A Liquidity-Based Resolution of the Uncovered Interest Parity Puzzle*

Seungduck Lee†

Sungkyunkwan University

Kuk Mo Jung‡

Sogang University

ABSTRACT

A new monetary theory is set out to resolve the "Uncovered Interest Parity (UIP)" Puzzle. It explores the possibility that liquidity properties of money and nominal bonds can account for the puzzle. A key concept in our model is that nominal bonds carry *liquidity premia*. We show that the UIP can fail to hold under the economic environment where collateral pledgeability and/or liquidity of nominal bonds and/or collateralized credit based transactions are relatively bigger. Our liquidity based theory can help understanding many empirical observations that risk based explanations find difficult to reconcile with.

JEL Classification: E4, E31, E52, F31

Keywords: uncovered interest parity puzzle, monetary search models, FOREX market

* We are grateful to Bong Hyun Baek, Athanasios Geromichalos, Espen Henriksen, Jaewook Jung, Jiwon Lee, Junsang Lee, Zhe Li, Guillaume Rocheteau, Antonio Rodriguez-Lopez, Katheryn Russ, and Ina Simonovska for useful comments and suggestions. We would also like to thank participants at Korea Foundation For Advanced Studies, UC Davis, UC Irvine, SHUFE, Tsinghua, Sungkyunkwan, and Yonsei University seminars as well as 1st Hen/INFER Applied Macroeconomics Conference. We also thank the editor and two anonymous reviewers for their constructive comments. Jung acknowledges that this work was supported by the research fund of Sogang University. Lee acknowledges that this paper was supported by Sungkyun Research Fund, Sungkyunkwan University, 2017. All errors are ours.

† Department of Economics, Sungkyunkwan University, Seoul, Republic of Korea. Email: seung.lee@skku.edu

‡ Corresponding Author: School of Economics, Sogang University 35 Baekbeom-Ro (Sinsu-Dong) Mapo-Gu, Seoul 04107, Phone: 86-(02)-705-8525, E-mail: *kmjung@sogang.ac.kr*

1 Introduction

In a seminal paper, Fama (1984) highlighted that relatively high interest rate currencies tend to appreciate on average. Interestingly, regardless of the increasing sophistication of the econometric techniques employed and of the increasing quality of the data sets utilized, researchers generally keep documenting similar results.[1] This empirical finding is considered to be an anomaly in the sense that high interest rate currencies have predictably positive excess returns, which contradicts the very foundation of the UIP condition. Accordingly, this anomaly has been widely referred to as the UIP or "forward premium" puzzle in the literature.

Yet, as Burnside et al. (2009) and Backus et al. (2010) explain, the vast majority of the literature on this puzzle is empirical, and very few theoretical attempts have been made to tackle the puzzle. Even among the theoretical literature, no consensus seems to have been reached. For instance, most prevailing theories revolve around the idea that the failure of the UIP has a close connection with the way the risk premium behaves.[2] Nevertheless, many recent studies have become critical of these risk-based explanations.[3] To that end, we take an alternative approach in this paper that the UIP violation might be attributed to endogenous liquidity properties of money and bonds.[4]

More precisely, we set out a new liquidity-based monetary model of international asset pricing, and study under what conditions the model rationalizes the UIP puzzle. Our methodology is novel for the following reason. We model the economy where nominal bonds and currencies explicitly play a liquidity role during the exchange process, and macro fundamentals, i.e.,

[1] See Bacchetta and Wincoop (2010), Backus et al. (2010), and the references therein for a recent survey of empirical findings and the literature

[2] Some recent studies offer non-risk-based explanations. For instance, see Corsetti et al. (2004), Burnside et al. (2009), Bacchetta and Wincoop (2010), and Ilut (2014).

[3] For instance, Burnside et al. (2009) point out, "It has been extremely difficult to tie deviations from uncovered interest parity to economically meaningful measures of risk". Also, see Burnside (2007) and Burnside et al. (2008) for a critical review of recent risk-based explanations of the puzzle.

[4] For instance, Atkeson et al. (2007) argue that standard monetary policy models are not suitable for studying exchange rate and therefore, call for a new monetary model of exchange rates in which time-varying liquidity drives fluctuations in the excess return on foreign bonds. In fact, Brunnermeier et al. (2008) is the first one that introduces a liquidity channel through which the UIP puzzle is rationalized. Linnenmann and Schabert (2015) offer a pioneering theoretical framework in which endogenous liquidity premia of U.S treasury securities play a pivotal role in rationalizing the UIP violation. Engel (2016) also argues that liquidity premium on government bonds may be an important foreign exchange rate determinant. We come back to these studies and discuss similarities and differences with ours later in the related literature part.

monetary policy, endogenously determine their liquidity values.[5] To reliably incorporate these novel features, our model adopts a microfounded monetary framework developed by Lagos and Wright (2005). The basic structure of the model goes as follows. There are two countries, and each country issues its own currency and nominal bonds. Unlike the conventional Walrasian framework, we allow goods trade to take place in a pairwise fashion with trading frictions such as anonymity and limited commitment, which are precisely what makes assets endogenously emerge as a medium of exchange (MOE), thereby creating asset liquidity values. Crucially, we allow bonds to serve as collateral in some pairwise meetings, while they compete with money as a direct means of payment in some other meetings. Unlike the goods trade, financial asset trade takes place in a perfectly integrated Walrasian market where agents regardless of the country frictionlessly reshuffle their portfolio of assets.

The key feature of this framework lies in the transmission mechanism of monetary policy, i.e., how changes in money supply affect nominal bond returns. Intuitively, monetary policy not only affects the real value of currency but also the rate of return on other assets that have direct or indirect liquidity properties, i.e., the nominal bonds. What is crucial is that this liquidity-based transmission mechanism opens up new possibilities for a richer set of joint dynamics between currency and nominal bond return in a way that the traditional Lucas (1982) model of international asset pricing cannot capture.[6]

To understand the rich joint dynamics of our liquidity-based model, first it is useful to explain why the aforementioned empirical facts, e.g., high interest currencies appreciate, constitute a puzzle, when viewed through the lens of the conventional Lucas (1982) model. In this model, a sudden increase in interest rate would lead to an instantaneous appreciation of the currency, followed by an expected depreciation. Technically speaking, the *nominal* intertemporal marginal rate of substitution (i.e., *nominal* bond price) of a country is negatively correlated with its inflation rate. Thus, the *Fisher effect* holds true in equilibrium, thereby implying the

[5] In doing so, we had to take a radical departure from the conventional approach by entirely abstracting from risk considerations. In other words, asset pricing in this framework is only driven by changes in liquidity values of assets. Although we take this approach to avoid complexity of the analysis, it would allows one to isolate the role of asset liquidity, if any, in solving the UIP puzzle.

[6] See Hu and Rocheteau (2015) for an extensive review on monetary search models where correlation between the currency and bond return could go either way.

UIP condition.[7] The key observation here is that the classical dichotomy, i.e., the separation between money supply and intertemporal marginal rate of substitution, *effectively* forces the nominal bond price to move in the opposite direction of inflation rate, which always ensures the UIP condition in equilibrium.

Our story builds upon the same premises as the conventional approach, namely, fully flexible prices and complete FX market. But, we break down the classical dichotomy since the intertemporal marginal rate of substitution is augmented by the exchange value or liquidity premia of assets, which in turn depends upon money supply in our model economy. What is of utmost importance is that our framework allows the correlation between anticipated inflation rate and nominal bond yield to critically hinge upon the *relative* exchange value of nominal bonds. For instance, suppose that nominal bonds are perfectly illiquid in the sense that no one in our model economy accepts bonds as an MOE in pairwise meetings. Then, our model is effectively equivalent to Lucas (1982), and the *Fisher effect* always holds true in equilibrium. Likewise, one can also consider another sub cases of our model in which bonds also play a transaction facilitator role. First, imagine a case where bonds and money are the perfect substitute, i.e., no collateralized credit transactions take place in any pairwise meeting and no exogenous liquidity differential between money and bonds exist as a direct means of payment. In this case, the no-arbitrage principle always guarantees a constant zero nominal interest rate.

Now, suppose that bonds serve the transaction facilitator role relatively more than money, i.e., bonds exhibit higher liquidity premia than money. For instance, consider a sub case of the model where some portion of pairwise meetings must require only bonds as collateral to settle the transaction. This has far-reaching implications on the *Fisher effects*, and therefore the UIP condition as follows. A lower anticipated inflation should undoubtedly decrease the nominal demand for money because agents can obtain the same consumption utility with less units of money. In other words, a lower anticipated inflation ought to cause money to become illiquid in a sense that the marginal consumption utility from holding additional unit of money becomes lower, if anticipated inflation falls. The point here is that the nominal demand for bonds ought

[7] For more detailed explanation on the rigorous relationship between monetary policy, pricing kernel, and the UIP puzzle, one can refer to Bekaert (1994), Bekaert and Hodrick (2001), Backus, Foresi, and Telmer (2001), and Alvarez et al. (2009).

to fall even further since the purchasing power of bonds in pairwise meetings here exceeds that of money by assumption. Put it differently, bonds should become even more illiquid than money in the anticipation of falling prices. As a result, the nominal rate of return on bonds must increase relatively higher than the nominal rate of return on money to compensate for the relatively higher illiquidity cost. Given that nominal rate of return on money is always zero, a lower anticipated inflation should therefore lead to an increase in nominal interest rates, breaking down the *Fisher effects*. Eventually, the UIP ends up being violated since a currency with a lower anticipated inflation is expected to appreciate.

The main message of this paper is well reflected upon the aforementioned examples. In our microfounded monetary model of international asset pricing, the UIP does not have to hold uniformly. In particular, the negative relationship between anticipated inflation and nominal bond yield is shown to be sufficient for the UIP deviation. Crucially, our framework implies that nominal bonds must exhibit relatively high enough liquidity premia in order to guarantee the sufficient condition. We rigorously show in the model that the sufficiently higher liquidity premia of bonds can be indeed achieved when the portion of collaterlized-credit-transaction-based pairwise meetings is large and/or the pledgeability of bonds as collateral is high and/or exogenous illiquidity discount on bonds as a direct means of payment is low.

One may question if our framework where bonds exhibit as high liquidity premia as money is empirically substantive. Yet, we argue that it is by no means a pure theoretical abstraction based on a recent empirical work by Krishnamurthy and Vissing-Jorgensen (2012), Nagel (2016), and Lee (2016) where high liquidity values of U.S Treasury bonds are confirmed. In particular, Lee (2016) empirically presents the effect of money growth rate on liquidity premia of the Treasuries. One can then address another potential concerns. First, not every nominal bonds, especially those issued by emerging economies, are same as the U.S. Treasury bonds. Second, the bond liquidity is surely time-varying, e.g., extreme dry-up of bond liquidity during the recent liquidity crunch episode.

Interestingly, these two issues are precisely what leads to the non-uniform UIP deviation in our framework. Put it differently, our model implies that the sufficient condition for the UIP de-

viation cannot be met whenever bonds are not liquid enough. This bond illiquidity is one of the defining characteristics of emerging market bonds and the liquidity crisis. Thus, our model predicts that the UIP should be confined to emerging economies and the liquidity crunch period. These two predictions are well supported by previous empirical studies. Bansal and Dahlquist (2000) and Frankel and Poonawala (2010) empirically confirm that the UIP deviation is pervasive only among developed currency pairs. In addition, Brunnermeier et al. (2008) demonstrate that interbank liquidity crunch has a strongly negative correlation with carry trade returns, i.e., the UIP tends to hold true when measures of market liquidity shrink. We empirically re-examine these predictions using updated data and our results reported in Table 1 reconfirm them. To the best of our knowledge, our model is the first one to rationalize these two empirical patterns simultaneously.

As for the related literature, we do not intend to thoroughly review a vast number of theories that have been proposed to make sense of the UIP evidence.[8] Broadly speaking, the theories can be assigned into two big categories, *non-rational expectation based models* and *rational expectation based models*. The former is relatively scarce and based on the idea that expectational errors or behavioral biases of investors drive the UIP deviation, e.g., behavioral biases based explanation by Froot and Thaler (1990) and *peso problems* based explanation by Lewis (1995). Yet, most theoretical attempts to solve the UIP puzzle have maintained the assumption of rational expectations. Our explanation also fits into this category. As mentioned already, most conventional rational expectation based theories argue that the failure of UIP is attributed to the behavior of the risk premium, e.g., Lustig and Verdelhan (2007) and Lustig et al. (2011) among others. Non-risk based models include, but are not limited to OTC FX market based explanation of Burnside et al. (2009), the rational inattention model of Bacchetta and Wincoop (2010), and the long run risk based explanation of Bansal and Shaliastovich (2013).

Among the rational expectation based models, Backus et al. (2010) have in common with our story to the extent that monetary policy jointly determines exchange and interest rates. The difference is they explicitly abandon the model of money in favor of the Taylor rule. They show

[8] One can refer to many excellent papers for the extensive review, e.g., Engel (2015) and Verdelhan (2010) among others.

that asymmetries in foreign and domestic Taylor rules can account for the UIP deviation. They admit, however, that their approach is partial where the consumption process and asymmetric Taylor rule coefficients are *exogenously* specified, while ours is a fully-fledged general equilibrium approach. Brunnermeier et al. (2008) also have a close connection to our theory in terms of the role that liquidity plays. They focus on *liquidity frictions* that carry traders face as a driving force behind why the UIP deviation cannot be eliminated quickly in the short run, and how marketwide liquidity crunch affects carry trade speculations and eventually a sudden shift in the UIP relation. Our story differs in that aggregate liquidity needs of the country rather than the behavior of particular carry trade speculators can also drive the UIP relation. In this respect, our theory is complementary to Brunnermeier et al. (2008). Lastly, Linnenmann and Schabert (2015) offer a liquidity-based theory of the UIP violation, closest to ours in spirit. Their key idea is essentially same as ours, i.e., the liquidity premia of nominal bonds augment the pricing kernel to the extent that certain economic environments cause positive movement between nominal interest rates and expected currency appreciation. While their pioneering work lays a foundation for the liquidity-based theory for the UIP violation, our mechanism substantially differs from them, and can in fact complement their work in many respects. First, we follow a tradition of money-search literature while they follow the interest rate rule based monetary policy literature. Second, we induce the asset liquidity value to arise within a decentralized goods market while they do by explicitly modeling the open market operation where nominal bonds serve as collateral. Finally, we work with a symmetric two-country environment where all nominal bonds endogenously exhibit liquidity premia, while they had to assume some reduced form of liquidity differential between the two country's bonds. This distinction is of particular importance because the Brunnermeier et al. (2008)'s evidence can be easily rationalized in our model framework.

In terms of methodology, our paper is also related to a growing body of money-search literature that studies how monetary policy affects asset prices through the liquidity of assets. Many find that higher nominal interest rates raise asset prices by fueling liquidity premia: for instance Geromichalos, Licari, and Suarez-Lledo (2007), Jacquet and Tan (2012), Lester, Postlewaite, and

Wright (2012) Geromichalos, Herrenbrueck, and Salyer (2015a), Geromichalos, Lee, Lee, and Oikawa (2015b), and Nosal and Rocheteau (2012). Recently, others such as Hu and Rocheteau (2015), Lagos and Zhang (2014), and Geromichalos and Jung (2019) found the opposite. Our framework can in fact nest both of these cases by allowing the correlation between nominal interest rates and inflation to go either way depending on the microstructure of decentralized market. This paper also contributes to the money-search literature that tackles traditional asset pricing puzzle through the notion of asset liquidity. Lagos (2010) explains the equity premium and risk-free rate puzzle under the explicit modeling of assets as facilitators of trade. Jung and Pyun (2016) argue that liquidity properties of U.S. government bonds generated by bilateral trading frictions in international capital markets might provide a fundamental reason behind a huge accumulation of reserves by emerging economies.

The rest of the paper is organized as follows. In Section 2, we describe the physical environment. Section 3 studies the agents' optimal behavior. In Section 4, we define a stationary, symmetric, and two-country equilibrium, and study how the UIP condition is related to monetary policy and market microstructure of pairwise meetings. Section 5 concludes.

2 Physical Environment

Time is discrete and infinite. Each period is divided into two subperiods. There are two counties, A and B. Each country has two types of agents, buyers and sellers, both of which are populated with a continuum of 1. The identity of buyers and sellers is fixed over time. All agents live infinitely and consider dynamics with a discount factor equal to $\beta \in (0, 1)$. They discount future only between periods, but not between subperiods. We will often refer to a buyer (seller) from country i as buyer i (seller i) for notational simplicity. There are three kinds of nonstorable and perfectly divisible goods: a general good produced by all agents and a special good i produced only by sellers in each country $i \in \{A, B\}$.

There are also two different types of (financial) assets in this model. First, a perfectly divisible and storable fiat currency is issued by each country's monetary authority. We denote this

asset as $money_i$, $i \in \{A, B\}$. The $money_i$ supply is stochastically determined by each country's monetary authority who injects or withdraws $money_i$ via lump-sum transfers or taxes to buyers of country i at the end of every period. Specifically the $money_i$ stock is initially given by $M_{i,0} \in \mathbb{R}_{++}$, and thereafter it grows at a stochastic rate $\gamma_{i,t}$ (i.e., $M_{i,t+1} = \gamma_{i,t} M_{i,t}$), which is assumed to follow a Markov process defined by its transition function $F(\gamma\prime, \gamma) = \Pr(\gamma_{i,t+1} \leq \gamma\prime \mid \gamma_{i,t} = \gamma)$ where $F : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}$ is continuous. Assume that the process defined by F has a stationary distribution $\Phi(\cdot)$ as well as a unique solution to $\Phi(\gamma') = \int F(\gamma', \gamma) d\Phi(\gamma)$, and that F has the Feller property. The second type is nominal bonds. In each country, a new set of Lucas (1978) trees are born every period. Each unit of the tree in country i delivers d_i units of a general good in the next period, and dies immediately afterwards.[9] We assume that d_i always equals the real value of $money_i$ in terms of a general good in every period as if each unit of the tree in country i delivered one unit of $money_i$ in the next period. This crucial assumption makes the share of these trees in country i equivalent to one-period nominal bonds of that country. For this reason we will hereinafter refer to shares of trees from country i as $(nominal)\, bond_i$. The supply of $bond_i$ is fixed over time and denoted by B_i.[10]

We now proceed to a detailed description of the subperiods characterized by different economic activities. We start with the second subperiod, and move backward. In the second subperiod, all agents have a linear technology that transforms a unit of labor into a unit of general good. All agents can then trade the general good and all types of financial assets, i.e., $money_i$ and $bond_i$, $\forall i$, within one single Walrasian or centralized market (henceforth, FM). $\varphi_{i,t}$ and $\psi_{i,t}$ respectively denotes the FM price of $money_i$ and $bond_i$ in terms of the general good at period t. Further, the nominal exchange rate at time t is defined as the FM price of $money_B$ in terms of

[9] One could introduce multi-period living trees in order to study the effects of coupon paying government bonds. However, it would simply make the analysis a little more complicated without drawing any new qualitative implication on the question at hand, i.e., the relationship between interest and exchange rates.

[10] Instead of privately as well as exogenously given Lucas tree, one could have adopted a standard government bond model as in Berentsen and Waller (2011) where the government actually issues IOU contracts promising to pay off one unit of money in the next period. However, the equilibrium outcome on asset prices would be qualitatively same as ours for the following reasons. Under the standard Berentsen and Waller (2011) framework, the change in the stock of money in period t would be given by $M_t - M_{t-1} = \tau M_{t-1} + B_{t-1} - \rho_t B_t$, where τM_{t-1} indicates the lump-sum money injections, B_t denotes the per-capita stock of newly issued bonds at time t, and ρ_t denotes the nominal bond price. Under this scenario, as already shown in Berentsen and Waller (2011), the supply of bonds in stationary equilibrium must grow at the same rate as the money growth rate γ. Moreover, the nominal price of bonds in stationary equilibrium would equal "β/γ + bond liquidity premium", which would be qualitatively identical to ours that will be presented later in Section 4.4.

$money_A$: $E_t = \varphi_{B,t}/\varphi_{A,t}$. Notice that the perfectly competitive FM assumption allows agents to trade two monies at the market clearing exchange rate. Thus the law of one price holds every period.

In the first subperiod, a decentralized goods market opens separately for each country (henceforth GM). We assume that agents can only trade in their 'domestic' market during the first subperiod. As a result, only buyer i and seller i can trade special good i in GM of country i.[11] Within any GM, trade is bilateral and anonymous. In addition, agents cannot make binding commitments, and trading histories are private in a way that precludes any borrowing and lending. This premise necessitates a medium of exchange (MOE) in any GM trade. On top of that, we make an additional assumption that GM in each country is differentiated into two types of sub-markets, depending on methods of payment: Goods Market 1 (henceforth $GM1$) and Goods Market 2 (henceforth $GM2$). Every agent visits $GM1$ ($GM2$) with probability θ $(1-\theta)$ where $\theta \in (0,1)$, and therefore all buyers and sellers match each other within each county.

In $GM1$, we assume that buyer i makes a take-it-or-leave-it (TIOLI) offer to her counterpart seller i under the *cash-in-advance* (CIA) type constraint that the buyer i can pay the seller i only with a combination of *domestic* assets, i.e., $money_i$ and $bond_i$, in exchange for a special good i. In order to incorporate the bond illiquidity as a direct MOE, we also assume that buyer i is able to transfer at most a fraction $g \in [0,1]$ of $bond_i$ holdings as an MOE. For examples, if $g = 0$, then bonds are completely illiquid, and so cannot be used as an MOE at all. If $g = 1$, then they are perfectly as liquid as money.

Unlike the $GM1$, $GM2$ only allows credit as a method of payment. Notice that the $GM2$ intuitively stands for a fraction of GM where transactions involve some form of credit, following Williamson (2012).[12] Specifically the credit in the $GM2$ means a promise that buyer i will

[11] This assumption precludes our model from considering international trade in goods, and studying its implications on the UIP puzzle. One could surely relax this assumption to make our model empirically more relevant. However doing so would greatly complicate the analysis, particularly trading protocols in Section 3.2, without providing any critical insight to the model. Furthermore, most studies that offered explanations for the UIP puzzle have stressed investment behavior in financial markets rather than trade-related factors. Given this emphasis, we also think that the no-international-trade assumption here is not a major caveat of our model.

[12] On top of that introduction of the $GM2$ in an empirically relevant way within this model is not an end itself. As will be analyzed later, it actually boosts liquidity properties of bonds to the extent that the comovement of

pay back to seller i a certain amount of general good in the coming FM in exchange for special good i. Due to anonymity and limited commitment, the buyer i cannot pay with unsecured credit (e.g., an IOU). Hence she needs to offer the seller i bonds held in a form of collateral to back the credit.[13] The credit limit is determined by the real value of bonds the buyer i places as collateral and the pledgeability parameter $h \in [0, 1)$ - the extent to which they can be used to secure loans. Similar to the $GM1$, we also adopt a TIOLI offer by buyer i as a pricing protocol. Further, we impose a similar CIA constraint as in $GM1$, i.e., the buyer i can only place domestic assets, i.e., $bond_i$, as collateral.

Note that these two CIA constraints in $GM1$ and $GM2$ are imposed for the sake of parsimony. They ensure zero equilibrium holdings of $money_{-i}$ and $bond_{-i}$ by buyer i and pin down the equilibrium nominal exchange rate. As a matter of fact, one could endogenize these two CIA constraints by introducing differential treatment of assets as a means of payment, e.g., differential recognizability costs of assets in Zhang (2014) and Lester, Postlewaite, and Wright (2012) and/or differential social norms on assets in Zhu and Wallace (2007), while keeping the main equilibrium results on asset prices unchanged. Recently, Gomis-Porqueras, Kam, and Waller (2017) show there is even no reason to treat foreign assets differently when endogenizing the CIA constraints. They show the threat of counterfeiting pins down the nominal exchange rate even if currencies are the perfect substitute. Appendix A.1. shows how to actually endogenize the two CIA constraints using Zhu and Wallace (2007) type mechanism design approach to the terms of trade determination in GMs. It also shows the equilibrium results on asset prices would be identical to those achieved under the TIOLI offer by buyers in GMs. For more detailed explanation, see Appendix A.1.

Following Lagos and Wright (2005) (LW henceforth), the utility of buyer i and seller i is

exchange rate and nominal interest rate can violate the UIP condition in equilibrium. Therefore modeling decentralized markets without credit secured by assets as collateral in this framework is *not* without loss of generality for studying the UIP puzzle.

[13] We chose to write a model where bonds serve only as collateral, not as a direct MOE in GM2, because we believe it's empirically more relevant. We believe it would be natural to categorize modern transactions that need an MOE into two: the one where bonds are partial substitute for money and the other where money cannot substitute for bonds at all. The latter, for instance, refers to REPO and/or wholesale funding markets where bonds serve as a sole transaction facilitator role, i.e., collateral. The former includes meetings where bonds compete with money as an MOE, but are at a disadvantage to money. For example, people can choose to directly buy a house(or a car) or to opt for financing by using it as collateral, which usually costs a little more. In sum, our GM1 is meant to represent the former type of transactions, while GM2 is supposed to reflect on the latter type.

respectively given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \{U(x_t) - h_t + u(q_t)\},$$
$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \{U(x_t) - h_t - q_t\},$$

where x_t and h_t stand for the consumption of general good and labor inputs to produce that good in the second subperiod of period t, respectively. q_t represents the amount of special good i produced by the seller i and consumed by the buyer i in the first subperiod of period t. Without loss of generality, we assume that disutility from producing q_t for the seller i is linear. In addition, we denote the utility function for the general good (the special good i) by $U : \mathbb{R}^+ \to \mathbb{R}^+$ ($u : \mathbb{R}^+ \to \mathbb{R}^+$). We also assume that both are twice continuously differentiable, increasing, strictly concave and bounded by B on support $\Xi \subseteq (-\infty, \infty)$ with $u(0) = U(0) = 0$, $u'(0) = U'(0) = \infty$ and $u'(\infty) = U'(\infty) = 0$. $\mathbb{E}_0$ denotes the expectation with respect to the probability measure induced by the random trading process in the GM_1 and GM_2. Figure 1 illustrates the timing of events.

3 Value Functions and Optimal Behavior

3.1 Value Functions in the Financial Market and Goods Market

First, let $\mathbf{w}_t = (m_{A,t}, a_{A,t}, m_{B,t}, a_{B,t})$ denote the portfolio of any agent at period t. Note that $m_{i,t}$ and $a_{i,t}$ represents units of $money_i$ and $bond_i$ at period t respectively. Define $\boldsymbol{\ell}_t = (\ell_{A,t}, \ell_{B,t})$ as a portfolio of credit (or loan) in terms of the general good which the buyer i borrowed from seller i she met in $GM2$ of previous subperiod, and should pay back at the beginning of the second subperiod of t. Note that $\ell_{i,t}$ means the credit borrowed against $bond_i$ in the $GM2$ of period t. In addition, let $\mathbf{s}_t = (\gamma_{A,t}, M_{A,t}, \gamma_{B,t}, M_{B,t})$ denote the aggregate state of the economy at period t. Then the Bellman's equation for buyer i, who enters FM with a portfolio $\mathbf{w}_t$ and $\boldsymbol{\ell}_t$

is given by[14]

$$W_i^B(\mathbf{w}_t, \boldsymbol{\ell}_t, \mathbf{s}_t) = \max_{x_t, h_t, \mathbf{w}_{t+1}} \left\{ U(x_t) - h_t + \beta \mathbb{E}_t \left[V_i^B(\mathbf{w}_{t+1}, \mathbf{s}_{t+1}) \right] \right\}$$
$$\text{s.t. } x_t + \boldsymbol{\phi}_t' \mathbf{w}_{t+1} = h_t + \boldsymbol{\phi}_t \mathbf{w}_t - \boldsymbol{\ell}_t + \varphi_{A,t} T(\gamma_{A,t}) \mathbb{I}_{\{i=A\}} + \varphi_{B,t} T(\gamma_{B,t}) \mathbb{I}_{\{i=B\}},$$

where $\boldsymbol{\phi}_t' = (\varphi_{A,t}, \psi_{A,t}, \varphi_{B,t}, \psi_{B,t})$, $\boldsymbol{\phi}_t = (\varphi_{A,t}, \varphi_{A,t}, \varphi_{B,t}, \varphi_{B,t})$, and $\boldsymbol{\phi}_t' \mathbf{w}_{t+1}$ ($\boldsymbol{\phi}_t \mathbf{w}_t$) denotes the dot product of $\boldsymbol{\phi}_t'$ ($\boldsymbol{\phi}_t$) and $\mathbf{w}_{t+1}$ ($\mathbf{w}_t$). $T(\gamma_{i,t})$ denotes the monetary transfers in country i, and equals $(\gamma_{i,t} - 1) M_{i,t}$. $\mathbb{I}_{\{i=n\}}, n \in \{A, B\}$, is an indicator function that equals 1 if $i = n$. The function $V_i^B(\mathbf{w}_{t+1}, \mathbf{s}_{t+1})$ represents the GM value function of the buyer i next period. We can easily verify that $x_t = \tilde{x}$, $\forall t$ at the optimum where $\tilde{x}$ is such that $U'(\tilde{x}) = 1$. Based on this fact, we can plug h_t in the budget constraint into W_i^B. It leads to

$$W_i^B(\mathbf{w}_t, \boldsymbol{\ell}_t, \mathbf{s}_t) = \boldsymbol{\phi}_t \mathbf{w}_t - \boldsymbol{\ell}_t + \Lambda_{i,t}^B, \tag{1}$$

where $\Lambda_{i,t}^B \equiv U(\tilde{x}) - \tilde{x} + \mathbf{T}_{i,t} + \max_{\mathbf{w}_{t+1}} \left\{ -\boldsymbol{\phi}_t' \mathbf{w}_{t+1} + \beta \mathbb{E}_t \left[V_i^B(\mathbf{w}_{t+1}, \mathbf{s}_{t+1}) \right] \right\}$.[15] In line with models based on LW, the buyer i's FM value function becomes linear in asset holdings as well as credit owing to quasi-linearity of the preference. This implies that there exists no wealth effects on the choice of $\mathbf{w}_{t+1}$.

Now we consider the FM value function of a seller i. She will never leave the FM with any money or bond holdings because she does not need any liquidity service from those assets in the forthcoming GM simply due to her fixed identity as a seller of the special good (see Rocheteau and Wright (2005) for a rigorous proof). Nevertheless, when she enters the FM, she will generally hold a portfolio of money, bonds, and credit received as payment in either $GM1$

[14] The budget constraint implies that the buyer i always pays back the credit borrowed in a previous subperiod. This is in fact not an *assumption* but an equilibrium outcome. In principle, she could instead default and let the seller i whom she met in a previous $GM2$ take collateral she placed, i.e., bonds. However, as will be seen in Section 3.2, this type of strategy becomes always inferior to paying back the credit due to the pledgeability parameter h, less than one. In other words, she would always lose $1 - h$ portion of her real bond balances by defaulting on the seller i. Hence, allowing for less than perfect pledgeability of bonds as collateral is not only empirically relevant, but prevents our model from considering more complex equilibrium default cases.

[15] $\mathbf{T}_{i,t}$ is a short expression for $\varphi_{A,t} T(\gamma_{A,t}) \mathbb{I}_{\{i=A\}} + \varphi_{B,t} T(\gamma_{B,t}) \mathbb{I}_{\{i=B\}}$

or $GM2$. The Bellman's equation for her is then given by

$$W_i^S(\mathbf{w}_t, \boldsymbol{\ell}_t, \mathbf{s}_t) = \max_{x_t, h_t} \left\{ U(x_t) - h_t + \beta \mathbb{E}_t \left[V_i^S(\mathbf{0}, \mathbf{s}_{t+1}) \right] \right\}$$
$$\text{s.t. } x_t = h_t + \boldsymbol{\phi}_t \mathbf{w}_t + \boldsymbol{\ell}_t.$$

Similar to the buyer i, the seller i will also choose $x_t = \tilde{x}$, $\forall t$. Replacing h_t from the budget constraint into W_i^S yields

$$W_i^S(\mathbf{w}_t, \boldsymbol{\ell}_t, \mathbf{s}_t) = \boldsymbol{\phi}_t \mathbf{w}_t + \boldsymbol{\ell}_t + \Lambda_{i,t}^S, \tag{2}$$

where $\Lambda_{i,t}^S \equiv U(\tilde{x}) - \tilde{x} + \beta \mathbb{E}_t \left[V_i^S(\mathbf{0}, \mathbf{s}_{t+1}) \right]$.

Next we consider the value functions in the GM. First, consider a value of the typical buyer i who enters the GM with a portfolio $\mathbf{w}_t$. Let $q_{i,t}^{GMj}$, $j \in \{1, 2\}$ denote the consumption of special good i that the buyer i obtained from GMj at period t. $\boldsymbol{p}_t = (p_t^{m_A}, p_t^{a_A}, p_t^{m_B}, p_t^{a_B})$ represents a portfolio of assets exchanged in a meeting with a seller in $GM1$ at period t. For instance, $p_t^{m_i}$ ($p_t^{a_i}$) denotes the units of $money_i$ ($bond_i$) handed over to the seller in $GM1$ at period t. All these terms will be determined in Section 3.2. Since the buyer i visits the $GM1$ ($GM2$) with the probability of θ ($1 - \theta$), her GM value function is given by

$$V_i^B(\mathbf{w}_t, \mathbf{s}_t) = \theta \left[u(q_{i,t}^{GM1}) + W_i^B(\mathbf{w}_t - \boldsymbol{p}_t, \mathbf{0}, \mathbf{s}_t) \right] + (1 - \theta) \left[u(q_{i,t}^{GM2}) + W_i^B(\mathbf{w}_t, \boldsymbol{\ell}_t, \mathbf{s}_t) \right]. \tag{3}$$

The typical seller i visits the $GM1$ or $GM2$ with the same probabilities as the buyer i. The GM value function of the seller i with no money and bonds carried over from a previous period is given by

$$V_i^S(\mathbf{0}, \mathbf{s}_t) = \theta \left[-q_{i,t}^{GM1} + W_i^S(\boldsymbol{p}_t, \mathbf{0}, \mathbf{s}_t) \right] + (1 - \theta) \left[-q_{i,t}^{GM2} + W_i^S(\mathbf{0}, \boldsymbol{\ell}_t, \mathbf{s}_t) \right].$$

3.2 Trading Protocols in Goods Market

In this section, we study the bargaining problem and the associated terms of trade for each GMj in detail. First, consider a meeting in $GM1$ between seller i and buyer i who carries a portfolio $\mathbf{w}_t$. The bargaining problem is then given by

$$\max_{q_{i,t},\boldsymbol{p}_t} \left\{u(q_{i,t}) + W_i^B(\mathbf{w}_t - \boldsymbol{p}_t, 0, \mathbf{s}_t) - W_i^B(\mathbf{w}_t, 0, \mathbf{s}_t)\right\}$$
$$\text{s.t.} - q_{i,t} + W_i^S(\boldsymbol{p}_t, \mathbf{0}, \mathbf{s}_t) - W_i^S(\mathbf{0}, \mathbf{0}, \mathbf{s}_t) = 0,$$

with the illiquidity augmented feasibility constraint $\boldsymbol{p}_t \leq \widetilde{\mathbf{w}}_t$ where $\widetilde{\mathbf{w}}_t = (m_{A,t}, ga_{A,t}, m_{B,t}, ga_{B,t})$ and $g \in [0,1]$ represents the bond illiquidity parameter[16], and the CIA constraint (that only domestic assets are used for GMj transactions): $p_t^{m_{-i}} = p_t^{a_{-i}} = 0$. Given the linearity of W_i^B and W_i^S, the bargaining problem simplifies to

$$\max_{q_{i,t}, p_t^{m_i}, p_t^{a_i}} \{u(q_{i,t}) - \varphi_{i,t}(p_t^{m_i} + p_t^{a_i})\}$$
$$\text{s.t.} - q_{i,t} + \varphi_{i,t}(p_t^{m_i} + p_t^{a_i}) = 0,$$

and the illiquidity augmented feasibility constraint, $p_t^{m_i} \leq m_{i,t},\ p_t^{a_i} \leq ga_{i,t}$. The next Lemma 1 summarizes the solution to the bargaining problem.

Lemma 1. *Define $q^* \equiv \{q : u'(q) = 1\}$. The total real balances of buyer i for GM1 transactions are denoted as $z_i(\mathbf{w}_t) \equiv \varphi_{i,t}(m_{i,t} + a_{i,t})$. Also, define z^* as the real balances of the portfolio $(m_{i,t}, a_{i,t})$ such that $\varphi_{i,t}(m_{i,t} + ga_{i,t}) = q^*$, and $(m_{i,t}^*, a_{i,t}^*)$ as any corresponding portfolio $(m_{i,t}, a_{i,t})$ for z^*. The*

[16] We keep g non-zero for the following reasons. It is true that money completely dominates bonds as an MOE in many transactions, i.e., $g = 0$. However, there are also many historical evidence on assets or capital goods being used as a direct MOE, especially among financially less developed countries and hyper-inflation-inflicted countries. Please refer to Einzig (1966) and Lagos and Rocheteau (2008). Our assumption on $g \in [0,1]$ in GM1 embeds all these empirical evidence in a unified way. In fact, the usefulness of g in this framework is not limited to its empirical relevance. This type of restriction on the illiquidity of bonds has been suggested as a most basic solution for the *rate-of-return dominance puzzle* (see Hu and Rocheteau (2013) for an extensive literature review). We adopt the g for a similar reason. Given the introduction of $GM2$, our model induces somewhat higher liquidity properties of nominal bonds than money to prevail in equilibrium, i.e., the zero nominal interest rate bound will be violated. However, as will be analyzed in Section 4, the g can potentially offset this effect by making money more liquid than nominal bonds in $GM1$, and therefore the nominal interest rate does not necessarily goes below zero in equilibrium.

terms of trade are given by $q_{i,t} \equiv q_i(\mathbf{w}_t)$*,* $p_t^{m_i} \equiv p^{m_i}(\mathbf{w}_t)$*, and* $p_t^{a_i} \equiv p^{a_i}(\mathbf{w}_t)$ *such that* $q_i(\mathbf{w}_t) = \varphi_{i,t}\left(p^{m_i}(\mathbf{w}_t) + p^{a_i}(\mathbf{w}_t)\right) = \min\{q^*, \varphi_{i,t}(m_{i,t} + ga_{i,t})\}$ *as follows.*

$$q_i(\mathbf{w}_t) = \begin{cases} q^*, & \text{if } z_i(\mathbf{w}_t) \geq z^*, \\ z_i(\widetilde{\mathbf{w}}_t), & \text{if } z_i(\mathbf{w}_t) < z^*, \end{cases} \qquad (p^{m_i}(\mathbf{w}_t), p^{a_i}(\mathbf{w}_t)) = \begin{cases} (m_{i,t}^*, a_{i,t}^*), & \text{if } z_i(\mathbf{w}_t) \geq z^*, \\ (m_{i,t}, ga_{i,t}), & \text{if } z_i(\mathbf{w}_t) < z^*, \end{cases}$$

and $p_t^{m_{-i}} = p_t^{a_{-i}} = 0$*.*

Proof. Trivial and therefore omitted. □

It is straightforward to interpret the solution to the bargaining problem in Lemma 1. Only the buyer i's domestic asset holdings determine the terms of trade $(q_{i,t}, p_t^{m_i}, p_t^{a_i})$. Importantly, when her bond-illiquidity augmented domestic real balances, $\varphi_{i,t}(m_{i,t} + ga_{i,t})$ exceeds the first best quantity, q^*, she receives the latter, and hands over any combination of $money_i$ and $bond_i$ whose $\varphi_{i,t}\left(m_{i,t} + ga_{i,t}\right)$ exactly equals z^*. On the other hand, if $\varphi_{i,t}(m_{i,t} + ga_{i,t})$ falls short of q^*, then she is liquidity constrained and, therefore, gives up all her $money_i$ and g fraction of $bond_i$ holdings. In return, she receives as much q_i as her bond-illiquidity augmented domestic real balances $\varphi_{i,t}(m_{i,t} + ga_{i,t})$ allow.

Now, let us look at the details in the $GM2$ where credit is only accepted in payments. Similarly, we assume that buyer i makes a TIOLI offer to seller i under the restriction that only domestic bond holdings $bond_i$ can be used as collateral to obtain credit, and the buyer i can acquire the credit only up to a fraction h of her real $bond_i$ balances. The bargaining problem is then given by

$$\max_{q_{i,t}, \boldsymbol{\ell}_t} \left\{u(q_{i,t}) + W_i^B(\mathbf{w}_t, \boldsymbol{\ell}_t, \mathbf{s}_t) - W_i^B(\mathbf{w}_t, 0, \mathbf{s}_t)\right\}$$
$$\text{s.t.} - q_{i,t} + W_i^S(\mathbf{0}, \boldsymbol{\ell}_t, \mathbf{s}_t) - W_i^S(\mathbf{0}, \mathbf{0}, \mathbf{s}_t) = 0,$$

and $\ell_{-i,t} = 0$ with the credit limit constraint $\ell_{i,t} \leq h\varphi_{i,t}a_{i,t}$. The linearity of W_i^B and W_i^S simpli-

fies the bargaining problem to

$$\max_{q_{i,t},\ell_{i,t}} \{u(q_{i,t}) - \ell_{i,t}\}$$
$$\text{s.t.} - q_{i,t} + \ell_{i,t} = 0,$$

with the same credit limit constraint above.

The following Lemma 2 summarizes the solution to the bargaining problem in $GM2$.

Lemma 2. *Define the buyer i's pledgeability-adjusted bond holdings as $z_i^a(\mathbf{w}_t) \equiv h\varphi_{i,t}a_{i,t}$. Define $\widetilde{a}_{i,t}$ as $a_{i,t}$ such that $q^* = h\varphi_{i,t}a_{i,t}$. The terms of trade are given by $q_{i,t} \equiv q_i(\mathbf{w}_t)$ and $\ell_{i,t} \equiv \ell_i(\mathbf{w}_t)$ such that*

$$q_i(\mathbf{w}_t) = \begin{cases} q^*, & \text{if } z_i^a(\mathbf{w}_t) \geq q^*, \\ z_i^a(\mathbf{w}_t), & \text{if } z_i^a(\mathbf{w}_t) < q^*, \end{cases} \qquad \ell_i(\mathbf{w}_t) = \begin{cases} \widetilde{a}_{i,t}, & \text{if } z_i^a(\mathbf{w}_t) \geq q^*, \\ a_{i,t}, & \text{if } z_i^a(\mathbf{w}_t) < q^*, \end{cases}$$

Proof. Trivial and therefore omitted. □

These results are intuitive, and admit almost identical interpretation as in Lemma 1. A key difference is that money holdings are irrelevant in $GM2$, and the liquidity restriction on bonds as collateral is now reflected by the pledgeability parameter h. Following this difference, the outcome is straightforward to understand. The terms of trade now depend on the buyer i's credit pledgeability augmented real $bond_i$ balances, i.e., $h\varphi_{i,t}a_{i,t}$. If this is less than the first best amount, q^*, then she becomes liquidity constrained, and therefore place all her $bond_i$ holdings as collateral to obtain as much credit as possible. Otherwise, she just borrows q^* by placing whatever amounts of $bond_i$ as collateral needed to obtain that q^*, i.e., $\widetilde{a}_{i,t}$.

3.3 Euler Equations

This section describes the optimal portfolio choice of buyers. The optimal behavior can be derived by solving the maximization problem in (1). To that end, lead eq.(3) by one period and substitute the emerging expression into (1). Notice that the buyer i's portfolio choice, i,e., $\mathbf{w}_{t+1}$,

does not depend on her private trading history. Furthermore, the fact that $V_i^B(\mathbf{w}_t, \mathbf{s}_t)$ is a concave function of $z(\mathbf{w}_t)$ implies that the distribution of the total real balances held by buyer i will be degenerate in equilibrium (see Lagos and Wright (2005) for a rigorous proof). The necessary and sufficient first-order conditions for the buyer i's choices of $\mathbf{w}_{t+1} = (m_{A,t+1}, a_{A,t+1}, m_{B,t+1}, a_{B,t+1})$ are given by

$$\varphi_{i,t} \geq \beta \mathbb{E}_t \frac{\partial V_i^B(\mathbf{w}_{t+1}, \mathbf{s}_{t+1})}{\partial m_{i,t+1}} \text{ with equality if } m_{i,t+1} > 0, \forall i \in \{A, B\},$$
$$\psi_{i,t} \geq \beta \mathbb{E}_t \frac{\partial V_i^B(\mathbf{w}_{t+1}, \mathbf{s}_{t+1})}{\partial a_{i,t+1}} \text{ with equality if } a_{i,t+1} > 0, \forall i \in \{A, B\}.$$

Substitute solutions from Lemma 1 and 2 into eq.(3), and lead the emerging function by one period again. Finally, by taking this function's first derivative with respect to $m_{i,t+1}$ and $a_{i,t+1}$, $\forall i \in \{A, B\}$, one could achieve the following Euler equations for the buyer i.

$$\varphi_{i,t} = \beta \int \left\{(1-\theta) + \theta u'\left(q_i^{GM1}(\cdot)\right)\right\} \varphi_{i,t+1} dF(\gamma_{-i,t+1}, \gamma_{-i,t}) dF(\gamma_{i,t+1}, \gamma_{i,t}), \tag{4}$$

$$\psi_{i,t} = \beta \int \left\{\tau_1 + \tau_2 u'\left(q_i^{GM1}(\cdot)\right) + \tau_3 u'\left(q_i^{GM2}(\cdot)\right)\right\} \varphi_{i,t+1} dF(\gamma_{-i,t+1}, \gamma_{-i,t}) dF(\gamma_{i,t+1}, \gamma_{i,t}), \tag{5}$$

$$\varphi_{-i,t} \geq \beta \int \varphi_{-i,t+1} dF(\gamma_{-i,t+1}, \gamma_{-i,t}) \tag{6}$$

$$\text{“}=\text{”if } m_{-i,t+1} > 0,$$

$$\psi_{-i,t} \geq \beta \int \varphi_{-i,t+1} dF(\gamma_{-i,t+1}, \gamma_{-i,t}) \tag{7}$$

$$\text{“}=\text{”if } a_{-i,t+1} > 0.$$

where $q_{i,t+1}^{GM1}(\cdot) \equiv q_i^{GM1}(\boldsymbol{\phi}_{t+1}\mathbf{w}_{t+1})$, $q_{i,t+1}^{GM2}(\cdot) \equiv q_i^{GM2}(h \cdot \varphi_{i,t+1} a_{i,t+1})$, $\tau_1 = (1-\theta)(1-h) + \theta(1-g)$, $\tau_2 = \theta g$, and $\tau_3 = (1-\theta)h$.

Interpretation of these Euler equations above is standard. The left side of each condition refers to a marginal cost of purchasing $money_i$ or $bond_i$, $\forall i$, while the right side represents the expected marginal benefit from carrying that asset into GM. For instance, condition (4) is the buyer i's Euler equation for $money_i$. The left side simply means the real cost of purchasing a unit of $money_i$. On the other hand, the right side represents the weighted average of the dis-

counted gain from carrying a unit of $money_i$ into the following period. Precisely, if she happens to visit $GM2$ with probability $1-\theta$, she can hold onto the unit of $money_i$ until the second subperiod in period $t+1$ to consume $\varphi_{i,t+1}$ units of general goods, i.e., $(1-\theta)\varphi_{i,t+1}$. Or, if she visits GM1 with probability θ, she can gain the consumption utility in $GM1$ from using that $money_i$ to purchase special good i, i.e., $\theta u'\left(q_i^{GM1}(\cdot)\right)\varphi_{i,t+1}$.

Condition (5) is the buyer i's Euler equation for $bond_i$. A key difference here is that the discounted expected benefit from carrying additional unit of $bond_i$ into the following period has three components. First, if she happens to visit $GM1$ in the next period, then she enjoys consumption utility from placing the $bond_i$ as means of payment, i.e., $\tau_2 u'\left(q_i^{GM1}(\cdot)\right)\varphi_{i,t+1}$. On the other hand, if she visits $GM2$ instead, she can gain the consumption utility by using that $bond_i$ as collateral, i.e., $\tau_3 u'\left(q_i^{GM2}(\cdot)\right)\varphi_{i,t+1}$. Lastly, regardless of which GM she enters, she will *effectively* face a certain restriction on the use of the $bond_i$ as a consequence of the pricing mechanism explained earlier, i.e., there always exists illiquid portion of the $bond_i$ that can not be liquidated. Then, she *effectively* carries that portion of the $bond_i$ into the second subperiod, and consume whatever amounts of general goods it allows her to purchase, i.e., $\tau_1\varphi_{i,t+1}$.

Condition (6) and (7) are respectively the buyer i's Euler equation for $money_{-i}$ and $bond_{-i}$. Notice that she never gains any benefit from carrying foreign assets into GM due to the two CIA constraints imposed in both $GM1$ and $GM2$. Thus, she only values them as the claim to the next period's general goods, i.e., the right side of (6) and (7) is only the discounted expected value of $\varphi_{i,t+1}$. Accordingly, agents in each country will hold only domestic assets in all states, i.e., condition (6) and (7) hold with strict inequality. Appendix A.1. also shows eq.(4), (5), (6), and (7) remain the same even under the Zhu and Wallace (2007) type of pricing protocol in GMs, confirming that our equilibrium results will still qualitatively go through even with more endogenous pricing protocols in GMs.

4 Equilibrium and Characterization

In this section, we describe the definition of a recursive equilibrium and then, derive expressions for functions of the equilibrium prices such as nominal interest rate for each country and exchange rate. Finally, we will discuss how these variables interrelate with each other, and specify conditions under which the UIP puzzle is resolved.

4.1 Definition of Equilibrium

Before we proceed for the definition of equilibrium, let us first define a few more variables. The transition function F along with the stochastic process for $\gamma_{i,t}$, $\forall i$ also yield a transition function for the aggregate state of the economy, $\mathbf{s}_t$. Specifically, if $\mathbf{s} = (\gamma_A, M_A, \gamma_B, M_B)$ and $\mathbf{s}' = (\gamma'_A, M'_A, \gamma'_B, M'_B)$ then, $\Pr(\mathbf{s}_{t+1} \leq \mathbf{s}' \mid \mathbf{s}_t = \mathbf{s}) = \prod_i \mathbb{I}_{\{\gamma_i M_i \leq M'_i\}} F(\gamma'_i, \gamma_i) \equiv \boldsymbol{F}(\mathbf{s}', \mathbf{s})$. Also let $\boldsymbol{\Psi}$ be the associated stationary distribution, i.e., let $\boldsymbol{\Psi}$ be the unique solution to $\boldsymbol{\Psi}(\mathbf{s}') = \int \boldsymbol{F}(\mathbf{s}', \mathbf{s}) d\boldsymbol{\Psi}(\mathbf{s})$. We define a recursive equilibrium where all prices are time-invariant functions of the aggregate state $\mathbf{s}_t$: $\boldsymbol{\phi}'_t = \boldsymbol{\phi}'(\mathbf{s}_t) = [\psi_A(\mathbf{s}_t), \varphi_A(\mathbf{s}_t), \psi_B(\mathbf{s}_t), \varphi_B(\mathbf{s}_t)]$ and $E_t = E(\mathbf{s}_t)$.

Definition 1. *A recursive equilibrium is a list of individual decision rules for buyer i, $\forall i \in \{A, B\}$, $\mathbf{w}_{t+1}$=$\mathbf{w}(\mathbf{s_t})$ =$[m_A(\mathbf{s}_t), a_A(\mathbf{s}_t), m_B(\mathbf{s}_t), a_B(\mathbf{s}_t)]$, pricing functions $\boldsymbol{\phi}'_t$=$\boldsymbol{\phi}'(\mathbf{s}_t)$ and $E_t = E(\mathbf{s}_t)$, bilateral terms of trade in $GM1$: $Q_i(\mathbf{s}_t) = q_i(\mathbf{w}(\mathbf{s}_t))$ and $\boldsymbol{P}(\mathbf{s}_t) = \boldsymbol{p}(\mathbf{w}(\mathbf{s}_t))$, $\forall i \in \{A, B\}$ where $q_i(\cdot)$ and $\boldsymbol{p}(\cdot)$ are given by Lemma 1, and bilateral terms of trade in $GM2$: $\widetilde{Q}_i(\mathbf{s}_t) = q_i(\mathbf{w}(\mathbf{s}_t))$ and $\boldsymbol{C}(\mathbf{s}_t) = \boldsymbol{\ell}(\mathbf{w}(\mathbf{s}_t))$, $\forall i \in \{A, B\}$ where $q_i(\cdot)$ and $\boldsymbol{\ell}(\cdot)$ are given by Lemma 2 such that:*

(i) the decision rule $\mathbf{w}(\cdot)$ solves the individual optimization problem (1), taking prices as given;

(ii) prices are such that the FM clears, i.e., $\mathbf{w}_{t+1} = [\gamma_{A,t} M_{A,t}, B_A, 0, 0]$ for buyer A and $\mathbf{w}_{t+1} = [0, 0, \gamma_{B,t} M_{B,t}, B_B]$ for buyer B;

(iii) the law of one price holds, i.e., $\varphi_A(\mathbf{s}_t) E(\mathbf{s}_t) = \varphi_B(\mathbf{s}_t)$.

In the remainder of the paper we only focus on a symmetric-recursive equilibrium case where all exogenously given parameters in this model are same across countries. That is θ, g, h

are identical across the two countries, and $B_A = B_B = \bar{B}$. This fact implies that the list of equilibrium objects does not depend on the agent's citizenship but only on the aggregate state of the economy, $\mathbf{s}_t$.

Definition 1 reveals some important properties of equilibrium. The fact that $m_i(\mathbf{s}_t) = \gamma_{i,t} M_{i,t}$ for all i in all states implies that the equilibrium is *always* monetary, meaning $\varphi_i(\mathbf{s}_t) > 0$ for all i and $\mathbf{s}_t$. Intuition is straightforward. By construction, a unit of $bond_i$ yields $\varphi_i(\mathbf{s}_t)$ units of general good. Therefore, if the non-monetary equilibrium prevails then, the $bond_i$ must yield no general goods in any states. This means that both $money_i$ and $bond_i$ are never valued so that no GM trade takes place in the equilibrium which would surely be inferior to any monetary equilibrium outcome, i.e., $money_i$, $\forall i$ is always essential in this economy. Secondly, as explained earlier, the competitive nature of the FM does not allow any arbitrage in currency trade to arise in equilibrium, i.e., $\varphi_A(\mathbf{s}_t)E(\mathbf{s}_t) = \varphi_B(\mathbf{s}_t)$ for all states. Lastly, the fact that $m_{-i}(\mathbf{s}_t) = 0$ and $a_{-i}(\mathbf{s}_t) = 0$, i.e., no international diversification in asset holdings occurs and it follows naturally from the optimality in Section 3.3.

In order to study equilibrium prices in the next section, one needs to consider "general equilibrium" counterparts of Euler equations in Section 3.3. To that end, let $Z_i(\mathbf{s}_t)$ denote "bond-illiquidity-augmented" equilibrium total real balances held by country i in state $\mathbf{s}_t$. Likewise, define $Z_i^a(\mathbf{s}_t)$ as "pledgeability-augmented" equilibrium real $bond_i$ balances held by the country i in state $\mathbf{s}_t$ as follows.

$$Z_i(\mathbf{s}_t) \equiv \varphi_i(\mathbf{s_t})[M_{i,t} + g\bar{B}], \text{ and } Z_i^a(\mathbf{s}_t) \equiv h \cdot \varphi_i(\mathbf{s}_t)\bar{B}.$$

In equilibrium, the Euler equations for $money_i$ and $bond_i$ holdings are then given by

$$\varphi_i(\mathbf{s}_t) = \beta \int \boldsymbol{L}[Z_i(\mathbf{s}_{t+1})]\varphi_i(\mathbf{s}_{t+1}) d\boldsymbol{F}(\mathbf{s}_{t+1}, \mathbf{s}_t), \tag{8}$$

$$\psi_i(\mathbf{s}_t) = \beta \int \boldsymbol{N}[Z_i(\mathbf{s}_{t+1}), Z_i^a(\mathbf{s}_{t+1})]\varphi_i(\mathbf{s}_{t+1}) d\boldsymbol{F}(\mathbf{s}_{t+1}, \mathbf{s}_t), \tag{9}$$

where the *stochastic liquidity factors* for $money_i$ and $bond_i$ are respectively given by $\boldsymbol{L}[Z_i(\mathbf{s}_{t+1})]$

and $\boldsymbol{N}[Z_i(\mathbf{s}_{t+1}), Z_i^a(\mathbf{s}_{t+1})]$ as below.

$$\boldsymbol{L}[Z_i(\mathbf{s}_{t+1})] \equiv (1-\theta) + \theta u'\left(\min\{Z_i(\mathbf{s}_{t+1}), q^*\}\right), \tag{10}$$

$$\begin{aligned}\boldsymbol{N}[Z_i(\mathbf{s}_{t+1}), Z_i^a(\mathbf{s}_{t+1})] \equiv\ & \boldsymbol{L}[Z_i(\mathbf{s}_{t+1})] - \theta(1-g)\left[u'\left(\min\{Z_i(\mathbf{s}_{t+1}), q^*\}\right) - 1\right] \\ & + (1-\theta)h\left[u'\left(\min\{Z_i^a(\mathbf{s}_{t+1}), q^*\}\right) - 1\right].\end{aligned} \tag{11}$$

Note that $\boldsymbol{L}\left[Z_i(\mathbf{s}_{t+1})\right] \geq 1$ for all $money_i$ growth rate realizations $\gamma_{i,t+1} \in \Xi_i$, with strict inequality if $\gamma_{i,t+1} \in \Gamma_{m_i}(\mathbf{s_t})$, where

$$\Gamma_{m_i}(\mathbf{s}_t) = \{\gamma_{i,t+1} \in \Xi_i : \varphi_i(\gamma_{i,t+1}, \gamma_{i,t}M_{i,t})[\gamma_{i,t}M_{i,t} + g\bar{B}] < q^*\},$$

Interpretation of $\boldsymbol{L}[Z_i(\mathbf{s}_{t+1})]$ is standard. It is stochastic and endogenously driven by the aggregate state of the economy. Most importantly, it captures the extra exchange value of $money_i$ in addition to its store of value. Thus, it is always bounded below by 1. It becomes unity, i,.e., the extra exchange value disappears, only when buyers already achieve the first best in the $GM1$, i.e., $Z_i(\mathbf{s}_{t+1}) \geq q^*$ in (10), or no opportunity to visit $GM1$ exists, i.e., $\theta = 0$ in (10).

On the other hand, the stochastic liquidity factor for $bond_i$, $\boldsymbol{N}[Z_i(\mathbf{s}_{t+1}), Z_i^a(\mathbf{s}_{t+1})]$ is richer and more interesting. If $money_i$ and $bond_i$ are perfect substitutes (e.g., $h = 0$ and $g = 1$) then, it should equal $\boldsymbol{L}[Z_i(\mathbf{s}_{t+1})]$ for all $\mathbf{s}_t$. This result complies with the rate-of-return dominance puzzle literature.

However, our proposed GM trade mechanism endogenously induces the equilibrium to nest imperfect substitutability cases as well. If social conventions dictate that $h > 0$ and $g < 1$ then, the stochastic liquidity value of $bond_i$ exhibits two offsetting components that potentially make $\boldsymbol{N}[Z_i(\mathbf{s}_{t+1}), Z_i^a(\mathbf{s}_{t+1})]$ deviate from $\boldsymbol{L}[Z_i(\mathbf{s}_{t+1})]$. First, since only up to g portion of $bond_i$ holdings can be fully liquidated in $GM1$, the net liquidity value of the $bond_i$ has to be proportionally discounted relative to that of $money_i$, i.e., $-\theta(1-g)\left[u'\left(\min\{Z_i(\mathbf{s}_{t+1}), q^*\}\right) - 1\right]$ in eq.(11). On the contrary, a unit of $bond_i$, serving as collateral in $GM2$, generates extra liquidity value relative to $money_i$, i.e., $(1-\theta)h\left[u'\left(\min\{Z_i^a(\mathbf{s}_{t+1}), q^*\}\right) - 1\right]$. Which of these two offsetting forces dominates critically determines nominal bond yields relative to the zero nominal money

return. In fact, structural parameters of the economy such as g, h, and θ turn out to be significant in this context. Suppose no $GM2$ exists, i,e., $\theta = 1$, or bonds are completely useless as collateral, i.e., $h = 0$, then, the second effect vanishes and therefore, the $bond_i$ is always traded at a discount compared to $money_i$, i.e., nominal bonds always dominate money in terms of the rate of return. Instead, let us imagine a economy where bonds are almost as liquid as money in $GM1$, i.e., $g \approx 1$. In this case, the $bond_i$ becomes an almost perfect substitute for $money_i$ in the $GM1$.

4.2 Inflation Rate

The price of $money_i$ in terms of general goods is $\varphi_i(\mathbf{s}_t)$. The nominal price of a general good is $1/\varphi_i(\mathbf{s}_t)$ in country i whose actual (gross) inflation rate between t and $t+1$ is then given by

$$\pi_i(\mathbf{s}_{t+1} = \mathbf{s}', \mathbf{s}_t = \mathbf{s}) \equiv \frac{\varphi_i(\mathbf{s})}{\varphi_i(\mathbf{s}')}.$$

One can also define expected (gross) inflation in country i as the rate at which the $money_i$ price of a general good, conditional on the information available at $\mathbf{s}_t$, changes. Let us denote the expected (gross) inflation rate for country i at state $\mathbf{s}_t$ as $\tilde{\pi}_i(\mathbf{s}_t)$. Without loss of generality, we define the latter as the harmonic mean of $\pi_i(\mathbf{s}_{t+1} = \mathbf{s}', \mathbf{s}_t = \mathbf{s})$ in the following way.

$$\frac{1}{\tilde{\pi}_i(\mathbf{s}_t)} \equiv \int \frac{1}{\pi_i(\mathbf{s}_{t+1} = \mathbf{s}', \mathbf{s}_t = \mathbf{s})} d\boldsymbol{F}(\mathbf{s}', \mathbf{s}).$$

4.3 Nominal Exchange Rate

Due to the law of one price as an equilibrium condition, $E(\mathbf{s}_t)$ can be rewritten as $\varphi_B(\mathbf{s}_t)/\varphi_A(\mathbf{s}_t)$. The expected exchange rate is then given by $E^e(\mathbf{s}_{t+1}, \mathbf{s}_t) = \mathbb{E}_t[\varphi_B(\mathbf{s}_{t+1}, \mathbf{s}_t)]/\mathbb{E}_t[\varphi_A(\mathbf{s}_{t+1}, \mathbf{s}_t)]$ owing to independent Markov process of $\gamma_{i,t}$ for each country. The expected depreciation (apprecia-

tion) of the $money_A$ ($money_B$) between t and $t+1$ is therefore given by

$$\frac{E^e(\mathbf{s}_{t+1},\mathbf{s}_t)}{E(\mathbf{s}_t)} = \frac{\mathbb{E}_t[\varphi_B(\mathbf{s}_{t+1},\mathbf{s}_t)]/\mathbb{E}_t[\varphi_A(\mathbf{s}_{t+1},\mathbf{s}_t)]}{\varphi_B(\mathbf{s}_t)/\varphi_A(\mathbf{s}_t)} = \frac{\int \frac{1}{\pi_B(\mathbf{s}_{t+1},\mathbf{s}_t)} d\boldsymbol{F}(\mathbf{s}_{t+1},\mathbf{s}_t)}{\int \frac{1}{\pi_A(\mathbf{s}_{t+1},\mathbf{s}_t)} d\boldsymbol{F}(\mathbf{s}_{t+1},\mathbf{s}_t)} = \frac{\tilde{\pi}_A(\mathbf{s}_t)}{\tilde{\pi}_B(\mathbf{s}_t)}. \tag{12}$$

Proposition 1. *The expected depreciation of a currency is positively correlated with its relative expected inflation rate to the partner country's, i.e., if $\tilde{\pi}_i(\mathbf{s}_t) > \tilde{\pi}_{-i}(\mathbf{s}_t)$ then, $E^e(\mathbf{s}_{t+1},\mathbf{s}_t)$ is higher (lower) than $E(\mathbf{s}_t)$ for $i = A$ ($i = B$).*

Proof. The proof is trivial, it it, therefore, omitted. □

The expected nominal exchange rate here behaves exactly same as in DSGE models with complete and Walrasian foreign exchange market.[17] Under such conventional models, a key equation that characterizes a joint stochastic process for nominal exchange rate and inflation is given by

$$\frac{E_{t+1}}{E_t} = \frac{m^*_{t+1}}{m_{t+1}} \frac{\pi_{t+1}}{\pi^*_{t+1}}, \tag{13}$$

where E_t denotes the nominal exchange rate (price of foreign currency in units of domestic), m_{t+1} denotes the inter-temporal marginal rate of substitution (IMRS) of the domestic representative agent (m^*_{t+1} for the foreign counterpart), and lastly, π_{t+1} (π^*_{t+1}) is the domestic (foreign) inflation rate. Notice that eq.(13) is identical to eq.(12) given that in our model, the IMRS for buyer i, $\forall i \in \{A, B\}$, equals a constant β. This constant IMRS is directly attributed to the fact that equilibrium general good consumption for buyers is fixed at $\tilde{x}$ for every period, which again is an artifact of the quasilinear preference in the FM.[18]

[17] A seminal paper in this line of research is Lucas (1982) who pioneered an international asset pricing model in a two-country DSGE setup. See Backus, Foresi, and Telmer (2001) for an extensive literature review.

[18] One could instead choose to work with stochastic IMRS by introducing a stochastic shock process for the general good as in Lagos (2011). However, this would be redundant in our model. As will be seen in 4.5, doing so would not affect the equilibrium relationship between currency and nominal bond prices, i.e., stochastic IMRSs can affect the level of equilibrium currency and nominal bond prices but not a joint dynamics between those two.

4.4 Nominal Interest Rate

Next, we characterize equilibrium nominal interest rate for each country. $\psi_i(\mathbf{s}_t)$ denotes the state $\mathbf{s}_t$ price of $bond_i$ in terms of general goods, i.e., real price of nominal $bond_i$ in state $\mathbf{s}_t$. Then, $\psi_i(\mathbf{s}_t)/\varphi_i(\mathbf{s}_t)$ is its $money_i$ denominated price, i.e., nominal price of the nominal $bond_i$ in state $\mathbf{s}_t$. Hence, we can define country i's (gross) nominal interest rate in state $\mathbf{s}_t$ as its reciprocal, $\varphi_i(\mathbf{s}_t)/\psi_i(\mathbf{s}_t)$. Using e.q.(8) and (9), it is given by

$$R_i(\mathbf{s}_t) \equiv \frac{\varphi_i(\mathbf{s}_t)}{\psi_i(\mathbf{s}_t)} = \frac{\int L\left[Z_i(\mathbf{s}_{t+1})\right]\varphi_i(\mathbf{s}_{t+1})d\boldsymbol{F}(\mathbf{s}_{t+1},\mathbf{s}_t)}{\int N\left[Z_i(\mathbf{s}_{t+1}), Z_i^a(\mathbf{s}_{t+1})\right]\varphi_i(\mathbf{s}_{t+1})d\boldsymbol{F}(\mathbf{s}_{t+1},\mathbf{s}_t)}. \tag{14}$$

Before we proceed, we restrict our model economy in a way that zero lower bound (ZLB) is never violated. It is important to note that this constraint will not *per se* affect equilibrium relationship between various variables, nominal interest rates and expected inflation in particular. This will be discussed more with examples in the following Proposition 2. However, what makes the zero nominal interest rate bound matter here is that it does put a limit on monetary policy. Specifically, we will hereinafter only consider the set of $money_i$ growth rate realizations, ensuring the greater numerator than the denominator in eq.(14), i.e., $L\left[Z_i(\mathbf{s}_{t+1})\right] \geq N\left[Z_i(\mathbf{s}_{t+1}), Z_i^a(\mathbf{s}_{t+1})\right], \forall i, \mathbf{s}_t$. Technically speaking, we only consider $\gamma_{i,t+1} \in \Gamma_{ZLB_i}(\mathbf{s_t})$, where

$$\Gamma_{ZLB_i}(\mathbf{s}_t) = \left\{\gamma_{i,t+1} \in \Xi_i : \frac{u'\left(\min\{h\varphi_i(\gamma_{i,t+1},\gamma_{i,t}M_{i,t})\bar{B}, q^*\}\right) - 1}{u'\left(\min\{\varphi_i(\gamma_{i,t+1},\gamma_{i,t}M_{i,t})[\gamma_{i,t}M_{i,t} + g\bar{B}], q^*\}\right) - 1} \leq \frac{\theta(1-g)}{(1-\theta)h}\right\}. \tag{15}$$

Next proposition reveals important equilibrium properties regarding the relationship between nominal interest rate and expected inflation in each country.

Proposition 2. *Consider an economy with the zero nominal interest rate bound, i.e.,* $\gamma_{i,t+1} \in \Gamma_{ZLB_i}(\mathbf{s_t})$ *for all* i *and* $\mathbf{s}_t$*. Then,* $R_i(\mathbf{s}_t)$ *and* $\tilde{\pi}_i(\mathbf{s}_t)$ *are related in the following way.*

a) If $\theta = 1$ *or* $h = 0$ *then,* $\Gamma_{ZLB_i}(\mathbf{s_t}) = \Xi_i$ *and* $\partial R_i(\mathbf{s}_t)/\partial\tilde{\pi}_i(\mathbf{s}_t) \geq 0$.

b) Otherwise, $\Gamma_{ZLB_i}(\mathbf{s_t}) \subset \Xi_i$*, and a sufficient condition for* $\partial R_i(\mathbf{s}_t)/\partial\tilde{\pi}_i(\mathbf{s}_t) < 0$ *is* $(1-\theta)h \geq (1-g)$.

Proof. See the appendix. □

In order to interpret these results, it is useful to rewrite the nominal price of $bond_i$ at $\mathbf{s}_t$, $\psi_i(\mathbf{s}_t)/\varphi_i(\mathbf{s}_t)$, in terms of three different values. Appendix A.2. shows the following.

$$\begin{aligned}\frac{\psi_i(\mathbf{s}_t)}{\varphi_i(\mathbf{s}_t)} &= 1 - \beta\theta(1-g)\int \frac{[u'(\min\{Z_i(\mathbf{s}_{t+1}), q^*\}) - 1]}{\pi_i(\mathbf{s}_{t+1}, \mathbf{s}_t)} d\boldsymbol{F}(\mathbf{s}_{t+1}, \mathbf{s}_t) \\ &\quad + \beta(1-\theta)h\int \frac{[u'(\min\{Z_i^a(\mathbf{s}_{t+1}), q^*\}) - 1]}{\pi_i(\mathbf{s}_{t+1}, \mathbf{s}_t)} d\boldsymbol{F}(\mathbf{s}_{t+1}, \mathbf{s}_t).\end{aligned} \tag{16}$$

The nominal value of a unit $bond_i$ at $\mathbf{s}_t$ can be thought of a sum of three different components as in (16). The first component which always equals a unity refers to the nominal value of a unit of $money_i$ at $\mathbf{s}_t$. The negative component in the first line of eq.(16) represents the expected nominal value of illiquidity discount on the $bond_i$ due to g in $GM1$. The third component in the second line of (16), on the other hand, captures the expected nominal premium value of the $bond_i$ as a result of its collateral role in $GM2$.

Part $a)$ of Proposition 2 characterizes the equilibrium relationship between $R_i(\mathbf{s}_t)$ and $\tilde{\pi}_i(\mathbf{s}_t)$ when the nominal $bond_i$ can not serve as collateral. A monetary policy in this case is never restricted since the $bond_i$ always has lower liquidity properties than the $money_i$, i.e., $\psi_i(\mathbf{s}_t)/\varphi_i(\mathbf{s}_t)$ always becomes less than or equal to unity regardless of $\gamma_{i,t+1}$. More importantly, it is straightforward to understand why $\partial R_i(\mathbf{s}_t)/\partial\tilde{\pi}_i(\mathbf{s}_t) \geq 0$. Since anticipated inflation acts as a tax on holding real balances, a higher $\tilde{\pi}_i(\mathbf{s}_t)$ reduces the $Z_i(\mathbf{s}_{t+1})$ and $Z_i^a(\mathbf{s}_{t+1})$. This in turn induces their expected nominal liquidity value to rise in $GM1$ and $GM2$ respectively.[19] Yet, the fact that the $GM2$ is now irrelevant makes the premium value of the $bond_i$ disappear. It is therefore the case that the higher expected nominal liquidity value of $Z_i(\mathbf{s}_{t+1})$ only amplifies the illiquidity discount of the $bond_i$, i.e., the second negative term in the first line of (16) gets bigger. Thus, the nominal $bond_i$ price falls, i.e., $R_i(\mathbf{s}_t)$ increases.

This Fisher effect no longer prevails universally as soon as the $bond_i$ has some liquidity properties as collateral in our model. Part $b)$ of Proposition 2 implies that relatively high

[19] A higher $Z_i(\mathbf{s}_{t+1})$ or $Z_i^a(\mathbf{s}_{t+1})$ surely causes a higher expected *real* liquidity value of itself due to the concavity of $u(\cdot)$. The fact that the nominal liquidity value also goes up can be understood intuitively as follows. The Euler equation for money holdings requires that (real) cost of carrying them must equal the (real) net benefit from doing so. This condition must hold true in nominal terms because one would divide both side of the Euler equation by the same real price of money to arrive the nominal Euler equation. Then, the higher anticipated inflation should raise both nominal cost and nominal benefit (nominal liquidity value) to ensure the optimality.

pledgeability of $bond_i$ both as means of payment and collateral, and a relatively high portion of credit based transactions in goods trade, i.e., $(1-\theta)h \geq (1-g)$, guarantee a negative correlation between $R_i(\mathbf{s}_t)$ and $\tilde{\pi}_i(\mathbf{s}_t)$. Intuition follows clearly from a previous paragraph. Since a higher $\tilde{\pi}_i(\mathbf{s}_t)$ now amplifies both the illiquidity discount and the premium value of the $bond_i$, what matters is their relative size of effects. As the eq.(16) reveals, the sufficient condition, $(1-\theta)h \geq (1-g)$ secures a higher weight on the premium value of the $bond_i$ than the illiquidity discount value. This intuitively explains why a higher $\tilde{\pi}_i(\mathbf{s}_t)$ makes the premium value of the $bond_i$ increase relatively more compared to the illiquidity discount. To see intuition from a different angle, consider *real* price change of $money_i$ and $bond_i$. When $\tilde{\pi}_i(\mathbf{s}_t)$ rises, the real price of both assets surely goes up due to higher marginal utilities in GM associated with a fall in real balances. Yet, the condition $(1-\theta)h \geq (1-g)$ induces the $bond_i$ to exhibit somewhat higher exchange value as a facilitator of special good i trade. Thus, no arbitrage condition makes sure that the real $bond_i$ price increases more than that of $money_i$, meaning the nominal $bond_i$ price should increase.[20]

Notice that allowing for no illiquidity constraint in $GM1$ $(g=1)$ would make the sufficient condition become redundant for $\partial R_i(\mathbf{s}_t)/\partial\tilde{\pi}_i(\mathbf{s}_t) < 0$. However, as seen in (15), this would in fact make the ZLB-inducing monetary policy in country i highly restrictive. Basically such policy should result in the first best outcome in $GM2$ at all states, but strictly less than the first best in $GM1$. A set of $\gamma_{i,t+1}$ that satisfies this condition would be particularly narrow when the supply of $bond_i$ is relatively small. In fact, it would be interesting to study what should be a family of optimal stochastic monetary policies in our framework. This, however, would be beyond the scope of this study. Thus, we leave this task for future research.[21]

Before we go on studying implications of these effects on the UIP condition, it is worth em-

[20] Another intuition for the sufficient condition $(1-\theta)h \geq (1-g)$ is that additional bond liquidity value generated from GM2 outweighs the loss of bond liquidity in GM1. Thus, the essence of this sufficient condition hinges upon the existence of GM2, where bonds serve as a sole MOE. Although one might feel that this model structure is unrealistic at first glance, our attempt is by no means the first one in the literature. Xiao, Wright, and Rocheteau (2017) similarly set up a theoretic model which allows for transactions where nominal bonds are only used as a means of payments. They support this assumption on the grounds that short term debt, such as Treasuries in the United States or Switzerland, is in strong demand, to the extent that investors endure negative yields on them, because they are useful in repo financing.

[21] Lagos (2011) studies a set of optimal stochastic monetary policies that implement the Friedman rule basically in a nested version of our economy, i.e., $g=0$ and $\theta=1$, with an inclusion of stochastic dividend paying equity.

phasizing that the mechanism in which a negative comovement between nominal interest rates and money supply (anticipated inflation) arises differs from the New Keynesian framework. This inverse relationship, often called as *liquidity effect* of monetary policy, is one major characteristics of traditional sticky-price New Keynesian models. Their key idea is that money supply has a direct positive effect on equilibrium real money balances in the short run due to some form of nominal rigidity. Since bonds only serve as a store of value in their framework, financial market equilibrium requires a positive movement between real bond price and real money balances. Finally, the price stickiness ensures that nominal bond price moves in the same direction as money supply.

On the contrary, our proposed model abandons the price stickiness assumption, and cause the liquidity effect through a different channel. In our model, a higher money growth rate boosts real bond price not because of price stickiness but due to bonds' role as a medium of exchange. Since the fully flexible price movement tends to depress the nominal value of bonds in the event of the higher money growth rate, the nominal bond price finally depends on the relative size of increase in the real bond price. When bonds play an extra facilitator role compared to money then, the real bond price effect tends to dominate, and the nominal bond price rises, i.e., the liquidity effect, otherwise the latter vanishes.[22]

4.5 Uncovered Interest Parity Puzzle

In this section, we state the main results of our paper. To ease presentation, let us rephrase the UIP puzzle in terms of predictable excess returns on currencies. In our framework, one can define a one-period excess $money_A$ return on $money_B$ at state $\mathbf{s}_t$ as

$$\mathcal{Q}(\mathbf{s}_t) = \ln E^e(\mathbf{s}_{t+1}, \mathbf{s}_t) - \ln E(\mathbf{s}_t) + \ln R_B(\mathbf{s}_t) - \ln R_A(\mathbf{s}_t). \tag{17}$$

[22] This difference at least points to the possibility that open market operations could potentially generate richer implications on monetary policy and asset prices than what nominal friction based New-Keynesian models imply. See Lagos, Rocheteau, and Wright (2014) for more extensive literature review on how micro-founded monetary economics based on search theory offers different monetary policy implications.

The UIP puzzle means that $\mathcal{Q}(\mathbf{s}_t)$ is actually predictable because the expected depreciation rate of $mone_A$ is positively correlated with the interest differential, i.e., $\ln R_B(\mathbf{s}_t) - \ln R_A(\mathbf{s}_t)$ on $\mathcal{Q}(\mathbf{s}_t)$. So the task here is to find conditions under which $\partial\mathcal{Q}(\mathbf{s}_t)/\partial\left[\ln R_B(\mathbf{s}_t) - \ln R_A(\mathbf{s}_t)\right] > 0$ for every $\mathbf{s}_t$ in our model. As an intermediate step, it is useful to define the *real* liquidity adjusted stochastic discount factor $\mathcal{M}_{i,t+1}$. Following the conventional international asset pricing model, inversing real (gross) return on $bond_i$ should generate the expected $\mathcal{M}_{i,t+1}$ in our model as well.[23] Therefore,

$$\mathbb{E}_t(\mathcal{M}_{i,t+1}) \equiv \mathcal{M}_i(\mathbf{s}_t) = \frac{\tilde{\pi}_i(\mathbf{s}_t)}{R_i(\mathbf{s}_t)}, \ \forall i \text{ and } \mathbf{s}_t. \tag{18}$$

Substituting (12) and (18) into (17) simplifies $\mathcal{Q}(\mathbf{s}_t)$ as follows.

$$\mathcal{Q}(\mathbf{s}_t) = \ln \mathcal{M}_A(\mathbf{s}_t) - \ln \mathcal{M}_B(\mathbf{s}_t). \tag{19}$$

Equation (19) reveals an important characteristic of excess returns on $money_B$. The latter is now completely driven by liquidity property differential. Notice that any IMRS differential between two countries could have never affected $\mathcal{Q}(\mathbf{s}_t)$ even if it existed. The reason is that its effects on interest rate differential and the expected appreciation of $money_B$ will be completely canceled out. In fact, this would be the same equilibrium property of conventional models where the IMRS simply equals the stochastic discount factor. What makes our model differ is that asset liquidity factors asymmetrically augment the stochastic discount factor for the interest and exchange rate. This mechanically gives rise to the liquidity-differential dependent excess returns on $money_B$ in (17). Next proposition finally states a sufficient condition for the UIP violation in equilibrium.

Proposition 3. *Under $\Gamma_{ZLB_i}(\mathbf{s_t}) \subset \Xi_i$ a sufficient condition for $\partial\mathcal{Q}(\mathbf{s}_t)/\partial\left[\ln R_B(\mathbf{s}_t) - \ln R_A(\mathbf{s}_t)\right] > 0$ for every $\mathbf{s}_t$ is $(1-\theta)h \geq (1-g)$.*

Proof. The proof is trivial given Proposition 2, and it is intuitively explained in the following

[23] As explained in footnote 18, this simplification is based on the fact that the standard IMRS, i.e., $\beta U'(X_{t+1})/U'(X_t)$, is a constant in our framework due to the fixed amount of general good consumption over time.

paragraph, therefore it is omitted. □

Proposition 2 states that the condition $(1-\theta)h \geq (1-g)$ under the zero nominal interest rate bound always guarantees a negative effect of $\tilde{\pi}_i(\mathbf{s}_t)$ on $R_i(\mathbf{s}_t)$ for all i and $\mathbf{s}_t$. Combining this result with (18) brings about a positive effect of anticipated inflation on the real liquidity adjusted stochastic discount factor. This implies that a relatively higher anticipated inflation in country A than B leads to higher excess returns on $money_B$. In the meanwhile, the country A's higher anticipated inflation induces the country B's nominal interest rate to become relatively higher than its counterpart since $\partial R_i(\mathbf{s}_t)/\partial\tilde{\pi}_i(\mathbf{s}_t) < 0$. Therefore, $\mathcal{Q}(\mathbf{s}_t)$, regardless of $\mathbf{s}_t$, is always increasing in $\ln R_B(\mathbf{s}_t) - \ln R_A(\mathbf{s}_t)$ under the sufficient condition.

This proposition implies that the UIP violation critically hinges upon the extent to which nominal bonds play a liquidity role. Under the conventional international asset pricing model, a sudden increase in one country's interest rate would lead to an expected depreciation of the currency, thereby holding the UIP condition. Again, this is because nominal bonds, playing no liquidity role, always yield inflation-only-dependent return. However, this mechanism is no longer pervasive when bonds play a liquidity role. For instance, if the bonds exhibit somewhat higher liquidity properties than money in a precise sense that $(1-\theta)h \geq (1-g)$ within our model, an increase in real return on money (i.e., fall in inflation rate) leads to a relatively bigger increase in real return on bonds (i.e., rise in nominal interest rate). Therefore, unlike the conventional model, a sudden increase in one country's interest rate would lead to an expected appreciation of the currency, thereby causing the UIP violation. This intuition naturally brings about the following corollary.

Corollary 1. *Under* $\Gamma_{ZLB_i}(\mathbf{s_t}) \subset \Xi_i$ $\partial\mathcal{Q}(\mathbf{s}_t)/\partial\left[\ln R_B(\mathbf{s}_t) - \ln R_A(\mathbf{s}_t)\right]$ *depends on* $\mathbf{s}_t$*, and the sign is ambiguous if* $(1-\theta)h < (1-g)$*.*

Corollary 1 states that the UIP violation becomes no longer pervasive when liquidity properties of nominal bonds are relatively lower. That is, the UIP no longer violates uniformly when the portion of credit based transactions are lower ($1-\theta$ is lower) and/or the bond pledgeability and liquidity are lower (h and g are lower). This prediction is consistent with a cou-

ple of empirical facts regarding the UIP puzzle. First, it is consistent with the evidence of Bansal and Dahlquist (2000) and Frankel and Poonawala (2010) and our empirical findings (Table 1). Caballero et al. (2008) argue that pledgeability and/or liquidity of assets for emerging economies are generally lower than developed economies. Furthermore, various measures for cross-country credit market and/or financial market development can confirm that credit based transactions are relatively scarce for emerging economies. Our model can capture a similar notion by assuming that g, h, and $1-\theta$ are lower for emerging economies. Consequently, the model implies that the UIP no longer violates uniformly for emerging economies.[24]

Table 1 reports panel regression estimates on the so-called UIP coefficients. In particular, the coefficients in regressions (1) and (2) support the model prediction we mentioned above. The coefficient for the developed countries over the whole sample period (regression (1)) is negative (-0.521), while for the emerging markets (regression (2)) positive (+0.537). This observation reconfirms the previous empirical finding that the forward premium puzzle appears to exist only among developed economies.

Corollary 1 also aligns with an empirical fact that the UIP does not violate when the effective liquidity of the economy suddenly shrinks. Among others, Brunnermeier et al. (2008) show that tightening interbank liquidity predicts carry trade losses (i.e., the UIP suddenly holds). One can capture the sudden "drying up" of economy-wide liquidity by a shock that reduces g, h, and $1-\theta$ in our model. For instance, suppose that the liquidity shock changes $(1-\theta)h \geq (1-g)$ to $(1-\theta)h < (1-g)$. Proposition 3 and Corollary 1 then suggest that such a sudden liquidity shock can lead to a sudden carry trade return reversal in our model. The regressions (3) and (4) in Table 1 reports a strong empirical support for this model prediction. We divide the sample period of the developed countries into "during the period of 2008 and 2009" and "during the period except for 2008 and 2009" to investigate how the UIP coefficient changes during the period when financial assets become illiquid. The 2008-2009 coefficient presents a statistically significant and economically great number (+11.30). However, during the period when the two years are excluded from the sample period, the coefficient turns into a negative and statistically

[24] Note that the excess returns can be positive even in this case if the liquidity premia of bonds happen to be strong enough such that the depreciation of $money_A$ outweighs the magnitude of interest rate differential.

insignificant number (-1.017).

5 Conclusion

Recent monetary and finance theories tell that liquidity properties of assets can play a significant role for asset pricing. Furthermore, they also show that such liquidity aspects of assets interact with monetary policy. This insight is a point of departure for our liquidity-based explanation of the UIP puzzle. We have shown that monetary policy determines the liquidity premium on nominal bonds, which can account for non-uniform deviations from the UIP condition. Intuitively, the conventional wisdom says a high interest rate currency appreciates because it is riskier. We, on the other hand, argue that the high interest rate currency might be appreciating because it is less liquid when the economy is confined to an environment where bonds serve as a sole MOE in some goods transactions. This property of the model turns out to be consistent with some UIP evidence that many conventional risk-based models find hard to justify.

Last but not least, we admit that this liquidity-based explanation is certainly no *panacea* for all those decades long discussions on the UIP puzzle. Yet, we hope that our approach can shed new light on the debate by offering a new liquidity-based perspective. For instance, Backus et al. (2010) have speculated that carry trade returns are in some sense a mirror image of monetary policy implementation costs. We offer a complementary view that the arbitrage carry trade profits might reflect upon the cost of aggregate liquidity management.

References

ALVAREZ, F., A. ATKESON, AND P. J. KEHOE (2009): "Time-Varying Risk, Interest Rates, and Exchange Rates in General Equilibrium," *Review of Economic Studies*, 76, 851–878.

ATKESON, A., F. ALVAREZ, AND P. J. KEHOE (2007): "If Exchange Rates are Random Walks,

then Almost Everything We Say About Monetary Policy is Wrong," *American Economic Review, Papers and Proceedings*.

BACCHETTA, P. AND E. V. WINCOOP (2010): "Infrequent Portfolio Decisions: A Solution to the Forward Discount Puzzle," *American Economic Review*, 100, 870–904.

BACKUS, D. K., S. FORESI, AND C. I. TELMER (2001): "Affine Term Structure Models and the Forward Premium Anomaly," *Journal of Finance, American Finance Association*, 56, 279–304.

BACKUS, D. K., F. GAVAZZONI, C. TELMER, AND S. E. ZIN (2010): "Monetary Policy and the Uncovered Interest Parity Puzzle," NBER Working Papers 16218, National Bureau of Economic Research, Inc.

BANSAL, R. AND M. DAHLQUIST (2000): "The Forward Premium Puzzle: Different Tales from Developed and Emerging Economies," *Journal of International Economics*, 51, 115–144.

BANSAL, R. AND I. SHALIASTOVICH (2013): "A Long-Run Risks Explanation of Predictability Puzzles in Bond and Currency Markets," *Review of Financial Studies*, 26, 1–33.

BEKAERT, G. (1994): "Exchange Rate Volatility and Deviations form Unbiasedness in a Cash-in-Advance Model," *Journal of International Economics*, 36, 29–52.

BEKAERT, G. AND R. J. HODRICK (2001): "Expectations Hypotheses Tests," *Journal of Finance*, 56, 1357–1394.

BERENTSEN, A. AND C. WALLER (2011): "Outside Versus Inside Bonds: A Modigliani-Miller Type Result for Liquidity Constrained Economies," *Journal of Economic Theory*, 146, 1852–1887.

BRUNNERMEIER, M. K., S. NAGEL, AND L. H. PEDERSEN (2008): "Carry Trades and Currency Crashes," *NBER Macroeconomics Annual*, 23.

BURNSIDE, C. (2007): "The Cross Section of Foreign Currency Risk Premia and Consumption Growth Risk: A Comment," *NBER Working Paper*.

BURNSIDE, C., M. EICHENBAUM, I. KLESHCHELSKI, AND S. REBELO (2008): "Do Peso Problems Explain the Returns to the Carry Trade?" *NBER Working Paper*, 14054.

BURNSIDE, C., M. EICHENBAUM, AND S. REBELO (2009): "Understanding the Forward Premium Puzzle: A Microstructure Approach," *American Economic Journal: Macroeconomics*, 1:2.

CABALLERO, R. J., E. FARHI, AND P.-O. GOURINCHAS (2008): "An Equilibrium Model of "Global Imbalances" and Low Interest Rates," *American Economic Review*, 98, 358–393.

CORSETTI, G., A. DASGUPTA, S. MORRIS, AND H. S. SHIN (2004): "Does One Soros Make a Difference? A Theory of Currency Crises with Large and Small Traders," *Review of Economic Studies*, 71, 87–113.

EINZIG, P. (1966): *Primitive Money in its Ethnological, Historical and Economic Aspects*, Pergamon Press, Oxford.

ENGEL, C. (2015): "Exchange Rates, Interest Rates, and the Risk Premium," *NBER Working Paper*, 21042.

——— (2016): "Interest Rates, Exchange Rates and the Risk Premium," *American Economic Review*, 106, 436–474.

FAMA, E. F. (1984): "Forward and spot exchange rates," *Journal of Monetary Economics*, 14, 319 – 338.

FRANKEL, J. A. AND J. POONAWALA (2010): "The Forward Market in Emerging Currencies: Less Biased in Major Currencies," *Journal of International Money and Finance*, 29, 585–598.

FROOT, A. K. AND H. R. THALER (1990): "Anomalies: Foreign Exchange," *Journal of Economic Perspectives*, 4, 179–92.

GEROMICHALOS, A., L. HERRENBRUECK, AND K. SALYER (2015a): "A Search-Theoretic Model of the Term Premium," *Journal of Money, Credit and Banking, forthcoming*.

GEROMICHALOS, A. AND K. M. JUNG (2019): "Monetary Policy and Efficiency in Over-the-Counter Financial Trade," *Canadian Journal of Economics*, forthcoming.

GEROMICHALOS, A., J. LEE, S. LEE, AND K. OIKAWA (2015b): "Over-the-Counter Trade and the Value of Assets as Collateral," *Economic Theory, forthcoming*.

GEROMICHALOS, A., J. M. LICARI, AND J. SUAREZ-LLEDO (2007): "Monetary Policy and Asset Prices," *Review of Economic Dynamics*, 10, 761–779.

GOMIS-PORQUERAS, P., T. KAM, AND C. WALLER (2017): "Nominal Exchange Rate Determinacy under the Threat of Currency Counterfeiting," *American Economic Journal: Macroeconomics*, 9, 256–273.

HU, T.-W. AND G. ROCHETEAU (2013): "On the Coexistence of Money and Higher Return Assets and its Social Role," *Journal of Economic Theory*, 148, 2520–2560.

——— (2015): "Monetary Policy and Asset Prices: A Mechanism Design Approach," *Journal of Money, Credit and Banking*, Forthcoming.

ILUT, C. (2014): "Ambiguity Aversion: Implications for the Uncovered Interest Rate Parity Puzzle," *American Economic Journal: Macroeconomics*, 4, 33–65.

JACQUET, N. L. AND S. TAN (2012): "Money and asset prices with uninsurable risks," *Journal of Monetary Economics*, 59, 784–797.

JUNG, K. M. AND J. H. PYUN (2016): "International Reserves for Emerging Economies: A Liquidity Approach," *Journal of International Money and Finance*, 68, 230–257.

KAREKEN, J. AND N. WALLACE (1981): "On the Indeterminacy of Equilibrium Exchange Rates," *The Quarterly Journal of Economics*, 96, 207–22.

KRISHNAMURTHY, A. AND A. VISSING-JORGENSEN (2012): "The Aggregate Demand for Treasury Debt," *Journal of Political Economy*, 120, 233–267.

LAGOS, R. (2010): "Asset prices and liquidity in an exchange economy," *Journal of Monetary Economics*, 57, 913–930.

——— (2011): "Asset Prices, Liquidity, and Monetary Policy in an Exchange Economy," *Journal of Money, Credit and Banking*, 43, 521–552.

LAGOS, R. AND G. ROCHETEAU (2008): "Money and capital as competing media of exchange," *Journal of Economic Theory*, 142, 247–258.

LAGOS, R., G. ROCHETEAU, AND R. WRIGHT (2014): "The Art of Monetary Theory: A New Monetarist Perspective," *Mimeo*.

LAGOS, R. AND R. WRIGHT (2005): "A Unified Framework for Monetary Theory and Policy Analysis," *Journal of Political Economy*, 113, 463–484.

LAGOS, R. AND S. ZHANG (2014): "A Model of Monetary Exchange in Over-the-Counter Markets," *New York University, mimeo*.

LEE, S. (2016): "Money, Asset Prices and the Liquidity Premium," MPRA Paper 73707, University Library of Munich, Germany.

LESTER, B., A. POSTLEWAITE, AND R. WRIGHT (2012): "Information, Liquidity, Asset Prices, and Monetary Policy," *Review of Economic Studies*, forthcoming.

LEWIS, K. K. (1995): "Puzzles in International Finance Markets," *Handbook of International Economics*, 1913–1971.

LINNENMANN, L. AND A. SCHABERT (2015): "Liquidity Premia and Interest Rate Parity," *Journal of International Economics*, forthcoming.

LUCAS, ROBERT E, J. (1978): "Asset Prices in an Exchange Economy," *Econometrica*, 46, 1429–45.

LUCAS, R. J. (1982): "Interest rates and currency prices in a two-country world," *Journal of Monetary Economics, Elsevier*, 10, 335–359.

LUSTIG, H., N. ROUSSANOV, AND A. VERDELHAN (2011): "Common Risk Factors in Currency Markets," *Review of Financial Studies*, 24, 3731–3777.

LUSTIG, H. AND A. VERDELHAN (2007): "The Cross Section of Foreign Currency Risk Premia and Consumption Growth Risk," *American Economic Review*, 97, 89–117.

NAGEL, S. (2016): "The Liquidity Premium of Near-Money Assets," *Quarterly Journal of Economics*, 131, 1927–1971.

NOSAL, E. AND G. ROCHETEAU (2011): *Money, Payments, and Liquidity*, MIT Press Books, The MIT Press, The MIT Press.

——— (2012): "Pairwise trade, asset prices, and monetary policy," *Journal of Economic Dynamics and Control*.

ROCHETEAU, G. AND R. WRIGHT (2005): "Money in search equilibrium, in competitive equilibrium, and in competitive search equilibrium," *Econometrica*, 73, 175–202.

VERDELHAN, A. (2010): "A Habit-Based Explanation of the Exchange Rate Risk Premium," *Journal of Finance*, 65, 123–146.

WALLACE, N. (2001): "Whither Monetary Economics?" *International Economic Review*, 847–869.

——— (2010): "The Mechanism-Design Approach to Monetary Approach," *Handbook of Monetary Economics*, 3, 3–23.

WILLIAMSON, S. (2012): "Liquidity, Monetary Policy, and the Financial Crisis: A New Monetarist Approach," *American Economic Review*, 2570–2605.

XIAO, S., R. WRIGHT, AND G. ROCHETEAU (2017): "Open Market Operations," 2017 Meeting Papers 345, Society for Economic Dynamics.

ZHANG, C. (2014): "An information-based theory of international currency," *Journal of International Economics*, 93, 286–301.

ZHU, T. AND N. WALLACE (2007): "Pairwise Trade and Coexistence of Money and Higher-Return Assets," *Journal of Economic Theory*, 133, 524–535.

A Appendix

A.1 Trading Mechanism in Goods Market

In this appendix, we propose a trading mechanism where the cash-in-advance constraint and the illiquidity parameters of bonds, which are exogenously imposed in Section 3.2, are fully endogenized in equilibrium. To that end, we adopt the mechanism design approach pioneered by Wallace (2001) and Zhu and Wallace (2007). The trading mechanism that we study here induces allocations to be pairwise Pareto optimal, but treats assets asymmetrically depending on their types as well as nationality. This is meant to capture two intuitive notions. The first thing is that agents receive better terms of trade in a country by using domestic assets rather than foreign ones. The second is that bonds can be accepted as a method of payment by sellers but for less output than what the buyer could have obtained with money, i.e., the illiquidity of bonds. A main advantage of the proposed trading mechanism is that despite asymmetric treatment of the assets, it leaves no gains from trade unexploited, i.e., allocations are socially efficient, which is not generally the case for other mechanisms (e.g., Nash Bargaining). On top of that, it yields nominal exchange rate determinacy without imposing any cash-in-advance type restrictions.[25]

First, consider a meeting in $GM1$ between seller i and buyer i who carries a portfolio $\mathbf{w}_t$. Given the above intuition, one can conceptually understand the proposed mechanism in two stages. In the first stage, the buyer i makes a take-it-or-leave-it offer to the seller i under restrictions that the former can only use $money_i$ and $bond_i$ for payment, and is able to transfer at most a fraction $g \in [0, 1]$ of her $bond_i$ holdings as exogenously imposed in Section 3.2. Then, they move together to the second stage where no restrictions on the use of any asset exist. The final terms of trade, which are actually implemented, is determined by maximizing the surplus of the seller i, while maintaining the buyer i's surplus at the first stage payoff level. As a result, the final allocation will be pairwise Pareto efficient. Nevertheless, the buyer i can obtain

[25] Kareken and Wallace (1981) showed that the nominal exchange rate indeterminacy is pervasive in monetary models unless *ad-hoc* frictions such as the cash-in-advance constraint, i.e., agents trade only with their domestic currency in their home market, are imposed. Yet, as Nosal and Rocheteau (2011) have argued, the cash-in-advance constraint seems particularly odd when the two currencies have different rates of return, i.e, inflation. Also see Wallace (2010) for various disadvantages of cash-in-advance type models from a monetary theorist's point of view.

additional gain neither from using foreign assets nor from paying with more $bond_i$ and less $money_i$. Thus, our mechanism can in fact allows for asset-specific transaction restrictions i.e., the cash-in-advance constraint (only domestic assets used in domestic markets) and the exogenous illiquidity of bonds, to be fully endogenized in equilibrium.

Let us now look at the buyer i's surplus at the first stage of the mechanism, $\mathbb{U}^b_{11}(\mathbf{w}_t)$. Following intuitive descriptions above, it can be expressed as

$$\mathbb{U}^b_{11}(\mathbf{w}_t) = \max_{q_{i,t}, \boldsymbol{p}_t} \left\{ u(q_{i,t}) + W^B_i(\mathbf{w}_t - \boldsymbol{p}_t, 0, \mathbf{s}_t) - W^B_i(\mathbf{w}_t, 0, \mathbf{s}_t) \right\}$$
$$\text{s.t.} - q_{i,t} + W^S_i(\boldsymbol{p}_t, \mathbf{0}, \mathbf{s}_t) - W^S_i(\mathbf{0}, \mathbf{0}, \mathbf{s}_t) = 0,$$

with the illiquidity augmented feasibility constraint $\boldsymbol{p}_t \leq \widetilde{\mathbf{w}}_t$ where $\widetilde{\mathbf{w}}_t = (m_{A,t}, ga_{A,t}, m_{B,t}, ga_{B,t})$ and the cash-in-advance constraint $p^{m_{-i}}_t = p^{a_{-i}}_t = 0$. Note that the subscript 'kj' of $\mathbb{U}^b_{kj}$ denotes the $k\ th$ stage in the GMj, $k, j \in (1, 2)$. Given the linearity of W^B_i and W^S_i, $\mathbb{U}^b_{11}(\mathbf{w}_t)$ simplifies to

$$\mathbb{U}^b_{11}(\mathbf{w}_t) = \max_{q_{i,t}, p^{m_i}_t, p^{a_i}_t} \{u(q_{i,t}) - \varphi_{i,t}(p^{m_i}_t + p^{a_i}_t)\}$$
$$\text{s.t.} - q_{i,t} + \varphi_{i,t}(p^{m_i}_t + p^{a_i}_t) = 0,$$

and $p^{m_i}_t \leq m_{i,t},\ p^{a_i}_t \leq ga_{i,t}$.

Interpretation of the problem above is standard. The buyer i's payoff is obtained by choosing her consumption and the transfer of her domestic money and bonds in order to maximize her surplus. It is important to note that while she can transfer $money_i$ up to her entire $money_i$ holdings, an upper bound for her $bond_i$ transfers is a fraction g of her $bond_i$ holdings. Furthermore, the aim of the first stage is to pin down a payoff level for the buyer i. It is worth emphasizing that the terms of trade chosen in this stage are not necessarily the ones that are finally implemented.

Next, we move to the second stage where the buyer i is allowed to use any of her assets to pay without any exogenous liquidity restrictions. The actual terms of trade for $GM1$ are determined such that the seller i maximizes her surplus, taking the predetermined surplus level

of the buyer i from the first stage as given. The seller i's surplus at the second stage of the mechanism, $\mathbb{U}^s_{21}(\mathbf{w}_t)$ where $\mathbf{w}_t$ denotes the buyer i's portfolio holdings, is then expressed as

$$\mathbb{U}^s_{21}(\mathbf{w}_t) = \max_{q_{i,t},\boldsymbol{p}_t} \left\{-q_{i,t} + W_i^S(\boldsymbol{p}_t, \mathbf{0}, \mathbf{s}_t) - W_i^S(\mathbf{0}, \mathbf{0}, \mathbf{s}_t)\right\}$$
$$\text{s.t. } u(q_{i,t}) + W_i^B(\mathbf{w}_t - \boldsymbol{p}_t, 0, \mathbf{s}_t) - W_i^B(\mathbf{w}_t, 0, \mathbf{s}_t) = \mathbb{U}^b_{11}(\mathbf{w}_t),$$

and the feasibility constraint $\boldsymbol{p}_t \leq \mathbf{w}_t$. Given the linearity of W_i^B and W_i^S, $\mathbb{U}^s_{21}(\mathbf{w}_t)$ again simplifies to

$$\mathbb{U}^s_{21}(\mathbf{w}_t) = \max_{q_{i,t},\boldsymbol{p}_t} \{-q_{i,t} + \boldsymbol{\phi}_t \boldsymbol{p}_t\}$$
$$\text{s.t. } u(q_{i,t}) - \boldsymbol{\phi}_t \boldsymbol{p}_t = \mathbb{U}^b_{11}(\mathbf{w}_t),$$

and $\boldsymbol{p}_t \leq \mathbf{w}_t$.

Notice that the buyer i is never restricted to use any of her assets as means of payment. Furthermore, the feasibility constraint does not impose any asymmetric liquidity restrictions (e.g., bonds are now fully liquid, i.e., $g = 1$). Further, the constraint that the buyer i's surplus must equal $\mathbb{U}^b_{11}$ guarantees that the final allocation is pairwise Pareto efficient. The next lemma describes the results of the proposed pricing mechanism in $GM1$.

Lemma 3. *Define $q^* \equiv \{q : u'(q) = 1\}$. The total real balances of buyer i are denoted as $z(\mathbf{w}_t) \equiv \boldsymbol{\phi_t}\mathbf{w}_t$. Finally, define $z^* \equiv u(q^*) - \mathbb{U}^b_{11}$ and $\tilde{\boldsymbol{p}}(\mathbf{w}_t)$ as the set of $(m_{A,t}, a_{A,t}, m_{B,t}, a_{B,t})$ such that $\boldsymbol{\phi_t} \cdot \tilde{\boldsymbol{p}}(\mathbf{w}_t) = z^*$. When the buyer i with a portfolio $\mathbf{w}_t$ meets seller i in $GM1$, the proposed pricing mechanism yields the following results. The terms of trade in the first stage are given by $q_{i,t} \equiv q_i(\mathbf{w}_t)$, $p_t^{m_i} \equiv p^{m_i}(\mathbf{w}_t)$, and $p_t^{a_i} \equiv p^{a_i}(\mathbf{w}_t)$ such that $q_i(\mathbf{w}_t) = \varphi_{i,t}\left(p^{m_i}(\mathbf{w}_t) + p^{a_i}(\mathbf{w}_t)\right) = \min\{q^*, \varphi_{i,t}(m_{i,t} + ga_{i,t})\}$. The actual terms of trade determined in the second stage are given by*

$$q_i(\mathbf{w}_t) = \begin{cases} q^*, & \text{if } z(\mathbf{w}_t) \geq z^*, \\ b_t, & \text{if } z(\mathbf{w}_t) < z^*. \end{cases} \qquad \boldsymbol{p}(\mathbf{w}_t) = \begin{cases} \tilde{\boldsymbol{p}}(\mathbf{w}_t), & \text{if } z(\mathbf{w}_t) \geq z^*, \\ \mathbf{w}_t, & \text{if } z(\mathbf{w}_t) < z^*, \end{cases}$$

where $b_t \equiv u^{-1}\left[z(\mathbf{w}_t) + \mathbb{U}^b_{11}(\mathbf{w}_t)\right]$. *The surplus for the buyer* i *and the seller* i *is respectively given by*

$$\mathbf{U}^b_{11}(\mathbf{w}_t) = \begin{cases} u(q^*) - q^*, & \text{if } \varphi_{i,t}(m_{i,t} + ga_{i,t}) \geq q^*, \\ u(\varphi_{i,t}(m_{i,t} + ga_{i,t})) - \varphi_{i,t}(m_{i,t} + ga_{i,t}), & \text{if } \varphi_{i,t}(m_{i,t} + ga_{i,t}) < q^*, \end{cases}$$

$$\mathbf{U}^s_{21}(\mathbf{w}_t) = \begin{cases} 0, & \text{if } \varphi_{i,t}(m_{i,t} + ga_{i,t}) \geq q^*, \\ u(q^*) - q^* - \mathbf{U}^b_{11}(\mathbf{w}_t), & \text{if } \varphi_{i,t}(m_{i,t} + ga_{i,t}) < q^* \textit{ and } z(\mathbf{w}_t) \geq z^*, \\ u(b_t) - b_t - \mathbf{U}^b_{11}(\mathbf{w}_t), & \text{if } \varphi_{i,t}(m_{i,t} + ga_{i,t}) < q^* \textit{ and } z(\mathbf{w}_t) < z^*. \end{cases}$$

Proof. See the Appendix. A.2. □

For the second stage outcome notice again that only the buyer i's total real balances determine the actual terms of trade. When $z(\mathbf{w}_t)$ exceeds z^* that guarantees the first best outcome for the seller i, the buyer i also receives the first best, q^* in return for any combination of her asset holdings whose real value equals z^*, i.e., $\tilde{\boldsymbol{p}}(\mathbf{w}_t)$. Otherwise, the buyer i is liquidity constrained. Hence, she gives up her entire portfolio in order to obtain as much $q_{i,t}$ as possible, subject to the constraint that her net consumption utility of $q_{i,t}$ in $GM1$ equals $\mathbf{U}^b_{11}(\mathbf{w}_t)$. The seller i's second stage payoff $\mathbf{U}^s_{21}(\mathbf{w}_t)$ in Lemma 1 then immediately follows by replacing the buyer i's participation constraint into her objective function.

There are three key observations worth emphasizing here. First, the fact that $\mathbf{U}^s_{21}(\mathbf{w}_t)$ is non-negative given the same level of buyer surplus as in the first stage indicates that the proposed mechanism yields a pairwise Pareto efficient outcome.[26] Second, $\mathbf{U}^b_{11}(\mathbf{w}_t)$ is never affected by foreign asset holdings of the buyer. Thus, it is immediate that she will never choose to hold any foreign asset in equilibrium with positive asset holding costs, i,e., the cash-in-advance constraint rises endogenously. Lastly, although no restrictions on the illiquidity of bonds are imposed, i.e., $g = 1$, in the second stage, the buyer i gets exactly the same payoff that she would have obtained in a model with the exogenous liquidity constraint, i.e., g affects the level of $\mathbf{U}^b_{11}(\mathbf{w}_t)$. Hence, our mechanism also endogenously derive, rather than impose, the bond illiquidity constraint.

[26] See Nosal and Rocheteau (2011) for detailed graphical illustration of the Pareto improvement from the first stage to the second, as well as the shapes of Pareto frontiers for the two stages.

Now, let us look at the details in the $GM2$ where credit is only accepted in payments. We take the same steps as in the $GM1$. In the first stage, the buyer i makes a TIOLI offer to the seller i under restrictions that only $bond_i$ can be used as collateral to obtain credit, and the buyer i can acquire that credit only up to a fraction h of her real $bond_i$ balances as in Section 3.2. In the second stage, we remove the restriction that only domestic bonds should be used as collateral. Then, we let the seller i choose the actual terms of trade by maximizing her surplus subject to the credit constraint that now applies to both $\ell_{i,t}$ and $\ell_{-i,t}$. Importantly, she also has to make sure that the buyer i's surplus remains at the first stage payoff level.

The first stage surplus for the buyer i is then given by

$$\mathbb{U}_{12}^b(\mathbf{w}_t) = \max_{q_{i,t},\boldsymbol{\ell}_t} \left\{ u(q_{i,t}) + W_i^B(\mathbf{w}_t, \boldsymbol{\ell}_t, \mathbf{s}_t) - W_i^B(\mathbf{w}_t, 0, \mathbf{s}_t) \right\}$$
$$\text{s.t.} - q_{i,t} + W_i^S(\mathbf{0}, \boldsymbol{\ell}_t, \mathbf{s}_t) - W_i^S(\mathbf{0}, \mathbf{0}, \mathbf{s}_t) = 0,$$

and $\ell_{-i,t} = 0$ with the credit limit constraint $\ell_{i,t} \leq h\varphi_{i,t}a_{i,t}$. The linearity of W_i^B and W_i^S simplifies $\mathbb{U}_{12}^b(\mathbf{w}_t)$ to

$$\mathbb{U}_{12}^b(\mathbf{w}_t) = \max_{q_{i,t},\ell_{i,t}} \{u(q_{i,t}) - \ell_{i,t}\}$$
$$\text{s.t.} - q_{i,t} + \ell_{i,t} = 0,$$

with the same constraints above.

In the second stage, the restriction on the use of foreign bonds as collateral, i.e., $\ell_{-i,t} = 0$, is removed. Thus the pricing mechanism is given by

$$\mathbb{U}_{22}^s(\mathbf{w}_t) = \max_{q_{i,t},\boldsymbol{\ell}_t} \left\{ -q_{i,t} + W_i^S(\mathbf{0}, \boldsymbol{\ell}_t, \mathbf{s}_t) - W_i^S(\mathbf{0}, \mathbf{0}, \mathbf{s}_t) \right\}$$
$$\text{s.t. } u(q_{i,t}) + W_i^B(\mathbf{w}_t, \boldsymbol{\ell}_t, \mathbf{s}_t) - W_i^B(\mathbf{w}_t, 0, \mathbf{s}_t) = \mathbb{U}_{12}^b(\mathbf{w}_t),$$

and the credit limit constraints $\ell_{i,t} \leq h\varphi_{i,t}a_{i,t}$, $\forall i$. Using the linear value functions, $\mathbb{U}_{22}^s(\mathbf{w}_t)$ is

again simplified to

$$\mathbb{U}_{22}^{s}(\mathbf{w}_t) = \max_{q_{i,t},\ell_{i,t},\ell_{-i,t}} \{-q_{i,t} + \ell_{i,t} + \ell_{-i,t}\}$$
$$\text{s.t. } u(q_{i,t}) - \ell_{i,t} - \ell_{-i,t} = \mathbb{U}_{12}^{b}(\mathbf{w}_t),$$

and $\ell_{i,t} \leq h\varphi_{i,t}a_{i,t}$, $\forall i$.

The following Lemma 4 summarizes the solutions to the proposed mechanism in $GM2$.

Lemma 4. *Define the total real value of the buyer i's bond holdings as $z_a(\mathbf{w}_t) \equiv \varphi_{A,t}a_{A,t} + \varphi_{B,t}a_{B,t}$, and $\boldsymbol{a}_t^h$ as a set of $(h\varphi_{A,t}a_{A,t},\ h\varphi_{b,t}a_{B,t})$. Finally, let $\tilde{\boldsymbol{\ell}}$ denote the set of $(\ell_{A,t}, \ell_{B,t})$ such that $\ell_{A,t} + \ell_{B,t} = u(q^*) - \mathbb{U}_{12}^{b}(\mathbf{w}_t)$, and define $z_a^* = u(q^*) - \mathbb{U}_{12}^{b}$. When the buyer i with a portfolio $\mathbf{w}_t$ meets seller i in $GM2$, the proposed pricing mechanism yields the following results. The terms of trade in the first stage are given by $q_{i,t} \equiv q_i(\mathbf{w}_t)$ and $\ell_{i,t} \equiv \ell_i(\mathbf{w}_t)$ such that $q_i(\mathbf{w}_t) = \ell_i(\mathbf{w}_t) = \min\{q^*, h\varphi_{i,t}a_{i,t}\}$. The actual terms of trade determined in the second stage are given by*

$$q_i(\mathbf{w}_t) = \begin{cases} q^*, & \text{if } z_a(\mathbf{w}_t) \geq z_a^*, \\ c_t, & \text{if } z_a(\mathbf{w}_t) < z_a^*. \end{cases} \qquad \boldsymbol{\ell}(\mathbf{w}_t) = \begin{cases} \tilde{\boldsymbol{\ell}}, & \text{if } z_a(\mathbf{w}_t) \geq z_a^*, \\ \boldsymbol{a}_t^h, & \text{if } z_a(\mathbf{w}_t) < z_a^*, \end{cases}$$

where $c_t = u^{-1}\left[z_a(\mathbf{w}_t) + \mathbb{U}_{12}^{b}(\mathbf{w}_t)\right]$. The surplus for the buyer i and the seller i is respectively given by

$$\mathbf{U}_{12}^{b}(\mathbf{w}_t) = \begin{cases} u(q^*) - q^*, & \text{if } h\varphi_{i,t}a_{i,t} \geq q^*, \\ u(\ h\varphi_{i,t}a_{i,t}) - \ h\varphi_{i,t}a_{i,t}, & \text{if } h\varphi_{i,t}a_{i,t} < q^*, \end{cases}$$

$$\mathbf{U}_{22}^{s}(\mathbf{w}_t) = \begin{cases} 0, & \text{if } h\varphi_{i,t}a_{i,t} \geq q^*, \\ u(q^*) - q^* - \mathbf{U}_{12}^{b}(\mathbf{w}_t), & \text{if } h\varphi_{i,t}a_{i,t} < q^* \textit{ and } z_a(\mathbf{w}_t) \geq z_a^*, \\ u(c_t) - c_t - \mathbf{U}_{12}^{b}(\mathbf{w}_t), & \text{if } h\varphi_{i,t}a_{i,t} < q^* \textit{ and } z_a(\mathbf{w}_t) < z_a^*. \end{cases}$$

Proof. The proof follows similar steps as in Lemma 3, and it is, therefore, omitted. □

These results are intuitive, and admit almost identical interpretation as in Lemma 1. A key difference is that in $GM2$ money holdings are irrelevant, and the liquidity restriction on bonds

as collateral is now reflected by the pledgeability parameter h. Following these differences, outcomes in each stage are straightforward to understand. The terms of trade in the first stage now depend on the buyer i's credit pledgeability augmented real $bond_i$ balance, i.e., $h\varphi_{i,t}a_{i,t}$. If this is less than the first best amount, q^* then, she becomes liquidity constrained, and therefore place all her $bond_i$ holdings as collateral to get as much credit as possible. Otherwise, she just borrows q^* by placing whatever amounts of $bond_i$ as collateral needed to obtain that q^*. In the second stage no restrictions on the use of $bond_{-i}$ exist any more. Therefore, whether the buyer i is liquidity constrained or not depends on the relative value of her total real bond balances, $z_a(\mathbf{w}_t)$, to the first best amount which is now z_a^*. Then, the solution to the second stage terms of trade again follows trivially. Similar to the $GM1$ mechanism, the actual allocation in $GM2$ is also pairwise Pareto efficient, i.e., $\mathbf{U}_{22}^s(\mathbf{w}_t)$ is non-negative. Likewise, it implies endogenously driven cash-in-advance constraint in equilibrium, i.e., $bond_{-i}$ has no effect on $\mathbf{U}_{12}^b(\mathbf{w}_t)$.

Lastly, we show why eq.(4), (5), (6), and (7) remain the same under this pricing protocol in GMs. To that end, it is useful to rewrite the terms within the max operator in the buyer i's portfolio choice problem (1). Substitute (3) into (1) and rearrange terms within the max operator using solutions to $\mathbb{U}_{1j}^b$ from Lemma 3 and 4. Then, one can arrive the following expression.

$$\max_{\mathbf{w}_{t+1}} (-\boldsymbol{\phi}_t + \beta\mathbb{E}_t\boldsymbol{\phi}_{t+1})\mathbf{w}_{t+1} + \beta\mathbb{E}_t\left[\theta\mathbb{U}_{11}^b(\mathbf{w}_{t+1}) + (1-\theta)\mathbb{U}_{12}^b(\mathbf{w}_{t+1})\right].$$

Notice that her discounted expected benefit from GM trade in the next period is pinned down by $\beta\mathbb{E}_t\left[\theta\mathbb{U}_{11}^b(\mathbf{w}_{t+1}) + (1-\theta)\mathbb{U}_{12}^b(\mathbf{w}_{t+1})\right]$ which is never affected by her $money_{-i}$ or $bond_{-i}$ holdings according to Lemma 3 and 4. Thus, the buyer i never appreciates the liquidity value of foreign assets in the forthcoming GM. One can also relate this intuition directly to the pricing mechanism proposed in the GM trade. Our mechanism induces the buyer i to obtain worse terms of trade if she chooses to purchase special good i with foreign assets than domestic ones in both $GM1$ and $GM2$. For instance, without loss of generality, consider buyer i who enters $GM1$. From the constraint for $\mathbb{U}_{11}^b(\mathbf{w}_t)$ earlier, one can show that she can obtain a unit of special good i in return for an additional unit of real $money_i$ ($bond_i$), i,.e., $1/\varphi_{i,t}$ ($1/(g\varphi_{i,t})$). Yet, according to the constraint for $\mathbb{U}_{21}^s(\mathbf{w}_t)$, she gets less than or equal to a unit of the good, i.e., $1/u'(q_{i,t}^{GM1}) \leq 1$,

with an additional real $money_{-i}$ ($bond_{-i}$). This implies that domestic assets are always superior to foreign ones in terms of the marginal surplus generated in the $GM1$. Same results follow trivially for the $GM2$. This explains why all those Euler equations should remain the same even under this mechanism design approach to the terms of trade determination.

A.2 Proof of Statements

Proof. Proof of Lemma 3.

Consider first the second stage in the trading mechanism. We substitute the real balances term in the buyer's participation constraint (A.1) into the maximization objective function (A.1). Then, bargaining problem in the second stage where the implemented terms of trade is determined is given by

$$\begin{aligned}
\mathbb{U}^s_{21}(\mathbf{w}_t) = \max_{q_{i,t},\boldsymbol{p}_t} & \; u(q_{i,t}) - q_{i,t} - \mathbb{U}^b_{11}(\mathbf{w}_t) \\
\text{s.t. } & \; u(q_{i,t}) - \boldsymbol{\phi_t}\boldsymbol{p}_t = \mathbb{U}^b_{11}(\mathbf{w}_t), \\
& \; \boldsymbol{p}_t \leq \mathbf{w}_t
\end{aligned}$$

If $z(\mathbf{w}_t) \geq z^*$, it is obvious that $q_{i,t}$ will always be equal to q^*, and $\tilde{\boldsymbol{p}}_t(\mathbf{w}_t)$ can be any combination of money and assets such that $\boldsymbol{\phi}_t \cdot \tilde{\boldsymbol{p}}_t(\mathbf{w}_t) = u(q^*) - \mathbb{U}^b_{11}(\mathbf{w}_t)$. In addition, $\mathbb{U}^s_{21}(\mathbf{w}_t)$ will be equal to $u(q^*) - q^* - \mathbb{U}^b_{11}(\mathbf{w}_t)$, which is zero in the case where $\varphi_{i,t}(m_t + ga_t) \geq q^*$, because $\mathbb{U}^b_{11}(\mathbf{w}_t) = u(q^*) - q^*$. However, if $z(\mathbf{w}_t) < z^*$, the first best choice q^* cannot be achieved, and so a seller i will make an offer to have the buyer hand over all of her real balances in order to sell the special goods as much as possible. In this case, $\boldsymbol{p}_t(\mathbf{w}_t) = \mathbf{w}_t$ and $q_{i,t} = b_t \equiv u^{-1}\left[z(\mathbf{w}_t) + \mathbb{U}^b_{11}(\mathbf{w}_t)\right]$. Lastly, $\mathbb{U}^s_{21}(\mathbf{w}_t)$ will be equal to $u(b_t) - b_t - \mathbb{U}^b_{11}(\mathbf{w}_t)$.

Likewise, the bargaining problem in the first stage is given by plugging the seller's partici-

pation constraint (A.1) into the objective function (A.1) as follows.

$$\mathbb{U}_{11}^{b}(\mathbf{w}_t) = \max_{q_{i,t},\, p_t^{m_i},\, p_t^{a_i}} u(q_{i,t}) - q_{i,t}$$
$$\text{s.t. } -q_{i,t} + \varphi_{i,t}(\, p_t^{m_i} + \, p_t^{a_i}) = 0$$
$$p_t^{m_i} \leq m_t, \;\; p_t^{a_i} \leq ga_t$$

If $\varphi_t(m_t + ga_t) \geq q^*$, $q_{i,t}$ will be equal to q^* and $\mathbb{U}_{11}^{b}(\mathbf{w}_t)$ is to $u(q^*) - q^*$. If $\varphi_{i,t}(m_t + ga_t) < q^*$, then $q_{i,t}$ will be equal to $\varphi_{i,t}(m_t + ga_t)$, because the buyer will give up all of her domestic money and assets to purchase as much $q_{i,t}$ as she can. In this case, $\mathbb{U}_{11}^{b}(\mathbf{w}_t)$ is $u(\varphi_{i,t}(m_t + ga_t)) - \varphi_{i,t}(m_t + ga_t)$. □

Proof. Proof of Proposition 2.

Dividing equation (8) by $\varphi(\mathbf{s}_t)$ leads to

$$1 = \beta \int \frac{\boldsymbol{L}[Z_i(\mathbf{s}_{t+1})]}{\pi_i(\mathbf{s}_{t+1}, \mathbf{s}_t)} d\boldsymbol{F}(\mathbf{s}_{t+1}, \mathbf{s}_t) = \frac{\beta}{\tilde{\pi}_i(\mathbf{s}_t)} + \beta\theta \int \frac{[u'(min\{Z_i(\mathbf{s}_{t+1}), q^*\}) - 1]}{\pi_i(\mathbf{s}_{t+1}, \mathbf{s}_t)} d\boldsymbol{F}(\mathbf{s}_{t+1}, \mathbf{s}_t)$$
$$= \frac{\beta}{\tilde{\pi}_i(\mathbf{s}_t)} + \beta\theta \mathbf{X}_i(\mathbf{s}_t)$$

where $\mathbf{X}_i(\mathbf{s}_t) = \int \frac{[u'(min\{Z_i(\mathbf{s}_{t+1}), q^*\}) - 1]}{\pi_i(\mathbf{s}_{t+1}, \mathbf{s}_t)} d\boldsymbol{F}(\mathbf{s}_{t+1}, \mathbf{s}_t)$. Then,

$$\beta\theta \mathbf{X}_i(\mathbf{s}_t) = 1 - \frac{\beta}{\tilde{\pi}_i(\mathbf{s}_t)}.$$

By the implicit function theorem,

$$\frac{\partial \mathbf{X}_i(\mathbf{s}_t)}{\partial \tilde{\pi}_i(\mathbf{s}_t)} = \frac{1}{\theta[\tilde{\pi}_i(\mathbf{s}_t)]^2} > 0.$$

On the other hand, we rewrite the nominal bond price at $\mathbf{s}_t$ in country i (equation (16)) as

follows.

$$\frac{\psi_i(\mathbf{s}_t)}{\varphi_i(\mathbf{s}_t)} = 1 - \beta\theta(1-g)\mathbf{X}_i(\mathbf{s}_t) + \beta(1-\theta)h\int \frac{[u'(\min\{Z_i^a(\mathbf{s}_{t+1}), q^*\}) - 1]}{\pi_i(\mathbf{s}_{t+1}, \mathbf{s}_t)} d\boldsymbol{F}(\mathbf{s}_{t+1}, \mathbf{s}_t).$$

Then, the partial derivative of the nominal bond price is

$$\begin{aligned}\frac{\partial[\psi_i(\mathbf{s}_t)/\varphi_i(\mathbf{s}_t)]}{\partial\tilde{\pi}_i(\mathbf{s}_t)} &= \beta\theta(1-g)\frac{\partial\mathbf{X}_i(\mathbf{s}_t)}{\partial\tilde{\pi}_i(\mathbf{s}_t)} + \beta(1-\theta)h\frac{\partial\mathbf{Y}_i(\mathbf{s}_t)}{\partial\tilde{\pi}_i(\mathbf{s}_t)} + \frac{\beta(1-\theta)h}{[\tilde{\pi}_i(\mathbf{s}_t)]^2} \\ &= \frac{\beta[(1-\theta)h - (1-g)]}{[\tilde{\pi}_i(\mathbf{s}_t)]^2} + \beta(1-\theta)h\frac{\partial\mathbf{Y}_i(\mathbf{s}_t)}{\partial\tilde{\pi}_i(\mathbf{s}_t)}\end{aligned}$$

where $\mathbf{Y}_i(\mathbf{s}_t) = \int \frac{[u'(\min\{Z_i^a(\mathbf{s}_{t+1}), q^*\}) - 1]}{\pi_i(\mathbf{s}_{t+1}, \mathbf{s}_t)} d\boldsymbol{F}(\mathbf{s}_{t+1}, \mathbf{s}_t)$. Now, it is obvious from the definitions of $Z_i(\mathbf{s}_t)$ and $Z_i^a(\mathbf{s}_t)$ that $\frac{\partial\mathbf{Y}_i(\mathbf{s}_t)}{\partial\tilde{\pi}_i(\mathbf{s}_t)} > 0$, because $\frac{\partial\mathbf{X}_i(\mathbf{s}_t)}{\partial\tilde{\pi}_i(\mathbf{s}_t)} > 0$. Consequently, the sufficient condition for $\frac{\partial[\psi_i(\mathbf{s}_t)/\varphi_i(\mathbf{s}_t)]}{\partial\tilde{\pi}_i(\mathbf{s}_t)} > 0$ is $(1-\theta)h \geq (1-g)$. □

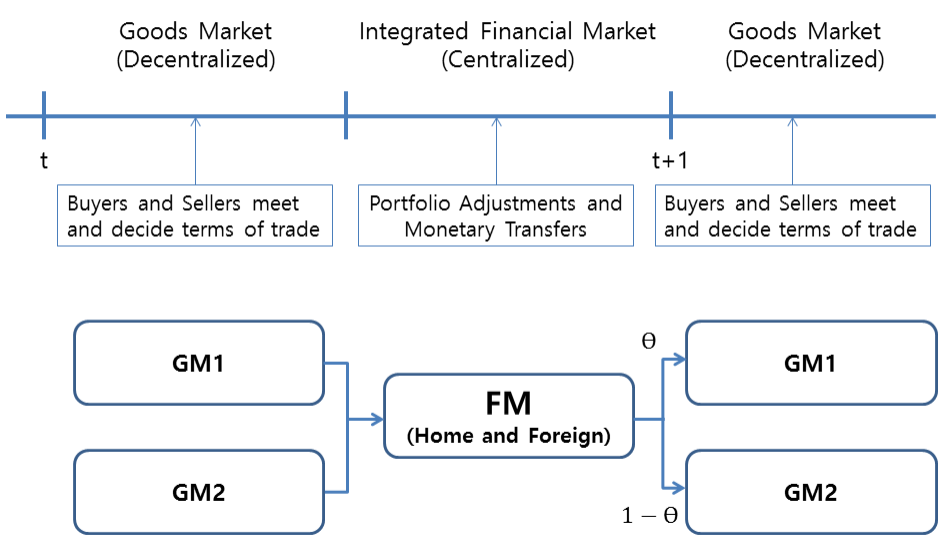

Figure 1: Timing of Events

$$\text{Regression Equation}: \left[\frac{E_{i,t+1} - E_{i,t}}{E_{i,t}}\right] = \alpha_0 + \alpha_1 \left[\frac{F_{i,t} - E_{i,t}}{E_{i,t}}\right] + u_{i,t+1}$$

	(1) Developed Economies	(2) Emerging Economies	(3) Developed Economies during 2008-9	(4) Developed Economies during the period except for 2008-9
α_1	-0.521	0.537***	11.30***	-1.017
	(0.414)	(0.0741)	(3.390)	(0.611)
α_0	8.52e-05	-0.000972	0.0139**	-0.000778
	(0.000934)	(0.00146)	(0.00473)	(0.00144)
Period	1/1996-2/2015	1/1996-2/2015	1/2008-12/2009	excl. 08-09
Observations	2,339	1,667	238	2,101
R-squared	0.002	0.192	0.102	0.008
Number of state	11	9	11	11
State FE	Yes	Yes	Yes	Yes

Note: Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. Monthly data on spot exchange rates, and 3-month forward rates for 20 countries from *Bloomberg* are used in the regressions. Fixed effects are included in all of the regressions, i.e., a country-specific intercept is added to each regression. The developed countries include Switzerland, Hong Kong, Singapore, Japan, Belgium, Austria, Denmark, Canada, UK, Australia, Sweden, and the emerging countries include Czech Republic, Malaysia, Argentina, Mexico, Thailand, Philippines, Indonesia, India, Turkey, Korea according to the *International Finance Corporation (IFC)* of the *World Bank*. The exchange rates for the developed countries are the US Dollar prices per unit and for the emerging countries the Malaysian Ringgit prices per unit.

Table 1: Forward Premium Regression