Liquidity Risk and Time-Varying Correlation between Equity and Currency Returns

Kuk Mo Jung*

School of Economics
Henan University, China

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Abstract

Using the data of 20 major Organization for Economic Co-operation and Development countries over time, this paper documents new evidence on real equity and real currency prices: higher real returns in the home equity market relative to its foreign counterparts are generally associated with real home currency depreciation at monthly frequency, but this negative correlation breaks down or even reverses during times of relatively higher aggregate economic uncertainty or volatility. This paper also proposes one plausible explanation for this time-varying correlation structure. The suggested model is based on a long-run risks-type model with time-varying liquidity risks in stock markets. With recursive preference for the early resolution of uncertainty and a negative link between the level of short-run (SR) economic growth and equity market liquidity volatility, the model demonstrates that severe SR economic uncertainty reverses the otherwise negative link between real currency and real relative equity returns.

Keywords: foreign exchange rates, long-run risks models, liquidity risks

JEL Classification Numbers: E43, F31, G12, G15

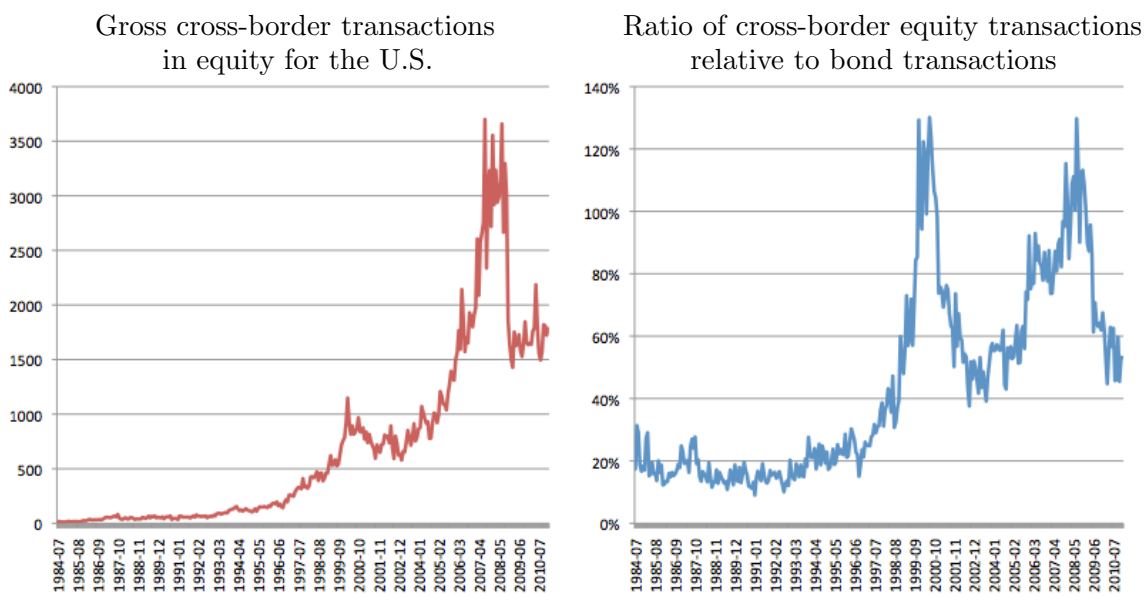
*Address: Ming Lun Street, Kaifeng, Henan 475001, China, Phone Number: +86-187-4986-2307, e-mail: *kmjung@ucdavis.edu*. I am indebted to Paul Bergin for continuous advice and support for this project. I am also grateful to Katheryn Russ and Kevin Salyer for their useful comments and suggestions. I would also like to thank the Korea Development Institute and Henan University Brownbag seminar participants for their helpful comments. I am especially indebted to William Branch and two anonymous referees for their help and many valuable suggestions. All errors are mine.

1 Introduction

Since Meese and Rogoff (1983)'s study, a long-standing challenge in international economics has been the difficulty in tying the floating exchange rates to macroeconomic fundamentals such as money supplies, outputs, and interest rates. While numerous studies have claimed success on various versions of fundamental-based exchange rate determination models (sometimes at longer horizons and over different time periods), the success of these models has not been robustly proven.

Thus, the recent exchange rate determination theory has advanced mostly outside the scope of traditional fundamental-based models. One important strand of this new literature views the link between equity and foreign exchange (FX) markets as a potential solution to this puzzle. Traditionally, international equity markets have been largely overlooked in the exchange-rate determination literature. However, the rapidly growing portion and size of equity flows reported in Figure 1 as well as the technical development in solving dynamic stochastic general equilibrium (DSGE) models involving portfolio choice have recently shown the need for a new exchange rate theory determining the exchange rates and equity-market returns jointly.

Figure 1: International Equity Transaction Trend



Source: Treasury International Capital System, U.S. Treasury

As regards the previous literature, the empirical relationship between exchange rates and stock markets has been studied for a long time, but the results are inconclusive. Most cointegration and standard Granger causality tests show no LR relationship between stock

prices and exchange rates.[1] On the other hand, for the relationship between *relative* equity returns and exchange rates, studies have generally revealed a negative relationship at the short and medium frequencies; see *2000 BIS Quarterly Review*, Brooks, Edison, Kumar, and Slock (2001), Cappiello and Santis (2007), and Hau and Rey (2006). In other words, previous studies have essentially shown that for a pair of countries, one country's currency appreciation tends to be associated with a fall in relative equity returns of the other country at the short to medium frequency.

The first objective of this study is to re-examine the correlation between equity returns and exchange rates in *real* terms; that is, the correlation between *real* currency returns and *real* relative equity returns. This is a new approach in the literature. Our sample includes a cross-section of 20 major Organization for Economic Co-operation and Development (OECD) countries with flexible exchange rate regimes. The time span of the data is from 1991/1 to 2014/12. Using standard panel estimators, this study finds the empirical results generally in line with the findings of previous studies even in real terms. However, a novel aspect of my new findings is that the generally observed negative correlation tends to disappear or even turn positive for an extended period of time, especially during times of economic uncertainty. To rigorously test this hypothesis, that is, to test whether the structural correlations are conditional on the degree of economic uncertainty, I include the interaction between (real) relative equity returns and a proxy for such aggregate economic uncertainty as an additional regressor into the otherwise standard regression equation of previous studies. Following several robustness checks, I find the coefficient on the interaction term almost always positive and often statistically significant. In sum, this study documents new evidence that the correlation between real relative equity and real FX returns is conditional on the degree of economic uncertainty.[2]

To the best of my knowledge, no previous study has accounted for this newly observed evidence. Thus, the second objective of this study is to provide a potential explanation for this new empirical evidence, and to that end, I develop a long-run risk-based model.

[1] See Granger, Huang, and Yang (2000) for details

[2] *Nominal* correlations tend to generate similar time-varying patterns as well. Appendix Table 7 illustrates these results with a smaller set of sample countries. However, the correlation results could still be driven by nominal factors such as the monetary policy. Nevertheless, I consider only real returns in this study for the following reasons. First, constructing a monetary model where the inflation process affects the real pricing kernel and the equilibrium price to dividend ratio so that the time-varying correlation mechanism works similarly to the current empirical evidence would require very demanding empirical tasks. For instance, one would have to empirically support the inflation-sensitive real pricing kernel, for example, through a jointly distributed process for monetary and dividend shocks. However, this is beyond the scope of this study. Further, a monetary model with monetary policy, uncertainty, and real pricing kernel evolving jointly would be implausible under a partial equilibrium framework, as in the current model. While worth pursuing, I leave all such (nominal) correlation issues to a further research.

The model features two uncertainty channels: the volatility of short-run (SR) consumption innovation, and the idiosyncratic volatility of equity cash flows. The former operates in the usual manner. First, the model assumes the Epstein-Zin (EZ) preference of agents, as in Bansal and Shaliastovich (2013). The introduction of EZ preference with risk aversion and intertemporal elasticity of substitution, both greater than 1, implies that the agents prefer the early resolution of uncertainty. Under this condition, a relative increase in SR uncertainty at home lowers the aggregate wealth and domestic asset prices in this model. At the same time, high SR uncertainty acts as a negative supply shock and raises the value of domestic goods, thus leading to an appreciation of the domestic currency. This is what the current model suggests as the main mechanism for the negative co-movement between relative equity returns and exchange rates.

The second offsetting channel comes from the movement in idiosyncratic volatility. A key point in this channel is that an increase in idiosyncratic volatility increases the valuation of contemporaneous assets through a convexity effect, often referred to as the Pastor and Veronesi (2006) effect. By its nature, idiosyncratic volatility does not impact the aggregate consumption, pricing kernel, and exchange rates. However, the model crucially assumes high idiosyncratic volatility in bad times of low SR consumption growth. Thus, the times of low SR consumption growth at home are associated with high domestic currency value (supply effect), high idiosyncratic volatility (model assumption), and high equity valuation (convexity effect). This eventually leads to a positive co-movement between relative equity returns and exchange rates.

Thus, the final conditional correlation depends on which of the two effects dominates. The model implies that the sum of SR uncertainty at home and abroad plays a pivotal role. In short, when the sum of two SR uncertainties is relatively high, the idiosyncratic volatility effect dominates. The reason is straightforward. The higher the sum of SR uncertainty, the higher is the above-mentioned supply effect. Thus, the positive co-movement gets relatively strengthened. This summarizes why and how the model accounts for the empirical facts found in this paper, that is, why the correlations tend to become positive during a crisis associated with higher degree of SR economic uncertainty.

Note importantly that I use calibration to show that the magnitudes of these effects are consistent with the data. First, I link idiosyncratic uncertainty to the movements in equity market liquidity following Acharya and Pedersen (2005). Specifically, the current study primarily focuses on one particular definition of equity market liquidity, that is, the ease of trading equities in stock markets. Thus, idiosyncratic volatility and equity market liquidity volatility are used interchangeably in the current study; these show the degree to which trading costs in the stock markets fluctuate. Using the liquidity proxies in the data, I

finally show that the calibration output of the model can successfully match salient features of the macroeconomic and asset-price data and quantitatively support the model channels described above.

2 Related Literature

Unlike the vast literature on exchange rate movements researched through the lens of the Uncovered Interest Parity (UIP) condition, studies on the link between currency and equity returns are relatively scarce in the literature. Nevertheless, the major strands of such studies generally show a negative link between currency and relative equity returns, as mentioned earlier. The recent studies in line with these findings include Kim (2011) and Melvin and Prins (2015). Yet, the *perfectly* negative correlation implied by existing theoretical models do not account for my new empirical results that show a time-varying correlation between equity and FX returns. Again, the main theoretical contribution of this paper is a plausible explanation for this newly observed evidence.

An alternative strand of the literature shows that the correlation between relative equity and currency returns can be non-negative; see Griffin, Nardari, and Stultz (2004), Pavlova and Rigobon (2007), Chabot, Ghysels, and Jagannathan (2014), and Cenedese, Payne, Sarno, and Valente (2015). In particular, Pavlova and Rigobon (2007) offer a portfolio-balance model theoretically endogenizing the dynamics of *real* equity prices and *real* exchange rates, which is relevant to this study.[3] Compared to the literature predicting *perfectly* negative correlations, Pavlova and Rigobon (2007) provide richer implications on how the equity and foreign exchange markets co-move in response to shocks. To state their results briefly, supply shocks generate a negative co-movement between FX and equity returns whereas demand shocks generate exactly opposite results. Therefore, their model predicts the relationship between FX and equity returns to critically hinge on the dominant relationship between demand and supply shocks. The terms of trade act as driving force behind this interesting mechanism. Unlike their study, this paper's main mechanism involves two different *volatility* shocks: liquidity volatility shocks and SR consumption volatility shocks.

This paper also relates to the broader asset-pricing literature on structural models and exchange rates. For example, the recent works of Colacito and Croce (2011) and Colacito and Croce (2013) build on the long-run risk model, which is relevant to the current paper. Verdelhan (2010), Heyerdahl-Larsen (2014), and Stathopoulos (2012), on the other hand,

[3] Heathcote and Perri (2013) provided numerical impulse responses of excess equity returns and real exchange rates to supply and demand shocks within their theoretical model. However, the numerical results basically follow Pavlova and Rigobon (2007) and Hau and Rey (2006)'s analytical predictions.

utilize habit models. Some other studies focus on rare disasters and crashes; for example, Farhi and Gabaix (2016) and Chernov, Graveline, and Zviadadze (2014). Lastly, Geromichalos and Jung (2015) study the exchange rates incorporating the fact of over-the-counter FX trading.

The paper is organized as follows. Section 3 documents the newly found empirical evidence using updated data. Sections 4 and 5 present the model in a rigorous manner. Section 6 discusses the model predictions on FX and equity returns. Section 7 directly tests whether the model quantitatively replicates the empirical evidence with a calibration exercise. Section 8 concludes the paper.

3 New Evidence

3.1 Data

This empirical study focuses on 20 OECD countries (including a reference country, i.e., the home country U.S.) having a flexible exchange rate regime against the U.S. dollar and data on the variables used in this study. The sample of countries includes Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, Norway, Portugal, Korea, Spain, Sweden, Switzerland, U.K., and the U.S. Since this analysis focuses also on the relationship between the variables at a monthly frequency, all the related data are based on average monthly values.

Data on the home countrys nominal FX rates per unit of foreign currency, that is, $\$/£$, were collected from the *Board of Governors of the Federal Reserve System*. As for the monthly aggregate stock market index data for each sample country, the "Total Share Prices for All Shares" index constructed by the *Federal Reserve Bank of St.Louis* was used. In order to convert these values into real terms, the monthly consumer price index (CPI) data, again collected by the *Federal Reserve Bank of St.Louis*, were used.

For the proxy of aggregate equity market liquidity, the Amihud measure of illiquidity, the most basic and widely accepted measure for aggregate liquidity in the literature (see Amihud (2002)), was constructed for each sample country. This is a ratio of the absolute value of monthly stock market returns to monthly stock market trade volume. It basically gives the price shift due to one unit of trade (indirect transaction cost measures in stock markets). Again, for comparison, this study uses an alternative proxy for such stock market liquidity, namely, the TED spread, or the difference between the LIBOR and government bond interest rates. This particular measure is widely referred to as a proxy for "funding liquidity" (the ease of trading using leverage) in the literature; see Amihud, Mendelson,

and Pedersen (2013). For macro-volatility-based uncertainty measures, this study uses the data on each country's industrial production index (IPI) and private final consumption expenditure (PFCE) from the *Federal Reserve Bank of St.Louis*.[4] I use these two variables to construct the ex-post volatility measures based on macro fundamentals, which is explained in section 3.2.

Government bond yields (three-month bond rates) and the three-month LIBOR rates data were obtained from the *Federal Reserve Bank of St.Loius*, and the stock market trade volume data were obtained from *Yahoo Finance*. All the data are from 1991/1 to 2014/12. Since this paper explicitly deals with monthly frequency data, I use the data of monthly changes in stock and FX returns to analyze the correlation structure.

3.2 Variables

The *real* monthly stock market returns between months t and $t+1$ for country i, that is, R_t^i, is calculated as follows:

$$R_t^i = \ln(SI_{t+1}^i) - \ln(SI_t^i) - \left\{\ln(CPI_{t+1}^i) - \ln(CPI_t^i)\right\},$$

where SI_t^i is the stock market index for month t of country i and CPI_t^i is the CPI of country i for month t. Similarly, the monthly change in *real* FX rates of country i's currency relative to the U.S. dollar from t to $t+1$, that is, Δq_t, is calculated as

$$\Delta q_t^i = \left\{\ln(FX_{t+1}^i) - \ln(FX_t^i)\right\} + \left\{\ln(CPI_{t+1}^i) - \ln(CPI_t^i)\right\} - \left\{\ln(CPI_{t+1}^{U.S}) - \ln(CPI_t^{U.S})\right\},$$

where FX_t^i is the nominal U.S. dollar price per unit of country i's currency, that is, $\$/£$, for month t.

The main equity market liquidity proxy used in this study is the Amihud illiquidity, that is, lq_t^i for country i at month t, calculated as

$$lq_t^i = \frac{|R_t^i|}{TV_t^i},$$

where TV_t^i is the measure for aggregate stock market trade volume for country i at time t.

IPI_t^i and $PFCE_t^i$ refer respectively to country i's proxy for aggregate production and consumption levels at time t.

Finally, $\sigma^2_{PD_t^i}$, $\sigma^2_{AC_t^i}$, $\sigma^2_{R_t^i}$, and $\sigma^2_{lq_t^i}$ are defined as the volatility of IPI_t^i, $PFCE_t^i$, R^i, and

[4] The highest frequency of data for PFCE was quarterly. Furthermore, I use the PFCEs measured in constant prices to obtain real values.

lq^i, respectively, for month t. For the calculation of these measures, I use a two-year rolling variance of measures for ΔIPI_t^i, $\Delta PFCE_t^i$, R_t^i, and lq_t^i respectively. Table 1 reports the summary statistics of these variables.[5] For the computation of these statistics, I use all data across the sample countries and time; I also use them extensively for the quantitative analyses that follow.

Table 1: Summary Statistics

Variables	# of Observations	Mean	Std. Dev
IPI_t^i	5128	94.2883	19.5192
$\sigma^2_{PD_t^i}$	5127	0.0541	0.07177
R_t^i	5740	0.0028	0.0528
$\sigma^2_{R_t^i}$	5280	0.0273	0.0255
lq_t^i	1874	0.2304	0.2241
$\sigma^2_{lq_t^i}$	1576	0.0408	0.0466
Δq_t^i	5453	0.0069	0.2012

3.3 Econometric Analyses

Table 2 provides the empirical estimates of monthly correlations between foreign currency values relative to the U.S. dollar and excess foreign stock index returns over the U.S. counterpart. It shows two standard panel estimates on β (fixed effects (FE) estimates controlling for country-specific FE and pooled ordinary least squares (OLS) estimates) for four different time periods, 1991/1 to 1998/12, 1999/01 to 2001/12, 2002/01 to 2010/12, and 2011/01 to 2014/12.

Table 2: $\Delta q_t^i = \alpha_i + \beta[R_t^i - R_t^{U.S}] + \varepsilon_t^i$

Periods	1991/01-1998/12	1999/01-2001/12	2002/01-2010/12	2011/01-2014/12
Panel with FE	-0.171^{***}	1.22^{**}	-0.19^{***}	-0.035
Pooled OLS	-0.176^{***}	1.159^{**}	-0.187^{***}	-0.037
# of cross-section	19	19	19	19
# of periods	95	36	108	48

Note: Cluster-robust standard errors are used for all the four regressions. *, **, and *** indicate that the coefficient is significant at the 10%, 5%, and 1% levels, respectively.

First, both the pooled OLS and FE estimates on β during the periods 1991/011998/12 and 2002/122010/12 are statistically significant and negative. This regression evidence even

[5] I use the first difference for IPI_t^i and $PFCE_t^i$ to calculate the two-year rolling variance measures because the macro data on industrial production and consumption are non-stationary. I do not report the summary statistics for $PFCE_t^i$ and $\sigma^2_{AC_t^i}$ here because of the quarterly frequency. In fact, they are not used for the quantitative analyses that follow.

in real terms is in line with what Hau and Rey (2006) and others found in the data, a negative correlation. However, this particular negative correlation certainly does not appear to hold universally over time, as Table 2 shows. The table shows that the correlation reverses its sign to positive, and this positive correlation is statistically significant during the period 1999/012001/12. Furthermore, the beta coefficients also become statistically insignificant during the 2011/012014/12 period.

What is interesting is that these two periods are closely linked to the times of global economic uncertainty. For instance, the 1999/012001/12 period coincided with a series of various world-wide economic crises, such as the Asian financial crisis, Russian default crisis, Long-Term Capital Management (LTCM) crisis, and dot-com bubble crisis. The 2011/012014/12 period also contributed to the current European debt crisis, which mainly appears as financial market turmoil in the major euro countries. In fact, appendix Table 8 also shows the individual OLS estimates for the major euro countries during that period. The beta coefficient of 8 out of 11 euro countries turn out to be positive, although not statistically significant.

In sum, all these interesting findings lead to a null hypothesis if the correlation between the currency and relative equity returns are conditional on the degree of economic uncertainty or volatility such that the correlation tends to become positive during periods of uncertainty. In order to test this hypothesis, I include an interaction term between $[R_t^i - R_t^{U.S}]$ and five different proxies for the sum of SR economic uncertainty measures for a pair including country i, that is, $X_{j,t}^i\, j \in \{1, 2, 3, 4, 5\}$, into the baseline regression equations in Table 2 as an additional regressor.

$X_{1,t}^i$ is the Chicago Board Options Exchange Volatility Index (VIX), a popular measure of the implied volatility of the S&P 500 index options. The VIX time-series are widely used as a proxy for short-term global market uncertainty trend. Similarly, the TED spread, a popular indicator of SR credit uncertainty risk in the U.S. economy, is used for $X_{2,t}^i$. Since these two U.S. measures are known to be highly correlated across countries, they are chosen as the proxy for *total* SR uncertainty for country pair i. $X_{3,t}^i$ is based on the sum of equity return volatility of the two countries; that is, $X_{3,t}^i = [\sigma^2_{R_t^i} + \sigma^2_{R_t^{U.S}}]$. $X_{4,t}^i$ captures the aggregate production volatility measures for the pair i; that is, $X_{4,t}^i = [\sigma^2_{PD_t^i} + \sigma^2_{PD_t^{U.S}}]$. Lastly, $X_{5,t}^i$ uses the aggregate consumption volatility measures for pair i; that is, $X_{5,t}^i = [\sigma^2_{AC_t^i} + \sigma^2_{AC_t^{U.S}}]$.

Table 3 reports the estimation results. All the beta coefficients become insignificant, except when the VIX measure is used to proxy uncertainty, that is, $X_{2,t}^i$. In the latter case, the beta coefficient is significantly negative at the 5% level. In contrast, the gamma coefficients become positive and statistically significant when the uncertainty proxy is based on financial indicators; that is, $X_{1,t}^i$, $X_{2,t}^i$, and $X_{3,t}^i$. Thus, the null hypothesis cannot be rejected at

Table 3: $\Delta q_t^i = \alpha_i + \beta[R_t^i - R_t^{U.S}] + \gamma[R_t^i - R_t^{U.S}]X_{j,t}^i + \varepsilon_t^i$

Estimator	Panel with FE		Pooled OLS	
	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\beta}$	$\hat{\gamma}$
With $X_{1,t}^i \equiv$ TED Spread	0.062	**0.143***	0.056	**0.140***
# of observation	5453		5453	
Estimator	Panel with FE		Pooled OLS	
	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\beta}$	$\hat{\gamma}$
With $X_{2,t}^i \equiv$ VIX	**−0.294****	**0.02***	**−0.294****	**0.02***
# of observation	5453		5453	
Estimator	Panel with FE		Pooled OLS	
	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\beta}$	$\hat{\gamma}$
With $X_{3,t}^i \equiv$ Equity Return Vol	-0.168	**9.938***	-0.175	**9.976***
# of observation	5016		5016	
Estimator	Panel with FE		Pooled OLS	
	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\beta}$	$\hat{\gamma}$
With $X_{4,t}^i \equiv$ Production Vol	0.132	0.439	0.13	0.354
# of observation	5109		5109	
Estimator	Panel with FE		Pooled OLS	
	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\beta}$	$\hat{\gamma}$
With $X_{5,t}^i \equiv$ Consumption Vol	0.037	0.001	0.011	0.001
# of observation	1823		1823	

Note: Cluster-robust standard errors are used for all five regressions, and quarterly variables are used for the regression with $X_{5,t}^i$.

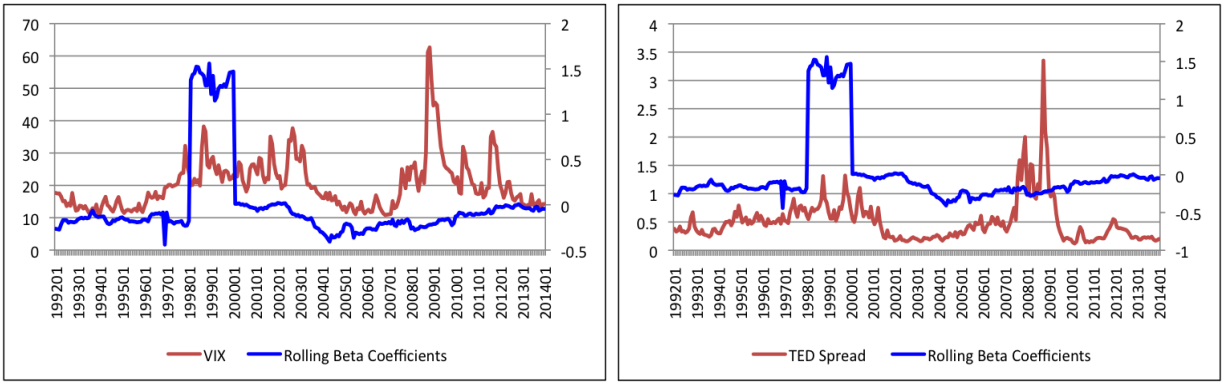
the 10% significance level.[6] In short, this result indicates that higher SR uncertainty of an economy has a strong tendency to generate positive correlations between the currency and the relative equity returns.

Unfortunately, all gamma estimates with uncertainty proxies based on macro volatility measures, that is, $X_{4,t}^i$ and $X_{5,t}^i$, become insignificant, although their signs still remain positive. These results might harm the robustness of the results. However, I argue that these insignificant results might not be serious caveats, especially for models built on fundamental macro volatilities, such as the one developed in this paper. First, note that the slope, $\hat{\gamma}$, has the right sign for both cases and that these measures are generally more noisy than the market ones accurately computed from prices. Second, and more importantly, the ex-post macro volatility measures might also include long-run volatility risks. This is well known in the long-run risks literature, for example, Bansal and Yaron (2004). Therefore, some mismeasurement issues could have caused such insignificant coefficients. The latter, nevertheless, reinforces the importance of SR uncertainty risks in deriving a model for the time-varying correlations between equity and currency returns. Accordingly, section 4 precisely develops an SR uncertainty-driven time-varying correlation model.

[6] In fact, the *p-value* for both estimation cases with $X_{2,t}^i$ and $X_{3,t}^i$ are close to 5%; that is, it could be that the null hypothesis cannot be rejected even at the 5% significance level.

For comparison, Tables 9, 10, 11, 12, and 13 report respectively the estimation results with five different $X^i_{j,t}$ using OLS estimators for the individual countries. The results are basically in line with Table 3. Most countries with $X^i_{1,t}$, $X^i_{2,t}$, and $X^i_{3,t}$ volatility proxies show positive gamma estimates, although most of them are also statistically insignificant. For cases with $X^i_{4,t}$ and $X^i_{5,t}$, most estimates, even those with beta coefficients, turn out to be insignificant, and their signs show no consistent patterns either.

Figure 2: Two-Year Rolling Beta Coefficients with VIX and TED Spread



Finally, Figure 2 illustrates the two-year rolling estimates of the aggregate beta coefficients in pooled regression along with the VIX measure and TED spread respectively. Three notable patterns can be found. First, the rolling beta coefficients remain negative for most of the time, except for the late 1990s global crisis periods. Second, the coefficients exhibit a rising trend after the 2008 global financial crisis. Lastly, the two SR uncertainty measures tend to spike up during these two crisis periods as well. On the whole, these patterns provide illustrative support for the validity of the null hypothesis.

Appendix Figure 8 also plots the two-year rolling correlation between Δq^i_t and $[R^i_t - R^{U.S}_t]$ with $X^i_{3,t}$ for 19 country pairs. This figure clearly shows that the correlation is neither perfectly positive nor perfectly negative for most country pairs. Further, they are certainly time-varying and show a strong tendency to turn negative during relatively *tranquil* periods. However, these trends tend to reverse during times of economic stress for many pairs. To sum up, all these evidence call for an SR uncertainty-based model that accounts for this sign-switching correlation structure, which I pursue in the following sections.

4 The Model

4.1 EZ Recursive Utility

The representative investor preference for the uncertain aggregate consumption stream C_t is assumed to have a functional form of the EZ utility function:

$$U_t = [(1-\beta)C_t^{\frac{1-\gamma}{\theta}} + \beta(E_t U_{t+1}^{1-\gamma})^{\frac{1}{\theta}}]^{\frac{\theta}{1-\gamma}}, \tag{1}$$

where β, ψ, and γ are the time discount factor, intertemporal elasticity of substitution (IES), and risk aversion parameter, respectively. Parameter θ is defined as $\theta = (1-\gamma)/(1-1/\psi)$. As pointed out in Bansal and Shaliastovich (2013), the logarithm of intertemporal marginal rate of substitution (IMRS) for these preferences is given by

$$m_{t+1} = \theta \log \beta - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta - 1) r_{c,t+1}, \tag{2}$$

where $\Delta c_{t+1} = \log(C_{t+1}/C_t)$ is the growth rate of aggregate consumption and $r_{c,t+1}$ is the log of the return on an *imaginary* asset delivering aggregate consumption as its dividend for each time period. This return is not observed in the data.

4.2 Aggregate Consumption Process

I adopt exactly the same consumption process in Bansal and Shaliastovich (2013), where the home and foreign countries differ only in consumption volatility, that is, $\sigma_{g,t}$, $\sigma^*_{g,t}$, and consumption growth innovations, that is, η_{t+1}, η^*_{t+1}. From now on, the foreign country variables are indexed by a superscript *. The consumption dynamics for the home country are as follows:

$$\begin{aligned} \Delta c_{t+1} &= \mu_g + x_t + \sigma_{g,t}\eta_{t+1} \\ x_{t+1} &= \rho x_t + \sigma_{x,t} e_{t+1} \\ \sigma^2_{g,t+1} &= v_g \sigma^2_{g,t} + \omega_{g,t+1} \\ \sigma^2_{x,t+1} &= v_x \sigma^2_{x,t} + \omega_{x,t+1}, \end{aligned} \tag{3}$$

where x_t is the persistent long-run expected growth component. The fact that the home and foreign countries share the same long-run component reflects the historical point that the long-run growth prospects across countries are similar. Note that this long-run consumption shock (e_t) is persistently transmitted to the future consumption process whereas the SR consumption shock (η_t) is not. The consumption growth differences between two countries in this model are captured by the difference in SR consumption shocks and volatilities.

For tractability, η_{t+1} and e_{t+1} are assumed to follow the standard normal distribution. The innovations in volatility processes $\omega_{g,t+1}$ and $\omega_{x,t+1}$ are assumed to follow a gamma distribution with a mean of $\bar{\omega}_g$ and $\bar{\omega}_x$ and a variance of σ^2_{gw} and σ^2_{xw}, respectively. The study assumes no contemporaneous correlation between $\omega_{g,t+1}$ and $\omega_{x,t+1}$ for simplicity. Finally, I present empirical justification for the time-varying volatilities of consumption and long-run components in Bansal and Shaliastovich (2013).

4.3 Aggregate Dividend Process

Following Bansal and Shaliastovich (2013), the aggregate dividend in this economy follows the following process.

$$\Delta d_{t+1} = \mu_g + \phi x_t + \varphi_d \sigma_{g,t} \eta_{d,t+1}, \tag{4}$$

where the average dividend growth rate is equal to the aggregate consumption rate, that is, μ_g, and the volatility of dividend growth is simply φ_d times greater than the consumption counterpart. For tractability, I assume the independence of consumption and dividend growth shocks.

4.4 Aggregate Equity Market Liquidity Process

Since this is (to the best of my knowledge) the first long-run risks model explicitly incorporating the liquidity process in asset markets, some of the structural assumptions in this section might appear to be non-standard and hence deserve detailed explanation.

First, the *liquidity-adjusted* home and foreign country aggregate dividend processes take the following forms respectively.

$$\begin{aligned} \Delta D_{t+1} &= \mu_g + (\phi + a) x_t + \varphi_d \sigma_{g,t} \eta_{d,t+1} + \sigma_{l,t} \zeta_{t+1}, \\ \Delta D^*_{t+1} &= \mu_g + (\phi + a) x_t + \varphi_d \sigma^*_{g,t} \eta^*_{d,t+1} + \sigma^*_{l,t} \zeta^*_{t+1}. \end{aligned} \tag{5}$$

For simplicity, the dividend growth shocks, that is, $\eta_{d,t+1}$ and $\eta^*_{d,t+1}$, and liquidity shocks, that is, ζ_{t+1} and ζ^*_{t+1}, are assumed to be independent and identically distributed (i.i.d.) normal processes with no covariance. The volatility processes of liquidity shocks for each country are given by

$$\begin{aligned} \sigma^2_{l,t+1} &= v_l \sigma^2_{l,t} + \omega_{l,t+1}, \\ \sigma^{*2}_{l,t+1} &= v_l \sigma^{*2}_{l,t} + \omega^*_{l,t+1}. \end{aligned} \tag{6}$$

The innovations in volatility processes $\omega_{l,t+1}$ and $\omega^*_{l,t+1}$ are assumed to follow a gamma distribution with no contemporaneous correlations. $\bar{\omega}_l$ and σ^2_{lw} are respectively the mean and variance of $\omega_{l,t+1}$ (the same applies to the foreign case).

Note that the key difference between Δd_{t+1} and ΔD_{t+1} is the existence of a secondary dividend shock factor, $\sigma_{l,t}\zeta_{t+1}$. A large and growing literature interprets and estimates such factors as the volatility of future idiosyncratic profitability; for example, Pastor and Veronesi (2006), Schorfheide, Song, and Yaron (2014), and Johnson and Lee (2014). This paper takes a complementary approach and links such idiosyncratic volatilities to liquidity in equity markets. Economically, this is motivated by the recent studies of Acharya and Pedersen (2005) who, in a simplified setup, show that the liquidity component can be directly mapped into the dynamics of (cash and liquidity cost adjusted) dividends. Further, this approach allows for quantitatively calibrating the idiosyncratic component of the dividend into the observable liquidity proxies in the data, which is precisely what this paper does in the quantitative section.

In what follows, I provide a clear rationale as to why equity market liquidity can be alternatively understood as the idiosyncratic component of dividend. To begin with, this model defines (equity market) liquidity as the ability to buy or sell large quantities of assets quickly and at low costs in stock markets. Specifically, we follow Acharya and Pedersen (2005)'s precise definition, the per share cost of selling aggregate security, for example, f_t.[7] Moreover, following Acharya and Pedersen (2005), ΔD_{t+1} implicitly assumes that the process of $\{f_t\}_{t=0}^{\infty}$ is exogenously given by

$$\Delta f_{t+1} = -ax_t - \sigma_{l,t}\zeta_{t+1}, \tag{7}$$

where ζ_{t+1} is the liquidity (level) shock and $\sigma_{l,t}$ is the time-varying volatility of ζ_{t+1}.

The assumption that Δf_{t+1} has a persistent long-run growth component, as in Δc_{t+1}, is empirically supported. Acharya and Pedersen (2005) report highly persistent U.S. equity market liquidity with an autocorrelation of around 0.9 at monthly frequency. Brunnermeier and Pedersen (2008) demonstrate the pro-cyclical nature of the asset market liquidity provision and offer a theoretical explanation based on the funding liquidity-constrained investors' decisions. Hence, this evidence justifies the liquidity process in eq. (7), which contains a

[7] Potentially, one could introduce FX market liquidity into this model. Focusing on the last concept stated above, the aggregate liquidity of the FX market can be defined as the per currency unit cost of selling currencies. However, I *implicitly* assume that such costs are zero in my model. This simplification is due to two reasons. First, it would greatly reduce the complexity of the theoretical analysis of this paper. If one decided to introduce the liquidity of currencies as well as equities in this framework, she would have to be very precise in modeling the correlation between currency and equity liquidities. One cannot simply assume the i.i.d. process for each, since liquidity exhibits the "commonality effect" across different asset markets; please see Amihud et al. (2013). Second, and most importantly, the FX market has been perceived as arguably the most "liquid" market in the world by both academics and practitioners. In fact, the bid-ask spreads are razor thin in FX markets and the trading volume and market depth are by far the greatest among different kinds of asset markets; please see Evans and Lyons (2002).

persistent long-run component, that is, pro-cyclicality.

ζ_{t+1} could be viewed as shocks to aggregate transaction costs, such as broker fees and bid-ask spreads, in stock markets. In reality, these aggregate liquidity shocks can be easily linked to macroeconomic events that suddenly dry up equity market liquidity, in the sense that dealers dramatically widen their bid-ask spreads, take the phone off the hook, or close down operations as their trading houses run out of cash and take their money off the table. Clearly, much evidence on such events has been documented in the literature. Furthermore, these events are found to be recurring. Amihud, Mendelson, and Wood (1990) show that the stock market crash of October 19, 1987, can be partly explained by the decline in investor perceptions of market liquidity. Amihud et al. (2013) also show that equity market liquidity dried up during the hedge fund Long-Term Capital Management (LTCM) collapse and Russian default. In recent times, stock markets around the world also experienced the drying up of liquidity during the collapse of Lehman Brothers and Bear Sterns in 2008. Amihud et al. (2013) also argue that the "flash crash" of 2010 in the U.S. stock market is another recent example of a liquidity event.

Further, the model allows for time-varying liquidity volatility, as shown in $\sigma_{l,t}$. This specification is well supported by existing studies; see Amihud (2002) and Acharya and Pedersen (2005). Section 3 of this study directly calculates two proxies for such aggregate equity market liquidity, that is, the Amihud measure of illiquidity and TED spreads. Figures 5 and 6 clearly illustrate the time-varying trends (both in levels and volatility) of Amihud illiquidity measures for 11 selected OCED countries.[8]

Note importantly that my model is based only on exogenous liquidity shocks. In other words, the economy has no endogenous mechanism by which liquidity shocks can become more volatile during any recession period. However, in my model, this exogenous shock shows that the time-varying correlations between equity and FX returns are triggered by the magnitude of economic uncertainty. I leave the task of endogenizing the liquidity shock for future research.

The main advantage of this exogenous liquidity process in the equity market is that it allows one to compute the*liquidity-adjusted* asset prices in equilibrium easily, as suggested by Acharya and Pedersen (2005). Basically, the agents in this economy can buy the aggregate security at, for instance, P_t, but must sell it at $P_t - f_t$. Acharya and Pedersen (2005) show that the equilibrium asset price process $\{P_t\}_{t=0}^{\infty}$ under exogenously given Δd_{t+1} and $\Delta f_{t+1}, \forall t$ is equivalent to the one under the imaginary dividend process of $\Delta d_{t+1} - \Delta f_{t+1}, \forall t$.

This equivalence result holds true in the current framework as well for the following

[8] The other nine countries in the sample do not have a reasonably long span of time series data and hence are not reported here.

reasons. First, the assumption of EZ preference in my model does not affect the equivalence result since Acharya and Pedersen (2005) chose to work with constant relative risk aversion (CRRA) preferences for a tractability reason. According to them, the equivalence result holds for an arbitrary increase and the concave utility defined on $(-\infty, \infty)$ as long as the conditional expected net returns are normal, which is exactly the characteristics of Bansal and Shaliastovich (2013) upon which the current model is built.

However, the critical assumption to establish Acharya and Pedersen (2005)'s result in this framework is that the agents ought to sell the security every period. Therefore, they work on a simple overlapping generations model where the old always sell securities after one period (when they die). With an infinitely living representative agent, as in this framework, one can apply the equivalence result only to the case where the representative agent buys and sells the aggregate security every period.

As Acharya and Pedersen (2005) point out, deriving a general equilibrium equity price level in a more general setting with endogenous holding periods would be an onerous task. This study avoids this task and instead takes a reduced-form approach. Thus, the current model restricts equilibrium asset prices to the case where agents are exogenously assumed to trade assets every period.

Consequent to this equivalence result, the current study works on a (liquidity-adjusted) asset pricing model where the aggregate equity dividend follows an *imaginary* process of $\Delta D_{t+1} \equiv \Delta d_{t+1} - \Delta f_{t+1}, \forall t$, in which one can interchangeably consider Δf_{t+1} as idiosyncratic profitability parts, as in Pastor and Veronesi (2006). This summarizes why and how the current study links idiosyncratic volatility to liquidity in equity markets.

Finally, I impose one crucial structural assumption on shock processes. As explained so far, all the shock processes are assumed to be idiosyncratic. Yet, one exception is introduced as follows.

Assumption 1 *A variance-covariance matrix for $(\omega_{l,t}, \eta_t)$, Σ is given by*

$$\Sigma = \begin{bmatrix} \sigma_{lw}^2 & \tau \\ \tau & 1 \end{bmatrix}$$

where $\tau < 0$. The same applies to the foreign counterpart.

In other words, the SR consumption growth level and liquidity volatility shocks for each country follow a joint distribution with a negative contemporaneous correlation. This assumption plays a pivotal role in generating a time-varying correlation between currency and equity return differentials, as intuitively explained in section 1. This assumption is provided with empirical support in the quantitative section 7. Section 6 rigorously shows the

underlying mechanism in detail.

5 Asset Markets

5.1 Stochastic Discount Factor

First, Bansal and Shaliastovich (2013) show that the log-linearized return on the *imaginary* asset paying out the aggregate consumption every period is given by the following processes, which are linear in state variables.[9]

$$\begin{aligned} r_{c,t+1} &= \kappa_0 + \kappa_1 pc_{t+1} - pc_t + \Delta c_{t+1}, \\ pc_t &= A_0 + A_x x_t + A_{gs}\sigma_{g,t}^2 + A_{xs}\sigma_{x,t}^2, \end{aligned} \tag{8}$$

where pc_t is the ratio of log wealth or price to consumption. The solution coefficients for $\kappa_0, \kappa_1, A_0, A_{xs}$ are shown in the appendix of Bansal and Shaliastovich (2013). These coefficients are not important for the following analysis and hence are not reported here. The solution coefficients A_x and A_{gs} are given by

$$A_x = \frac{1 - \frac{1}{\psi}}{1 - \kappa_1 \rho}, \quad A_{gs} = \frac{(1-\gamma)(1-\frac{1}{\psi})}{2(1 - \kappa_1 v_g)}.$$

Note that A_x and A_{gs} would have been negative and positive values, respectively, if the CRRA preference with risk aversion was greater than 1. Under these conditions, a higher consumption volatility $\sigma_{g,t}^2$ today would raise the asset prices today, which is certainly counterintuitive. In contrast, if the intertemporal elasticity of substitution and risk aversion are greater than 1, A_{gs} becomes negative, resulting in a negative relationship between the contemporaneous consumption volatility and asset prices. This is precisely what Bansal and Yaron (2004) put forward to theoretically explain the relationship between consumption volatility and asset prices.

By combining the Euler condition in eq. (2) and the equilibrium price-consumption ratio in eq. (8), I obtain an analytical expression for the intertemporal marginal rate of substitution (IMRS) as follows.[10]

$$\begin{aligned} m_{t+1} = m_0 + m_x x_t + m_{gs}\sigma_{g,t}^2 + m_{xs}\sigma_{x,t}^2 \\ -\lambda_\eta \sigma_{g,t}\eta_{t+1} - \lambda_e \sigma_{x,t} e_{t+1} - \lambda_{gw}\omega_{g,t+1} - \lambda_{xw}\omega_{x,t+1}, \end{aligned} \tag{9}$$

[9] Bansal and Shaliastovich (2013) show that the log-linearized solution to the model is similar to the solution of the model based on numerical methods.

[10] The computation of IMRS is greatly facilitated because the expectations of the state variables exponential is linear in the current states. See Bansal and Shaliastovich (2013) for details

where λ_η and λ_e are the market prices of SR and long-run risks.

λ_{gw} and λ_{xw} are the market prices of SR and long-run volatility risks, respectively. They can be given by

$$\lambda_{gw} = -\left(\gamma - \frac{1}{\psi}\right)(\gamma - 1)\frac{\kappa_1}{2(1-\kappa_1 v_g)},$$
$$\lambda_{xw} = -\left(\gamma - \frac{1}{\psi}\right)(\gamma - 1)\frac{\kappa_1}{2(1-\kappa_1 v_g)}\left(\frac{\kappa_1}{1-\kappa_1\rho}\right)^2.$$

Note that these risk compensation parameters, λ_{gw} and λ_{xw}, are zero in the CRRA utility case, whereas they become negative when the agents prefer the early resolution of uncertainty, that is, $\gamma > 1$ and $\psi > 1$.

m_{gs} and λ_η are the two most important solution coefficients since the expected difference between the home and foreign stochastic discount factor (SDF) comes from different consumption volatility levels, that is, $\sigma^2_{g,t}$ and $\sigma^{*2}_{g,t}$, which become critical in FX movements.

$$m_{gs} = -\frac{1}{2}(\gamma - \frac{1}{\psi})(\gamma - 1), \quad \lambda_\eta = \gamma.$$

If both IES and risk aversion are greater than 1, the IMRS's sensitivity to current consumption volatility, that is, m_{gs}, becomes negative. In other words, the pricing kernel's negative sensitivity to consumption volatility is totally consistent with eq. (8), where asset prices fall when the consumption volatility increases. This is a unique feature of the EZ preference since the typical CRRA utility implies no impact of consumption volatility on the pricing kernel, that is, $m_{gs} = 0$.

With this analysis of the pricing kernel, I can obtain the equilibrium currency and equity prices as well as expected returns, as discussed in the following two sections.

5.2 Equilibrium Real Foreign Exchange Rate

Backus, Allan, and Chris (2001) show that with regard to complete markets, foreign currency investment amounts to shorting a claim paying off home SDF and going long in a claim paying off foreign SDF. In other words, the following condition holds:

$$s_{t+1} - s_t = m^*_{t+1} - m_{t+1}, \tag{10}$$

where s_t is the real FX rate in home currency per unit of foreign currency and m^*_{t+1} and m_{t+1} are the pricing kernel for the foreign and home countries respectively. Intuitively, a higher foreign SDF is consistent with foreign consumers who value tomorrow's consumption

goods compared to home consumers. This would in turn mean a higher relative real price for foreign goods tomorrow, that is, foreign currency appreciation.

Under the complete market assumption, eqs. (9) and (10) give the equilibrium real FX process in this economy as follows:

$$s_{t+1} = s_t + m_{gs}\left\{\sigma_{g,t}^{*2} - \sigma_{g,t}^2\right\} - \lambda_\eta\left\{\sigma_{g,t}^*\eta_{t+1}^* - \sigma_{g,t}\eta_{t+1}\right\} - \lambda_{gw}\left\{\omega_{g,t+1}^* - \omega_{g,t+1}\right\}. \tag{11}$$

As in Bansal and Shaliastovich (2013), the expected FX changes depend on the current consumption volatility difference, that is, $\sigma_{g,t}^{*2} - \sigma_{g,t}^2$.

$$E_t[s_{t+1} - s_t] = \frac{1}{2}(\gamma - \frac{1}{\psi})(\gamma - 1)(\sigma_{g,t}^2 - \sigma_{g,t}^{*2}). \tag{12}$$

The intuition behind eq. (12) is straightforward. A higher domestic consumption volatility today lowers the domestic pricing kernel under the EZ preference, with both γ and ψ being larger than 1. Thus, the relative price of home goods is expected to fall tomorrow, indicating expected home currency depreciation.

5.3 Equilibrium Real Equity Returns

From the appendix, the log-linearized real return on home equity is a linear process in state variables.

$$\begin{aligned} r_{d,t+1} &= \ell_0 + \ell_1 pd_{t+1} - pd_t + \Delta D_{t+1}, \\ pd_t &= B_0 + B_x x_t + B_{gs}\sigma_{g,t}^2 + B_{xs}\sigma_{x,t}^2 + B_{ls}\sigma_{l,t}^2, \end{aligned} \tag{13}$$

where pd_t is the ratio of log price to (imaginary) dividend and the solution coefficients for B_{gs} and B_{ls} are as follows:

$$B_{gs} = \frac{0.5(\varphi_d - \gamma)^2 - (\gamma - 1/\psi)(\gamma - 1)}{1 - \ell_1 v_g} < 0, \tag{14}$$

$$B_{ls} = \frac{1}{2(1 - \ell_1 v_l)} > 0. \tag{15}$$

Again, I do not report other equilibrium solution coefficients here because they do not affect the following analyses.

The sign of the two coefficients, B_{gs} and B_{ls}, needs to be interpreted. First, the sign of B_{gs} depends on the model and preference parameters. Nevertheless, its sign is most likely to be negative under the typical parameter values widely used in the long-run risks literature. This will be made clear in the calibration section. As already discussed, the assumption of both IES and risk aversion being larger than 1 is critical for the negative B_{gs}.

Second, B_{ls} is always positive. This implies that a higher liquidity volatility, that is, $\sigma_{l,t}^2$, *ceteris paribus* boosts equity prices, reducing the expected equity returns. This is one crucial finding of the current model. In contrast to consumption volatilities, liquidity volatility has a positive effect on asset prices. As previously explained, the mechanism behind this is similar to Pastor and Veronesi (2006). Higher liquidity volatility, assumed to be idiosyncratic in this framework, increases the contemporaneous price dividend ratio.

Finally, according to Bansal and Yaron (2004), the risk premium on the aggregate equity security in this model can be described as

$$RP_t \equiv E_t\left[r_{d,t+1} - r_{f,t+1}\right] + \frac{1}{2}var_t\left[r_{d,t+1}\right] = -cov_t\left[m_{t+1}, r_{d,t+1}\right],$$

where $r_{f,t+1}$ is the risk-free rate in this economy. The following lemma shows a closed-form solution to risk premium in this economy.

Lemma 1 *The risk premium takes the following form:*

$$RP_t = \lambda_e \ell_1 B_x \sigma_{x,t}^2 + \lambda_{gw} \ell_1 B_{gs} \sigma_{gw}^2 + \lambda_{xw} \ell_1 B_{xs} \sigma_{xw}^2 + \lambda_\eta \ell_1 B_{ls} \tau,$$

where both the IES and risk aversion are larger than 1, $\lambda_e \ell_1 B_x > 0$, $\lambda_{gw} \ell_1 B_{gs} > 0$, $\lambda_{xw} \ell_1 B_{xs} > 0$, *and* $\lambda_\eta \ell_1 B_{ls} \tau < 0$.

Proof. See the appendix. ■

First, the risk premium on the aggregate security is time-varying because the volatility of the long-run growth trend, $\sigma_{x,t}^2$, fluctuates. Second, the loadings in front of $\sigma_{x,t}^2$, σ_{gw}^2, and σ_{xw}^2 become positive when both the IES and risk aversion parameters are assumed to be greater than 1. This intuitively implies that during periods of high economic uncertainty, the risk premia will rise. All these are standard characteristics of long-run risk models.

Finally, note that this model assumes a fixed negative covariance, τ, between the liquidity volatility shock, ω_l, and SR consumption level shock, η. Thus, the changes in τ have a leveling effect on the risk premium, although they cannot influence the time-varying risk premium. The intuition is that a higher $|\tau|$ results in the aggregate liquidity volatility and aggregate consumption moving in opposite directions to a greater extent. Since the aggregate liquidity volatility *effectively* increases the asset returns through a mechanism as in Pastor and Veronesi (2006), a higher $|\tau|$ lowers the risk premium; that is, it lowers the constant level in risk premium.

6 Correlations on FX and Equity Returns

This section focuses on the time-varying correlations between the FX and relative equity returns implied by the model. What is critical for triggering the sign-switching behavior of correlation in this model is the magnitude of the SR consumption volatility or SR economic uncertainty. The following proposition summarizes the model prediction on the time-varying correlations.

Proposition 1 *The conditional covariance of unexpected (or realized) FX movements and relative equity returns has the following closed-form solution in this model economy.*

$$cov_t\left[FX_{t+1}, RD^*_{t+1}\right] = -\lambda_{gw}B_{gs}2\left[\sigma^2_{gw} + (\bar{\omega}_g)^2\right] - \tau\lambda_\eta B_{ls}\left[\sigma^*_{g,t} + \sigma_{g,t}\right], \tag{16}$$

where $FX_{t+1} = s_{t+1} - s_t$ *and* $RD^*_{t+1} = r^*_{d,t+1} - r_{d,t+1}$.

Assuming that $\gamma > 1$, $\psi > 1$, *and* $\tau < 0$, *a unique positive threshold level exists for* Q *such that if* $\sigma^*_{g,t} + \sigma_{g,t} > Q$, *the conditional covariance becomes positive, and otherwise, it becomes negative. The* Q *is given by*

$$Q = \frac{-\lambda_{gw}B_{gs}2\left[\sigma^2_{gw} + (\bar{\omega}_g)^2\right]}{\tau\lambda_\eta B_{ls}} > 0. \tag{17}$$

Proof. See the appendix. ■

In order to develop the intuition behind this result, one can work with the following equation instead.

$$cov_t\left[FX_{t+1}, RD^*_{t+1}\right] = E_t\left[(FX_{t+1} - E_t[FX_{t+1}])(RD^*_{t+1} - E_t[RD^*_{t+1}])\right]. \tag{18}$$

Thus, one can intuitively understand conditional covariance as how the realized FX movement, that is, $FX_{t+1} - E_t[FX_{t+1}]$, and realized equity return differentials, that is, $RD^*_{t+1} - E_t[RD^*_{t+1}]$, co-move in response to various different shocks.

From the appendix, the realized FX movement and realized equity return differentials can be expressed as

$$FX_{t+1} - E_t[FX_{t+1}] = -\lambda_{gw}\left\{\omega^*_{g,t+1} - \omega_{g,t+1}\right\} - \lambda_\eta\left\{\sigma^*_{g,t}\eta^*_{t+1} - \sigma_{g,t}\eta_{t+1}\right\}, \tag{19}$$

$$\begin{aligned} RD^*_{t+1} - E_t[RD^*_{t+1}] = {} & B_{gs}\left\{\omega^*_{g,t+1} - \omega_{g,t+1}\right\} + B_{ls}\left\{\omega^*_{l,t+1} - \omega_{l,t+1}\right\} \\ & \varphi_d\left\{\sigma^*_{g,t}\eta^*_{d,t+1} - \sigma_{g,t}\eta_{d,t+1}\right\} + \left\{\sigma^*_{l,t}\zeta^*_{t+1} - \sigma_{l,t}\zeta_{t+1}\right\}. \end{aligned} \tag{20}$$

First, note that the realized SR consumption volatility differentials, that is, $\omega^*_{g,t+1} - \omega_{g,t+1}$, cause the realized FX movement and realized equity return differentials to move in opposite directions. As regards FX movements, because the market price of SR volatility risks is negative, that is, $\lambda_{gw} < 0$, the higher realized SR consumption volatility in this framework makes the agents give a higher value for consumption. In other words, the realized pricing kernel rises as a response. This explains why higher realized SR consumption volatility differentials lead to a realized foreign currency appreciation, as shown in eq. (19). On the contrary, higher realized SR consumption volatility depresses the realized equity return differentials, as standard in the long-run risk models. This is shown in eq. (20), with a negative value for B_{gs}. In sum, $\omega^*_{g,t+1} - \omega_{g,t+1}$ always induces the conditional covariance to become negative. This effect is captured by the (negative constant) first term in eq. (16).

The differentials of other shocks do not affect the conditional covariance because they are i.i.d., except through the negative contemporaneous correlation between ω_l and η. Specifically, as shown in eq. (20) with $B_{ls} > 0$, the higher realized liquidity volatility differentials, that is, $\omega^*_{l,t+1} - \omega_{l,t+1}$, boost the realized equity return differentials *effectively* through the second independent dividend effect, as explained earlier. Importantly, the increase in $\omega^*_{l,t+1} - \omega_{l,t+1}$ is also likely to cause a fall in $\sigma^*_{g,t}\eta^*_{t+1} - \sigma_{g,t}\eta_{t+1}$, that is, the realized SR consumption growth differentials, through a negative τ. Since the market price of SR risks, that is, λ_η, is positive in this framework, a reduction in $\sigma^*_{g,t}\eta^*_{t+1} - \sigma_{g,t}\eta_{t+1}$ is equivalent to an increase in realized pricing kernel differentials. This, in turn, would lead to realized foreign currency appreciation. Furthermore, the magnitude of such realized foreign currency appreciation is amplified by the level of current SR consumption volatility, that is, $\sigma^*_{g,t}$ and $\sigma_{g,t}$, as implied in eq. (19). This summarizes why $\omega^*_{l,t+1} - \omega_{l,t+1}$ creates a positive pressure for the conditional covariance, and more importantly induces the covariance to be time-varying. This effect is captured by the (positive and time-varying) second term in eq. (16).

Eventually, the SR economic uncertainty level, that is, $\sigma^*_{g,t} + \sigma_{g,t}$ relative to the Q in Proposition 1, determines whether the correlations become positive or negative. Again, this model prediction is consistent with the empirical evidence found in section 3. The following section finally conducts various quantitative analyses to empirically support the model.

7 Quantitative Analysis

7.1 Empirical Evidence for Assumption 1

Empirical support for τ, that is, the negative correlation between SR consumption growth level shock η_t and liquidity volatility shock $\omega_{l,t}$, is provided in Table 4.

Table 4: Correlation between IPI growth rate and liquidity volatility

Countries	$corr(IPI_{Return}, \sigma^2_{TED})$	$corr(IPI_{Return}, \sigma^2_{lq})$
U.S.	-0.489***	-0.235**
Germany	-0.123**	0.011
U.K.	-0.086	-0.133*
South Korea	0.039	0.065
France	-0.058	-0.275**
Austria	-0.001	-0.15*
Netherlands	-0.003	-0.075
Belgium	-0.037	-0.1
Japan	-0.016	-0.13*
Canada	-0.202***	-0.036

Note: Only countries with available σ^2_{lq} are reported. σ^2_{TED} refers to the 2-year rolling variance of TED spreads. The same significance level applies to *, ** and *** as before.

The third column shows the Pearson's correlation coefficients between the two-year rolling variance of the Amihud measure of equity market illiquidity (proxy for $\omega_{l,t}$) and IPI returns (proxy for SR economic growth, i.e., η_t) for ten major OECD countries.[11] Half of them are negative and statistically significant. In terms of sign only, all of them except for Germany are negative too. For robustness check, the second column uses a different proxy for $\omega_{l,t}$; that is, a two-year rolling variance of TED spreads. It also shows similar results qualitatively, although only 30% of them are statistically significant.

7.2 Calibration

In this section, I show that the proposed model quantitatively characterizes the time-varying correlations. To that end, I undertake a calibration exercise. Note that I calibrate only the liquidity side of the model outlined in (6)(7) at a monthly frequency, to obtain model outputs from monthly simulations that match the key moments of relative equity returns, the exchange rate, and the liquidity proxy. As regards the consumption and (intrinsic) dividend side of the model outlined in (3) and (5), I adopt the parameter values of Bansal and Shaliastovich (2013) since the latter is identical to the current model, except for the idiosyncratic component of the dividend proxied by the equity market liquidity factor.

Table 5 reports the parameters used in this section. Most of them, except for aggregate

[11] σ^2_{TED} ranges from 1992/11 to 2014/01 and from 1992/01 to 2012/06 for Belgium and Canada, respectively. σ^2_{lq} ranges from 1992/06 to 2014/01, 1999/04 to 2014/01, 2000/01 to 2014/01, 2003/06 to 2014/01, 2003/12 to 2014/01, 2005/10 to 2014/01, and 2009/08 to 2014/01 for Canada, South Korea, U.K, Japan, Switzerland, (Austria, Belgium, France, and Netherlands), and Germany.

liquidity dynamics, were taken from Bansal and Shaliastovich (2013). Note that because the model had to be calibrated at monthly frequency, the parameter values from Bansal and Shaliastovich (2013) were transformed into monthly values (see Bansal and Shaliastovich (2013) for a detailed explanation). Basically, they chose these values such that the consumption processes in this model economy correspond well to the U.S. and U.K. (mostly U.S.) business-cycle data. As for preference parameters, nothing is at odds with regard to the standard values in the literature, except for the magnitude of the IES (greater than 1), which is still debatable. However, we choose the IES value of 1.5 to match the inverse relationship between asset values and consumption volatility, as in Bansal and Shaliastovich (2013).

The calibrated parameters of aggregate liquidity dynamics need to be explained because they are unique features of this model. Most importantly, the aggregate liquidity volatility level, σ_l, and the aggregate liquidity volatility of volatility, σ_{lw}, are chosen to match the data on the Amihud measure of equity market illiquidity volatility used in section 3 (see the summary statistics for $\sigma^2_{lq^i_t}$ in Table 1). The aggregate liquidity volatility's persistence parameter, v_l, is chosen to match the U.S. equity market illiquidity volatility having an autocorrelation of about 0.9, as reported by Acharya and Pedersen (2005). I choose a fixed parameter, τ, for the contemporaneous covariance between η and ω_l to match the average of correlation coefficients between IPI returns and the volatility of Amihud illiquidity measures, which is about -0.11 (see the third column in Table 4 for details). Finally, these parameter values confirm the negative marginal effect of consumption volatility on asset prices, which is about -145 for B_{gs}, and the positive marginal effect of liquidity volatility on asset prices, which is about 4 for B_{ls}.

Table 6 reports the calibration output of the model, based on 1,000 simulations over a 20-year period. As the Table shows, the model can reasonably match the salient features of the equity return differentials, exchange rate movements, and equity market liquidity data.[12] I do not compare the model output and data for consumption dynamics here because it is already shown in Bansal and Shaliastovich (2013), which indeed shows a very close connection between the model and data.

The calibration output of the model seems to match the equity market data very well. In particular, the equity return differentials and liquidity volatilities in the data and model are very close to each other with a difference of more or less just 1%. Yet, the difference between the model and data seems to be larger for exchange rate movements. The standard deviation of Δq from the model is about seven times greater than that of the data. One explanation for this large difference could be related to the well-known "exchange rate disconnect puzzle" of

[12] The moments for the data are identical to those in Table 1.

Table 5: Model parameter values

Consumption Dynamics	
Mean of consumption growth	$\mu_g = 0.0016$
Expected growth persistence	$\rho = 0.991$
SR volatility level	$\sigma_g = 0.0042$
SR volatility persistence	$v_g = 0.803$
SR volatility of volatility	$\sigma_{gw} = 1.57 * 10^{-5}$
Long-run volatility level	$\sigma_x = 1.67 * 10^{-4}$
Long-run volatility persistence	$v_x = 0.9799$
Long-run volatility of volatility	$\sigma_{xw} = 1.96 * 10^{-6}$
Aggregate Dividend and Liquidity Dynamics	
Aggregate dividend sensitivity to long-run news	$\phi + a = 1.25$
Aggregate dividend growth volatility level	$\varphi_d = 10$
Aggregate liquidity volatility level	$\sigma_l = 0.2$
Aggregate liquidity volatility persistence	$v_l = 0.9$
Covariance parameter for SR growth and liquidity volatility	$\tau = -0.01$
Aggregate liquidity volatility of volatility	$\sigma_{lw} = 2.16 * 10^{-3}$
Preference Parameters	
Discount factor	$\beta = 0.9978$
Intertemporal elasticity of substitution	$\psi = 1.5$
Risk aversion coefficient	$\gamma = 10$

Meese and Rogoff (1983). Put simply, the discrepancy between the implied model exchange rate volatility and its empirical counterpart is found in many international finance models.[13] Moreover, this difference can be attributed to the way the stock market liquidity process is calibrated. The fact that Bansal and Shaliastovich (2013) show a narrower gap indicates that this could be true.

Table 6: Model v.s Data

	Model		Data	
Variables	Mean	Standard Deviation	Mean	Standard Deviation
Equity Return Differential ($R_t^i - R_t^{U.S}$)	-0.0013	0.03	-0.0014	0.04
Real Exchange Rate (Δq)	0.00009	0.03	0.0069	0.2
Liquidity Volatility (σ_{lq}^2)	0.001	0.052	0.041	0.047

[13] The magnitude of discrepancy implied by the current model is by no means the biggest among similar calibration exercises in other studies, for example, Taylor (1995). However, some studies such as Colacito and Croce (2011) show that this gap can be reduced.

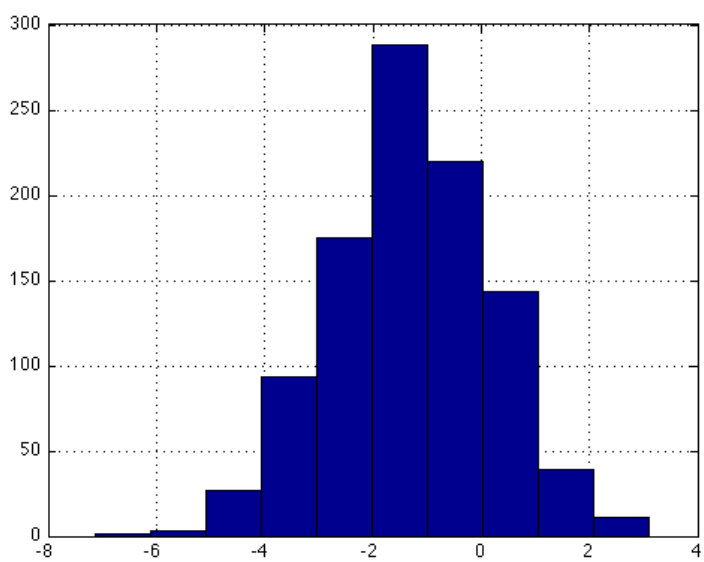
7.3 Model Implications on FX and Equity Returns

Finally, this section focuses on model implications on the time-varying correlations between FX and relative equity returns at the calibrated parameter values. To that end, I once again conduct 1000 model simulations covering a 20-year period under the parameter values specified in Table 5. In each simulation, I regressed the exchange rate movement (Δq_t) on the equity return differentials ($R_t^i - R_t^{U.S}$) similarly to Table 3.

Figure 3 reports a histogram of the estimated beta coefficients obtained from the simulation. It shows that the equity-currency correlation is negative on average (about -0.8). This is within the empirical evidence shown in Table 2. Further, only about 20% of the estimated coefficients are positive. This result also seems to be in line with the empirical evidence. From Figure 2, the beta coefficients in the pooled regression turn out be positive for 5 years (19982002) out of the whole 23 years (19912014).

Next, this paper examines the economic significance of the aggregate volatilities for the sign-switching correlation. For thispurpose, I compute the "uncertainty index", the average of $[\sigma_{gt}^i + \sigma_{gt}^{U.S}]$, for each simulation. The results show that the average of the uncertainty index conditional upon $\hat{\beta} > 0$ is about 0.012, whereas the average based on the entire sample is about 0.0084. Thus, an increase by about 43% in the uncertainty index from its average value can reverse the correlation signs from negative to positive on average.

Figure 3: Frequency distribution of regression coefficients, $\hat{\beta}$, based on 1000 simulations



8 Concluding Remarks

This study finds new evidence on the time-varying correlation between (real) equity and (real) currency returns. In particular, the negative correlation, as documented in several existing studies, becomes very weak or even overturns its sign during times of economic stress or uncertainty. Given this newly found evidence, this paper provides one plausible explanation for time-varying correlations. A key mechanism behind the possible positive link between equity and currency returns lies in the negative correlation between the *level* of SR economic growth and idiosyncratic volatility of dividends, proxied by the volatility of equity market liquidity in this paper. Since this positive link becomes stronger whenever the *volatility* of SR economic growth soars, the correlations exhibit a strong tendency to become positive during times of economic uncertainty.

This explanation is, however, not without limitations. "Uncertain" economic times are surely a combination of potentially interrelated economic events, for example, flight to quality episodes, unconventional monetary policy, and idiosyncratic components of the dividend process, which jointly create various adverse consequences for many aspects of the economy and asset markets. Equity market liquidity could be just one of those channels through which the relative prices are severely distorted during uncertain economic times. Thus, it would be interesting to endogenize the equity market liquidity process, especially in accordance with the various macroeconomic fundamentals such as monetary policy, idiosyncratic volatility, and endogenous portfolio choice of international investors.

Another avenue for future research could be the extension of the current model to the "Uncovered Interest Parity (UIP) Puzzle"; that is, high interest rate currencies tend to appreciate in the short run. Recent empirical studies point to the time-varying correlations between interest rate differentials and exchange rate movements. In particular, much evidence suggests that the UIP coefficient flips signs, especially when the measures of market volatility soar; see Brunnermeier, Nagel, and Pedersen (2008), for instance. Therefore, one could see bond market liquidity as a potential resolution to this interesting empirical pattern, as the current paper has done in the context of FX-Equity correlations.[14] I leave all these interesting exercises to future research.

[14] See Atkeson, Alvarez, and Kehoe (2007), Geromichalos and Simonovska (2014), and Jung and Lee (2015) for studies that build upon asset liquidity as a potential resolution for other international macro-finance puzzles.

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Theory Appendix

Proof for the equation (13), (14) and (15)

Define W_t as a price of equity before dividend at time t and then the formula for this price should be as below.

$$W_t = E_t \sum_{j=0}^{\infty} M_{t+j} D_{t+j},$$

where M_{t+j} is the stochastic discount factor at time $t+j$. The rate of return on this equity is then given by

$$R_{d,t+1} = \frac{W_{t+1} + D_{t+1}}{W_t} = \frac{D_{t+1}}{D_t}\frac{(1+Z_{t+1})}{Z_t},$$

where Z_t is defined as a price to dividend ratio. The standard log linearization of $R_{d,t+1}$ gives a following equation

$$r_{d,t+1} = \ell_0 + \ell_1 pd_{t+1} - pd_t + \Delta D_{t+1}. \tag{21}$$

Now the proof for the e.q.(13) follows as below.
First, the log price to dividend ratio, pd_t is conjectured as

$$pd_t = B_0 + B_x x_t + B_{gs}\sigma^2_{g,t} + B_{xs}\sigma^2_{x,t} + B_{ls}\sigma^2_{l,t}. \tag{22}$$

Second, a standard Euler equation for equities is given by

$$E_t[exp(m_{t+1} + r_{d,t+1})] = 1. \tag{23}$$

Third, substitute e.q.(22) into (21) and then into e.q.(23). This will give

$$\begin{aligned} E_t[exp(m_{t+1} + r_{d,t+1})] = E_t[exp\{&m_0 + m_x x_t + m_{gs}\sigma^2_{g,t} + m_{xs}\sigma^2_{x,t} \\ &-\lambda_\eta \sigma_{g,t}\eta_{t+1} - \lambda_e \sigma_{x,t} e_{t+1} - \lambda_{gw}\omega_{g,t+1} - \lambda_{xw}\omega_{x,t+1} \\ &+\ell_0 + \ell_1(B_0 + B_x x_{t+1} + B_{gs}\sigma^2_{g,t+1} + B_{xs}\sigma^2_{x,t+1} + B_{ls}\sigma^2_{l,t+1}) \\ &-(B_0 + B_x x_t + B_{gs}\sigma^2_{g,t} + B_{xs}\sigma^2_{x,t} + B_{ls}\sigma^2_{l,t}) \\ &+\mu_g + (\phi+\tau)x_t + \varphi_d \sigma_{g,t}\eta_{d,t+1} + \sigma_{l,t}\zeta_{t+1}\}] \\ &= 1. \end{aligned} \tag{24}$$

Even though the volatility shocks are non-Gaussian, this model specification belongs to the exponentially affine class. One of the nicest features of the exponentially affine function is that the expectations of the exponential of the state variables is exponentially linear in the current states. In consequence, solving for the equilibrium solution coefficients, B_{gs} would only require us to sum up all the loadings in front of $\sigma^2_{g,t}$ and to set them equal to zero. Similar logic applies to B_{ls} as well. The loadings in front of $\sigma^2_{g,t}$ and $\sigma^2_{l,t}$ are respectively given by

$$0 = m_{gs} + \ell_1 v_g B_{gs} - B_{gs} + \frac{1}{2}(\varphi_d - \gamma)^2,$$

$$0 = \ell_1 B_{ls} v_l - B_{ls} + \frac{1}{2}.$$

Finally, rearranging the two equations above gives e.q.(14) and (15). As mentioned already,

equilibrium solutions for all the other coefficients are omitted here because they are irrelevant for the purpose of this study. The exact derivation for those coefficients are almost identical as the ones in Bansal and Shaliastovich (2013). *Q.E.D.*

Proof for Lemma 1

First, $-cov_t\left[m_{t+1}, r_{d,t+1}\right] = -E_t\left[(m_{t+1} - E_t[m_{t+1}])(r_{d,t+1} - E_t[r_{d,t+1}])\right]$. From eq.(9) it is easy to construct $m_{t+1} - E_t[m_{t+1}]$ as

$$m_{t+1} - E_t[m_{t+1}] = -\lambda_\eta \sigma_{g,t}\eta_{t+1} - \lambda_e \sigma_{x,t} e_{t+1} - \lambda_{gw}\omega_{g,t+1} - \lambda_{xw}\omega_{x,t+1}. \tag{25}$$

By using eq.(13) and its expected value, one could derive $r_{d,t+1} - E_t[r_{d,t+1}]$ as

$$\begin{aligned} r_{d,t+1} - E_t[r_{d,t+1}] = {} & \varphi_d \sigma_{g,t}\eta_{d,t+1} + \sigma_{l,t}\zeta_{l,t+1} \\ & + \ell_1 \left\{ B_x \sigma_{x,t} e_{t+1} + B_{gs}\left(\omega_{g,t+1} - \bar{\omega}_g\right) + B_{xs}\left(\omega_{x,t+1} - \bar{\omega}_x\right) + B_{ls}\left(\omega_{l,t+1} - \bar{\omega}_l\right) \right\}, \end{aligned} \tag{26}$$

where $\bar{\omega}_g, \bar{\omega}_x$ and $\bar{\omega}_l$ are the unconditional mean of consumption growth volatility, long-run growth volatility and liquidity volatility respectively. Finally, by exploiting i.i.d shock processes and combining eq.(25) and (26), one can derive the closed form solution in Lemma 1. *Q.E.D.*

Proof for Proposition 1

Equation (19) can be easily obtained through using eq.(11). $RD^*_{d,t+1}$ can be computed using eq.(13). The result is given by

$$\begin{aligned} r^*_{d,t+1} - r_{d,t+1} = {} & - B_{gs}(1 - \ell_1 v_g)(\sigma^{*2}_{g,t} - \sigma^2_{g,t}) - B_{ls}(1 - \ell_1 v_g)(\sigma^{*2}_{l,t} - \sigma^2_{l,t}) \\ & + B_{gs}\left\{\omega^*_{g,t+1} - \omega_{g,t+1}\right\} + B_{ls}\left\{\omega^*_{l,t+1} - \omega_{l,t+1}\right\} \\ & + \varphi_d\left\{\sigma^*_{g,t}\eta^*_{d,t+1} - \sigma_{g,t}\eta_{d,t+1}\right\} + \left\{\sigma^*_{l,t}\zeta^*_{t+1} - \sigma_{l,t}\zeta_{t+1}\right\}. \end{aligned}$$

By taking expectation into this expression one could derive eq.(20).

By replacing the equation (19) and (20) into (18) and using the i.i.d. assumptions on relevant shocks one could finally get the following.

$$\begin{aligned} cov_t\left[FX_{t+1}, RD^*_{t+1}\right] = {} & E_t\Big[- \lambda_{gw} B_{gs} E_t[\omega^{*2}_{g,t+1} + \omega^2_{g,t+1}] \\ & \quad - \lambda_\eta B_{ls}\left\{E_t[\eta_{t+1}\omega_{l,t+1}]\sigma_{g,t} + E_t[\eta^*_{t+1}\omega^*_{l,t+1}]\sigma^*_{g,t}\right\}\Big] \\ = {} & - \lambda_{gw} B_{gs} 2\left[\sigma^2_{gw} + \bar{\omega}^2_g\right] - \tau\lambda_\eta B_{ls}\left\{\sigma^*_{g,t} + \sigma_{g,t}\right\}. \end{aligned}$$

Note that the second equation above uses two facts. First, $E_t[\eta_{t+1}\omega_{l,t+1}] = cov_t\left[\eta_{t+1}\omega_{l,t+1}\right] = \tau$ due to $E_t[\eta] = 0$ (the same applies to the foreign case). Second, $E_t[\omega^2_{g,t+1}] = Var_t[\omega_{g,t+1}] + (E_t[\omega_{g,t+1}])^2$ (the same applies to the foreign case). *Q.E.D*

Tables and Graphs for Individual Countries

Table 7: Monthly *nominal* correlations of foreign currency and excess foreign equity returns

$$\Delta\xi_t = \alpha + \beta[NR_t - NR_{U.S.,t}] + \epsilon_t$$

Periods	(1991/01 $\sim$ 2010/10)	(1998/01 $\sim$ 2000/12)	(2007/01 $\sim$ 2009/03)
Coefficients	$\hat{\beta}$	$\hat{\beta}$	$\hat{\beta}$
Australia	-0.004	0.006	0.201**
Germany	-0.066*	0.025	0.134***
UK	-0.119***	0.0775	-0.035
Swiss	-0.06***	0.055	-0.024
Norway	-0.038**	-0.005	0.163***
New Zealand	-0.115***	N/A	0.049
South Korea	-0.039***	N/A	0.188***
Sweden	-0.045*	N/A	0.049

Notes: $\Delta\xi_t$ denotes nominal exchange rate changes in log for a month t. NR_t stands for nominal stock index returns during a month t for each country. *,** and *** indicate that the coefficient is significant at 30%, 20% and 10% level respectively. Due to the lack of stock index data, New Zealand, South Korea and Sweden only have data starting from 2000. All the standard errors are Newey-West adjusted, and the number of lags, i.e., 7, are chosen following Andrews (1991).

Table 8: $\Delta q_t = \alpha + \beta[R_t - R_{US,t}] + \varepsilon_t$

Countries	$\hat{\beta}$
Austria	0.166
Belgium	-0.371^{**}
Finland	0.054
France	0.058
Germany	0.184
Greece	0.068
Ireland	-0.545^{***}
Italy	0.245^{**}
Netherlands	-0.455^{**}
Portugal	0.116
Spain	0.122

Note: The sample periods is for European Debt Crisis (2011/01-2012/12). All the standard errors are Newey-West adjusted, and the number of lags, i.e., 7, are chosen following Andrews (1991). The same significance level applies to *, ** and *** as before.

Table 9: $\Delta q_t = \alpha + \beta[R_t - R_{US,t}] + \gamma[R_t - R_{US,t}]X_{1,t} + \varepsilon_t$

Countries	$\hat{\beta}$	$\hat{\gamma}$	# of lags
Austria	-0.525	0.162	7
Belgium	0.122	0.232	7
Canada	0.039	-0.148	7
Denmark	-0.182^{**}	-0.072	7
Finland	0.086	0.061	7
France	0.257	-0.133	7
Germany	-0.035	-0.295	7
Greece	0.096	1.042	7
Ireland	0.819	0.609	7
Italy	0.434	0.331	7
Japan	0.033	-0.34	7
Netherlands	-0.578^{***}	0.341^{**}	7
Norway	-0.296^{***}	0.383^{***}	7
Portugal	0.47	-0.03	7
South Korea	-0.097	0.223	7
Spain	0.173	-0.162	7
Sweden	-0.133	-0.199	7
Switzerland	-0.585^{***}	0.063	7
U.K.	-0.44^{**}	-0.352^{**}	7

Note: All the standard errors are Newey-West adjusted, and the number of lags are chosen following Andrews (1991). The same significance level applies to *, ** and *** as before.

Table 10: $\Delta q_t = \alpha + \beta[R_t - R_{US,t}] + \gamma[R_t - R_{US,t}]X_{2,t} + \varepsilon_t$

Countries	$\hat{\beta}$	$\hat{\gamma}$	# of lags
Austria	-0.452^{*}	0.001	7
Belgium	-0.557	0.037	7
Canada	0 .132	-0.008	7
Denmark	-0.201^{**}	-0.001	7
Finland	-0.468	0.028	7
France	-0.482	0.031	7
Germany	-0.426^{**}	0.011	7
Greece	-1.022	0.078	7
Ireland	-1.589	0.128	7
Italy	-0.606	0.06	7
Japan	-0.005	-0.006	7
Netherlands	-0.883^{***}	0.021^{**}	7
Norway	-0.316^{***}	0.013^{**}	7
Portugal	-0.334	0.036	7
South Korea	0.876	-0.038	7
Spain	-0.329	0.02	7
Sweden	-0.296	0.003	7
Switzerland	-0.537^{***}	-0.0002	7
U.K.	-0.377^{**}	-0.012^{***}	7

Note: All the standard errors are Newey-West adjusted, and the number of lags are chosen following Andrews (1991). The same significance level applies to *, ** and *** as before.

Table 11: $\Delta q_t = \alpha + \beta[R_t - R_{US,t}] + \gamma[R_t - R_{US,t}]X_{3,t} + \varepsilon_t$

Countries	$\hat{\beta}$	$\hat{\gamma}$	# of lags
Austria	-0.629^{*}	6.551	7
Belgium	0.073	15.853	7
Canada	0.017	-3.437^{**}	7
Denmark	-0.225^{***}	-0.048	7
Finland	-0.022	3.115	7
France	-0.257	30.511	7
Germany	-0.250^{*}	4.238	7
Greece	-1.460^{**}	46.474^{***}	7
Ireland	0.910	16.042	7
Italy	-0.835	66.955	7
Japan	0.014	-7.140^{*}	7
Netherlands	-0.518^{***}	8.739	7
Norway	-0.268^{***}	5.075^{***}	7
Portugal	-0.427	44.809	7
South Korea	-0.138	3.424	7
Spain	-0.431	27.596	7
Sweden	-0.159^{*}	-3.119	7
Switzerland	-0.508^{***}	-0.867	7
U.K.	-0.498^{**}	-11.543	7

Note: All the standard errors are Newey-West adjusted, and the number of lags are chosen following Andrews (1991). The same significance level applies to *, ** and *** as before.

Table 12: $\Delta q_t = \alpha + \beta[R_t - R_{US,t}] + \gamma[R_t - R_{US,t}]X_{4,t} + \varepsilon_t$

Countries	$\hat{\beta}$	$\hat{\gamma}$	# of lags
Austria	-0.073	-8.520	7
Belgium	0.583	-5.112	7
Canada	0.059	-8.052^{**}	7
Denmark	-0.221^{**}	-0.011	6
Finland	0.200	-2.123	6
France	0.823	-39.805^{*}	6
Germany	0.1709	-16.748^{***}	7
Greece	0.944	-3.862	7
Ireland	2.220	-4.000	6
Italy	0.700	-5.296	6
Japan	-0.067	-1.560	7
Netherlands	-0.281	-1.519	7
Norway	0.076	-1.613^{**}	7
Portugal	1.978	-22.417	7
South Korea	0.145	-1.674	7
Spain	0.478	-14.502	7
Sweden	-0.434^{***}	5.074^{**}	7
U.K.	-0.439^{**}	-17.968^{*}	7

Note: All the standard errors are Newey-West adjusted, and the number of lags are chosen following Andrews (1991). The same significance level applies to *, ** and *** as before.

Table 13: $\Delta q_t = \alpha + \beta[R_t - R_{US,t}] + \gamma[R_t - R_{US,t}]X_{5,t} + \varepsilon_t$

Countries	$\hat{\beta}$	$\hat{\gamma}$	# of lags
Austria	0.141	-0.008	4
Belgium	-0.742	0.104	4
Canada	0.104	-0.048^{**}	4
Denmark	-0.285^{**}	0.002	4
Finland	0.001	0.019	4
France	0.242	-0.131	4
Germany	-0.161	-0.071	4
Greece	0.948^{*}	0.015	4
Ireland	0.956	-0.007	4
Italy	1.625	-0.318	4
Japan	-0.076	-0.019	4
Netherlands	-0.619^{**}	0.055	4
Norway	-0.167^{*}	0.031^{*}	4
Portugal	-0.676	0.038	4
South Korea	0.096	-0.003	4
Spain	-0.163	0.018	4
Sweden	-0.375^{**}	0.027	4
U.K.	-0.341	-0.071	4

Note: All the standard errors are Newey-West adjusted, and the number of lags are chosen following Andrews (1991). The same significance level applies to *, ** and *** as before.

Figure 4: Correlation between FX and equity returns with a rolling window of two-year periods

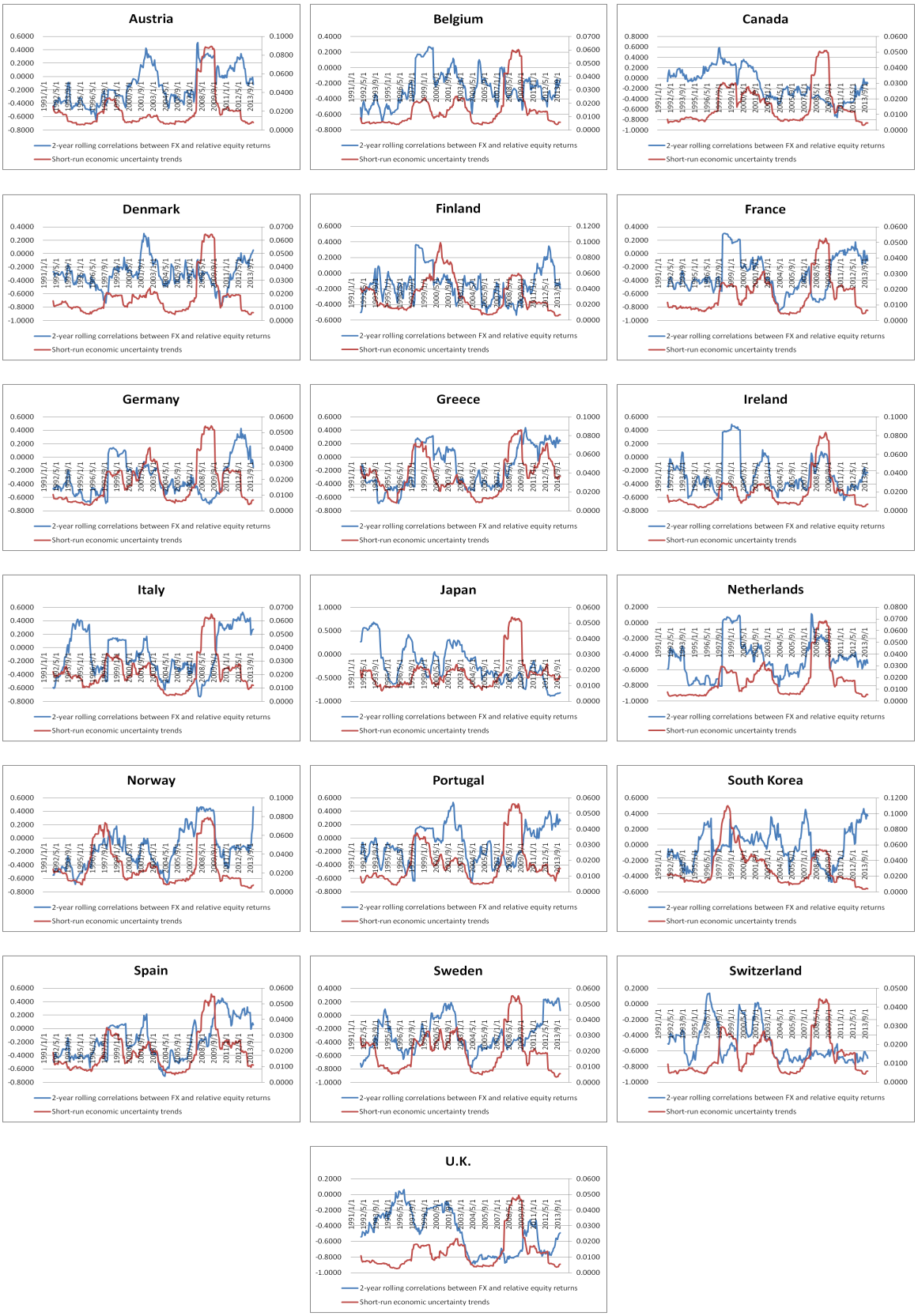

Figure 5: Amihud illiquidity measures (levels)

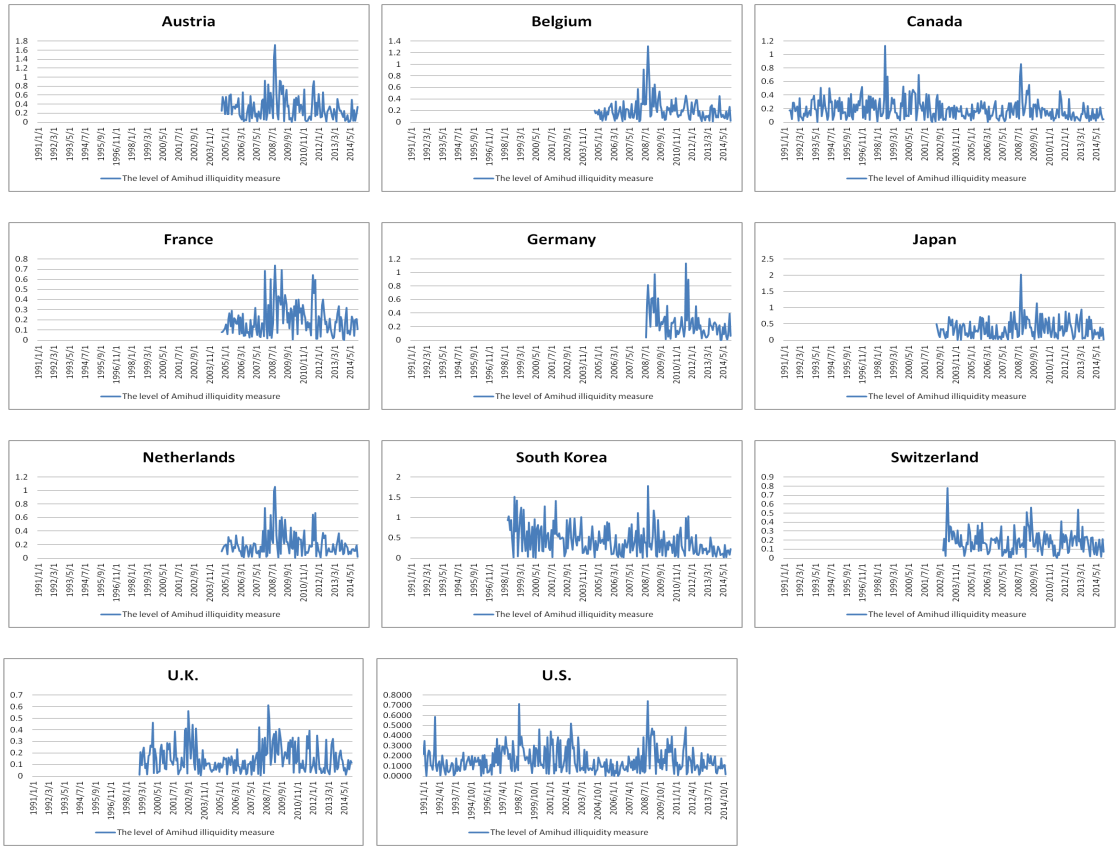

Figure 6: Amihud illiquidity measures (volatility)