}^* - \omega_{g,t+1}).
\]  
(14)

The appendix shows how the expected FX change is dependent on the expected inflation difference (i.e., \( z_t^* - z_t \)) and the current consumption volatility difference (i.e., \( \sigma_{gt}^2 - \sigma_{gt}^2 \)).

\[
E_t[s_{t+1}^{NM} - s_{t}^{NM}] = (z_t - z_t^*) + \frac{1}{2}(\gamma - \frac{1}{\psi})(\gamma - 1)(\sigma_{gt}^2 - \sigma_{gt}^2).
\]  
(15)

The intuition behind the eq.(15) is as follows. First, a higher expected inflation rate at home would indicate that the home currency will depreciate, which is in line with the classical fisher condition. Second, higher domestic consumption volatility today would lower the domestic pricing kernel under the EZ preference with \( \gamma \) and \( \psi \) both being larger than one.
In consequence, the relative price of home goods would fall tomorrow, indicating home currency depreciation.

### 4.3 Equilibrium Equity Return

The appendix shows that the log-linearized nominal return on home equity is a linear process in state variables.

\[
\begin{align*}
    r_{d,t+1}^{NM} &= \ell_0 + \ell_1 pd_{t+1}^{NM} - pd_t^{NM} + \Delta d_{t+1}, \\
    pd_t^{NM} &= B_0 + B_x x_t + B_z z_t + B_{gs} \sigma_{gt}^2 + B_{xs} \sigma_{xt}^2 + B_{ls} \sigma_{lt}^2,
\end{align*}
\]

where \(pd_t^{NM}\) is the log nominal price to dividend ratio and the solution coefficients for \(B_{gs}\) and \(B_{ls}\) are the following:

\[
\begin{align*}
    B_{gs} &= \frac{(\varphi_d - \gamma - \varphi_x + \ell_1 \varphi_{xg})^2 - (\gamma - \frac{1}{\psi})(\gamma - 1)}{2(1 - \ell_1(v_g + v_m))} < 0, \quad (17) \\
    B_{ls} &= \frac{1}{2(1 - \ell_1 v_l)} > 0. \quad (18)
\end{align*}
\]

Again, other equilibrium solution coefficients are not reported here since they do not affect the expected equity return difference between two countries.

It is worth noting the signs of the two coefficients, \(B_{gs}\) and \(B_{ls}\). First, the sign of \(B_{gs}\) depends on model and preference parameters. Nevertheless, its sign is most likely to be negative under the typical parameter values widely used in the long-run risks literature. This will become clear in the calibration section later. As already discussed, the assumption of IES and risk aversion both being larger than one is critical in bringing about the negative \(B_{gs}\).

Second, \(B_{ls}\) is always positive, meaning that the higher liquidity volatility (i.e., \(\sigma_{lt}^2\)) \textit{ceteris paribus} boosts equity prices, hence reducing expected equity returns in the future. This does not necessarily mean that a liquidity crisis would cause the asset prices to soar since the positive connection between consumption and liquidity volatilities would somewhat mitigate the positive price impact of the liquidity volatility as explained earlier. As a matter of fact, this model implies that the price impact of the liquidity shock is likely to be negative given the larger magnitude of the coefficient \(B_{gs}\) compared to \(B_{ls}\) under the typical parameter assumption. Hence the usual intuitive idea of there being a negative correlation between liquidity volatility and asset prices would generally hold.

Finally, the appendix shows that the expected equity return difference between the home
and foreign country would be as follows:

\[
E_t(r_{d,t+1}^{NM} - r_{d,t+1}^{*NM}) = (z_t - z_t^*)
+ B_{gs}^{NM}(\ell_1(v_g + v_m) - 1)(\sigma_{gt}^2 - \sigma_{gt}^{*2})
+ B_{ls}^{NM}(\ell_1v_l - 1)(\sigma_{lt}^2 - \sigma_{lt}^{*2}).
\] (19)

5 Model Predictions on FX and Equity Returns

The focus of the final analysis is on the time-varying correlation between FX and relative equity returns in this model economy. What is critical in triggering the sign-switching behavior of the correlation is the magnitude of liquidity volatility in this model.

First, it should be noted from eq.(15) and (19) that expected inflation difference and current consumption volatility difference would induce a negative correlation between FX and equity returns. Algebraically, this can be verified through the loadings on \((z_t - z_t^*)\) and \((\sigma_{gt}^2 - \sigma_{gt}^{*2})\), both positive under the usual parameter values in eq.(15) and eq.(19). Intuitively, higher expected inflation at home would lead to higher nominal equity returns at home as well. At the same time, home currency would depreciate given the usual fisher condition in eq.(15). Similar reasoning works in the consumption volatility case. Higher consumption volatility at home would lower home equity values more today, thus leading to higher expected return tomorrow. In the mean time, it would lead to a home currency depreciation tomorrow since the higher home volatility would lower the home pricing kernel under the EZ preference.

Second, the liquidity volatility difference only affects the expected equity return difference. However, it has nothing to do with the expected FX movement in this model economy. This is the key factor in deriving the time-varying correlation of FX and equity returns. As the negative loadings on the liquidity volatility difference in eq.(19) indicates, higher liquidity volatility at home would mean relatively lower expected home equity returns since the equity risk premium goes down in repose to the higher liquidity volatility. Of course, the expected FX change is 0 in this case since the dividend liquidity volatility has no impact on the pricing kernel. After all, the magnitude of the liquidity volatility difference would critically determine the correlation structure of the FX and equity returns.

Third, this model economy further assume a positive correlation between \(\sigma_{gt}^2\) and \(\sigma_{lt}^2\), which is in line with empirical evidence. This model assumption could potentially account for the empirical correlation structure. What the evidence shows is that a foreign currency appreciation and foreign excess returns (in foreign currency) over home market returns have a negative correlation on average. This evidence makes sense perfectly under this model economy. Since the correlation between \(\sigma_{gt}^2\) and \(\sigma_{lt}^2\) is positive, the \(\text{sign}(\sigma_{gt}^2 - \sigma_{gt}^{*2})\) is usually
same as the $\text{sign}(\sigma_{lt}^2 - \sigma_{lt}^{*2})$. Furthermore, under the usual parameter values the magnitude of loadings on $(\sigma_{gt}^2 - \sigma_{gt}^{*2})$ is greater than the one for $(\sigma_{lt}^2 - \sigma_{lt}^{*2})$

$$|B_{gs}^{NM}(\ell_1(v_g + v_m) - 1)| > |B_{ls}^{NM}(\ell_1 v_l - 1)|.$$ 

Eventually, the impact of the consumption volatility difference would dominate which would in turn imply the negative correlation structure. Of course, when the $\text{sign}(\sigma_{gt}^2 - \sigma_{gt}^{*2})$ is different from $\text{sign}(\sigma_{gt}^2 - \sigma_{gt}^{*2})$, the dominance of consumption volatility effects would be further enhanced due to different signs for the loadings. This explains well why the empirical evidence on the average sign of correlation across countries and over time appears to be negative.

Lastly, this model implies that the correlation sign turns positive when the following two conditions are met:

$$\begin{cases} 
\text{sign}(\sigma_{gt}^2 - \sigma_{gt}^{*2}) = \text{sign}(\sigma_{lt}^2 - \sigma_{lt}^{*2}) \\
(\sigma_{gt}^2 - \sigma_{gt}^{*2}) \ll (\sigma_{lt}^2 - \sigma_{lt}^{*2}).
\end{cases}$$

The first condition is usually met since the correlation between $\sigma_{gt}^2$ and $\sigma_{lt}^2$ is set to be positive in this model. What is important is the second condition, which occurs when the liquidity volatility difference is substantially larger than the consumption volatility difference, which in practice corresponds to what happens during a liquidity crisis. Under these conditions, the liquidity volatility effects are expected to dominate and are thus expected to bring about a positive correlation between FX and equity returns. In the calibration exercise, I will examine to what extent this model economy can replicate the empirical evidence on the time-varying correlation structure.

6 Calibration of FX and Equity Returns

6.1 Parameterization

The baseline calibration parameter values of Table 2, were adapted from Bansal and Shaliastovich (2013). Notice here that the model is calibrated at a monthly frequency, so these parameter values were transformed into monthly values. Bansal and Shaliastovich (2013) provided detailed explanation for these values.
Table 2: Model parameter values

<table>
<thead>
<tr>
<th>Consumption Dynamics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean of consumption growth</td>
<td>$\mu_g = 0.0016$</td>
</tr>
<tr>
<td>Expected growth persistence</td>
<td>$\rho = 0.991$</td>
</tr>
<tr>
<td>Short-run volatility level</td>
<td>$\sigma_g = 0.0042$</td>
</tr>
<tr>
<td>Short-run volatility persistence</td>
<td>$v_g = 0.803$</td>
</tr>
<tr>
<td>Short-run volatility of volatility</td>
<td>$\sigma_{gw} = 1.57 \times 10^{-5}$</td>
</tr>
<tr>
<td>Long-run volatility level</td>
<td>$\sigma_x = 1.67 \times 10^{-4}$</td>
</tr>
<tr>
<td>Long-run volatility persistence</td>
<td>$v_x = 0.9799$</td>
</tr>
<tr>
<td>Long-run volatility of volatility</td>
<td>$\sigma_{xw} = 1.96 \times 10^{-6}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Inflation Dynamics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean of inflation rate</td>
<td>$\mu_\pi = 0.0027$</td>
</tr>
<tr>
<td>Inflation sensitivity to short-run news</td>
<td>$\varphi_{\pi g} = 0$</td>
</tr>
<tr>
<td>Inflation sensitivity to long-run news</td>
<td>$\varphi_{\pi x} = -2$</td>
</tr>
<tr>
<td>Inflation shock volatility</td>
<td>$\sigma_\pi = 0.0031$</td>
</tr>
<tr>
<td>Expected inflation persistence</td>
<td>$\alpha_z = 0.87$</td>
</tr>
<tr>
<td>Expected inflation leverage on long-run news</td>
<td>$\alpha_x = -0.34$</td>
</tr>
<tr>
<td>Expected inflation sensitivity to short-run news</td>
<td>$\varphi_{\pi g} = 0$</td>
</tr>
<tr>
<td>Expected inflation sensitivity on long-run news</td>
<td>$\varphi_{\pi x} = -1$</td>
</tr>
<tr>
<td>Expected inflation shock volatility</td>
<td>$\sigma_z = 1.67 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Aggregate Dividend and Liquidity Dynamics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate dividend sensitivity to long-run news</td>
<td>$\phi + \tau = 1.25$</td>
</tr>
<tr>
<td>Aggregate dividend growth volatility level</td>
<td>$\varphi_d = 10$</td>
</tr>
<tr>
<td>Aggregate liquidity volatility level</td>
<td>$\sigma_l = 0.17$</td>
</tr>
<tr>
<td>Aggregate liquidity volatility level</td>
<td>$v_l = 0.803$</td>
</tr>
<tr>
<td>Consumption spillover effects parameter</td>
<td>$v_m = 0.5$</td>
</tr>
<tr>
<td>Aggregate liquidity volatility of volatility</td>
<td>$\sigma_{lw} = 0.0136$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Preference Parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor</td>
<td>$\beta = 0.9978$</td>
</tr>
<tr>
<td>Intertemporal elasticity of substitution</td>
<td>$\psi = 1.5$</td>
</tr>
<tr>
<td>Risk aversion coefficient</td>
<td>$\gamma = 10$</td>
</tr>
</tbody>
</table>
Basically, they chose these values such that consumption and inflation processes in this model economy correspond well to U.S and UK (mostly U.S) business-cycle data. As for the preference parameters, nothing is at odds with the standard values in the literature apart from the magnitude of the IES, which is still a debate. However, an IES value higher than 1 (i.e., 1.5) is chosen to match the inverse relationship between asset values and consumption volatility.

The parameter values for aggregate liquidity dynamics deserve an explanation since they are unique features of this model. Most importantly, the aggregate liquidity volatility level, (0.17) was adapted from Acharya and Pedersen (2005). Although their measure was technically based on the U.S market illiquidity series, it still provides to be a good approximation of liquidity volatility level. The persistence ($v_l$) and the liquidity volatility of volatility ($\sigma_{lw}$) were specified such that the unconditional mean of the time-varying variance of the aggregate liquidity process equal the empirical measure (i.e $\sigma_{l}^{2} = 0.17^2$).\textsuperscript{13} Finally, these parameter values confirm the negative marginal effect of consumption volatility on asset prices (i.e., $B_{gs} = -6.3$) and the positive marginal effect of liquidity volatility on asset prices (i.e., $B_{ls} = 2.1$). Moreover, a larger absolute value of $B_{gs}$ relative to $B_{ls}$ is indeed consistent with the model prediction section earlier.

### 6.2 Quantitative Results

Under the parameter values specified above, simulation of a 20-year period for the model was conducted. Based on 1000 simulations of a 20-year period, the average regression coefficient (i.e $\beta$) for the whole period was about $-0.03$, which is close to what has been empirically observed among eight countries. This result is entirely consistent with the model prediction. Since the impact of the consumption and inflation difference would generally dominate that of the liquidity volatility difference under the calibrated model, the negative correlation between foreign currency appreciation and foreign excess returns must hold on average. Panel A of Figure 4 supports this theoretical prediction well. In only 4% of cases is the regression coefficient is positive, which is well within the reach of the model.

Panel B of Figure 4 reports the frequency distribution of regression coefficients for a crisis period only. A conservative approach was taken to define crisis and liquidity crisis. In this calibration exercise, the liquidity crisis period is assumed as a period during which the liquidity volatility differential between two countries fluctuates beyond the usual level. This assumption is in line with the empirical evidence demonstrating that the asset market liquidity volatility differential between countries shows a huge fluctuation during a liquidity

\textsuperscript{13} Under the gamma distribution assumption of $\zeta_l$, $E(\zeta_{l,t}) = \sigma_{l}^{2}(1 - v_l)$ and $Var(\zeta_{l,t}) = \sigma_{gl}^{2}$. 

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crisis period.

For instance, in the most recent liquidity crisis in the US banking system, the relative US liquidity volatility initially showed a huge spike. Then, over the course of the crisis period, this liquidity volatility would spread all over the global financial markets, which would in turn reduced the relative US liquidity volatility dramatically again.

Given this assumption for a liquidity crisis, the liquidity crisis period in my model was calibrated using the following procedure. I first performed 1000 model simulations of exchange rates, equity return differential and liquidity volatility differential within a 20-year period window. Then, for each of the 1000 cases within the 20-year period, a conditional variance of liquidity volatility differential with a rolling window of 10-months periods was calculated for each month. I performed this step to measure how volatile the liquidity volatility differential is for every rolling 10-months period. I call this measure as the 'liquidity volatility index'. I then picked the month with the highest index number (i.e., the highest rolling variance of liquidity volatility differential) during the entire 20-year period. Finally, for each of the 1000 cases within the 20-year period, I defined the liquidity crisis period as a 10-months period in the middle of which the the month showing the highest index number is placed.

The average regression coefficient for this liquidity crisis period was about $-0.015$, which is not significantly different from 0. Furthermore, as the shape of the frequency distribution
indicates, the coefficient estimates are mostly centered around 0, and the regression coefficients turn out to be positive in almost half of the cases. All of this evidence indicates that the negative correlation during the liquidity crisis period certainly disappears in this model, which is consistent with empirical data.

Figure 5: Four model simulation rolling correlation and liquidity volatility

To enable a better comparison of the results based on the data reported in Figure 2, I have also reported four simulation results of the rolling correlation and liquidity volatility index. These pictures look very similar to their empirical counterparts. First, the model simulation certainly confirms the time-varying correlation. Second and more importantly, it also confirms that a positive correlation is mostly likely to occur under strong market liquidity.

While not reported here, the distribution skewness for crisis case turned out to be even positive in many occasions.
volatility. As shown in Figure 5, a rolling correlation usually turns positive whenever liquidity volatility soars (i.e., whenever the liquidity volatility index number spikes).

7 Concluding Remarks

This paper shows that a long run risks type model combined with the liquidity volatility can explain the time-varying correlation structure between equity and currency returns. A key intuition is that a severe market liquidity volatility breaks the positive connection between the intertemporal marginal rate of substitution and asset values and thus reverses the otherwise negative relationship between equity and currency returns. Using consumption, asset markets and aggregate liquidity data, I provide direct empirical evidence to support the key economic channels highlighted in the model.

This study could be extended in many directions. First, panel data and instrumental variable approaches controlling for country fixed effects and endogeneity issues could possibly enrich the empirical results on the correlation structure. Second, one could also compare the testable implications of the model in this paper to data. For instance, the main empirical findings in this study show that during the course of the 2008 US financial crisis, the U.S dollar values on average appreciated against many other major currencies at a monthly frequency and, very interestingly, that a fall in equity returns in the U.S at a monthly frequency was most of the time actually less severe than many other countries (i.e., a positive correlation between FX and relative equity returns during the 2008 financial crisis). According to the model in this paper, this evidence is consistent with the hypothetical model prediction that the U.S dollar appreciation was a result of relatively higher US consumption volatility and that the relatively less severe equity returns were caused by the relatively higher US liquidity volatility. Although, the FX behavior during the 2008 financial crisis is quite hard to explain by any economic theory, one could easily test this paper’s model hypothesis using data on the consumption and liquidity volatilities during the current financial crisis. Finally, it would be interesting to consider the endogenous aggregate liquidity process as mentioned earlier. This endogenized framework would open up new channels through which equity, FX rates and the market liquidity-driven financial crisis are all endogenously determined.

References


Appendix

Proof for the equation (17) and (18)

Define $W_t$ as a price of equity before dividend at time $t$ and then the formula for this price should be as below.

$$W_t = E_t \sum_{j=0}^{\infty} M_{t+j}^{NM} d_{t+j}$$

where $M_{t+j}^{NM}$ is the nominal stochastic discount factor at time $t+j$. The nominal rate of return on this equity is then given by

$$R_{d,t+1}^{NM} = \frac{W_{t+1}}{W_t - d_t} = \frac{d_{t+1} (1 + Z_{t+1})}{d_t Z_t}$$

where $Z_t$ is defined as a price to dividend ratio. The standard log linearization of $R_{d,t+1}^{NM}$ gives a following equation

$$r_{d,t+1}^{NM} = \ell_0 + \ell_1 p_{d,t+1}^{NM} - p_{d,t}^{NM} + \Delta d_{t+1} \quad (20)$$

Now the proof for the e.q.(17) and (18) follows as below.

First, the log nominal price to dividend ratio, $p_{d,t}^{NM}$ is conjectured as

$$p_{d,t}^{NM} = B_0 + B_x x_t + B_z z_t + B_{gs} \sigma_{gt}^2 + B_{xs} \sigma_{xt}^2 + B_{ls} \sigma_{lt}^2 \quad (21)$$

Second, a standard Euler equation for equities is assumed

$$E_t[exp(m_{t+1}^{NM} + r_{d,t+1}^{NM})] = 1 \quad (22)$$

where $m_{t+1}^{NM}$ is the log of nominal stochastic discount factor.

Third, substitute e.q.(12), (20) and (21) into e.q.(22)

$$E_t[exp(m_{t+1}^{NM} + r_{d,t+1}^{NM})] = E_t[exp\{m_0 - \mu_x + m_x x_t - z_t + m_{gs} \sigma_{gt}^2 + m_{xs} \sigma_{xt}^2 - (\lambda_\eta + \varphi \pi_g) \sigma_{gt} \eta_{t+1} - (\lambda_\nu + \varphi \pi_x) \sigma_{xt} \ell_{t+1} + \sigma_\pi \xi_{t+1} - \lambda_{gw} \omega_{g,t+1} - \lambda_{wx} \omega_{x,t+1} + \ell_0 + \ell_1 (B_0 + B_x x_{t+1} + B_z z_{t+1} + B_{gs} \sigma_{gt}^2 + B_{xs} \sigma_{xt}^2) \rho_{d,t+1} \sigma_{d,t+1}^2 - (B_0 + B_x x_t + B_z z_t + B_{gs} \sigma_{gt}^2 + B_{xs} \sigma_{xt}^2 + B_{ls} \sigma_{lt}^2) + \mu_g + (\phi + \tau) x_t + \varphi \sigma_{gt} \eta_{d,t+1} + \sigma_{dt} \xi_{t+1}\}] = 1.$$  

$25$
Even though the volatility shocks are non-Gaussian, this model specification belongs to the exponentially affine class. One of the nicest features of the exponentially affine function is that the expectations of the exponential of the state variables is exponentially linear in the current states. In consequence, solving for the equilibrium solution coefficients, $B_{gs}$ would only require us to sum up all the loadings in front of $\sigma_{gt}^2$ and to set them equal to zero. Similar logic applies to $B_{ls}$ as well. The loadings in front of $\sigma_{gt}^2$ and $\sigma_{lt}^2$ are respectively

$$0 = m_{gs} + \ell_1(v_g + v_m)B_{gs} - B_{gs} + \frac{1}{2}((\varphi_d - \gamma - \varphi_{zg} + \ell_1\varphi_{zg})^2$$

$$0 = \ell_1B_{ls}v_l - B_{ls} + \frac{1}{2}$$

Finally, rearranging the two equations above gives e.q. (17) and (18). As mentioned already, equilibrium solutions for all the other coefficients are omitted here because they are irrelevant for the purpose of this study. The exact derivation for those coefficients are almost identical as the ones in Bansal and Shaliastovich (2013).

**Proof for the equation (19)**

By taking expectation on $r_{d,t+1}$, the expected nominal return on home equity is as follows

$$E_t(r_{d,t+1}^{NM}) = \ell_0 - B_0 - B_x x_t - B_z z_t - B_{gs}\sigma_{gt}^2 - B_{xs}\sigma_{xt}^2 - B_{ls}\sigma_{lt}^2$$

$$+ \mu_g + (\phi + \tau)x_t + \ell_1B_0 + \ell_1B_x px_t + \ell_1B_z(\alpha_z z_t + \alpha_x x_t)$$

$$+ \ell_1B_{gs}(v_g + v_m)\sigma_{gt}^2 + \overline{\omega}_g) + \ell_1B_{xs}(v_x \sigma_{xt}^2 + \overline{\omega}_x) + \ell_1B_{ls}(v_l \sigma_{lt}^2 + \overline{\omega}_l)$$

where $\overline{\omega}_g$, $\overline{\omega}_x$ and $\overline{\omega}_l$ are the unconditional mean of consumption growth volatility, long-run growth volatility and liquidity factor volatility respectively. Finally based on the fact that $\ell_1B_z\alpha_z - B_z = 1$, the expected equity return differential should be

$$E_t(r_{d,t+1}^{NM} - r_{d,t+1}^{*NM}) = (z_t - z_t^*)$$

$$+ B_{gs}^{NM}(\ell_1(v_g + v_m) - 1)(\sigma_{gt}^2 - \sigma_{gt}^{*2})$$

$$+ B_{ls}^{NM}(\ell_1v_l - 1)(\sigma_{lt}^2 - \sigma_{lt}^{*2})$$