Technical Appendix: International Reserves for Emerging Economies: A Liquidity Approach

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A Auxiliary properties of the regions specific $J^H\left(\widehat{a}_h, \widehat{a}_h^*\right)$

Lemma 6. *Define $J_i^H(\widehat{a}_h, \widehat{a}_h^*), i = 1, \cdots, 6$ as H's objective function in region i. Then the partial derivative with respect to the second argument, $\widehat{a}_h^*$ in each region can be expressed as follows.*

$$\begin{aligned}
\frac{\partial J_1^H(\widehat{a}_h, \widehat{a}_h^*)}{\partial \widehat{a}_h^*} &= \frac{\partial J_3^H(\widehat{a}_h, \widehat{a}_h^*)}{\partial \widehat{a}_h^*} = \frac{\partial J_5^H(\widehat{a}_h, \widehat{a}_h^*)}{\partial \widehat{a}_h^*} = -\psi^* + \beta, \\
\frac{\partial J_2^H(\widehat{a}_h, \widehat{a}_h^*)}{\partial \widehat{a}_h^*} &= \frac{\partial J_4^H(\widehat{a}_h, \widehat{a}_h^*)}{\partial \widehat{a}_h^*} = -\psi^* + \beta + \chi_h \beta \left\{ \frac{u'(\kappa)}{c'(\kappa)} - 1 \right\}, \\
\frac{\partial J_6^H(\widehat{a}_h, \widehat{a}_h^*)}{\partial \widehat{a}_h^*} &= -\psi^* + \beta + \chi_h \beta \left\{ \frac{u'(\kappa)}{c'(\kappa)} \left[(1-\sigma) + \sigma u'(\beta(\widehat{a}_h^* + \widehat{a}_f^*)) \right] - 1 \right\}
\end{aligned}$$

Proof. See proofs of statements □

B The sign of cost terms in equilibrium

Lemma 7. *In any equilibrium, the following two conditions must hold.*

$$\psi \geq \beta \quad \text{and} \quad \psi^* \geq \beta.$$

Proof. See proofs of statements □

Notice that the β is the so-called *fundamental* asset value, i.e., the price that agents would be willing to pay for one unit of the asset if neither FIM nor DMs existed. The non-negative sign of the cost terms assigns a very intuitive meaning to the objective functions of agents. Agents wish to bring assets into either of the FIM and DM in order to facilitate trade. However, they face a trade-off because carrying these assets is not free, i.e., the first line in e.q.(10) and (12)). This eventually gives rise to the optimal portfolio choice problem of agents.

C Proofs of Statements

Proof of Lemma 2.

The participation constraint for the F during this bargaining also depends on how much foreign assets in reference to the first best in the subsequent DM ($\breve{a}$) that F has brought up. We consider three possible scenarios regarding the amount of F's foreign asset holdings.

Scenario 1: $a_f^* \geq \breve{a}$

In this scenario, the bargaining problem simplifies to

$$\max_{\{\kappa, b^*\}} \{u(\kappa) - \beta b^*\},$$
$$s.t. \quad c(\kappa) \leq \beta b^*,$$

with $b^* \leq a_h^*$. Solution for this problem is standard and straightforward. If $a_h^* \geq c(\tilde{\kappa})/\beta$ then, $\kappa = \tilde{\kappa}$ and $b^* = c(\tilde{\kappa})/\beta$. If instead $a_h^* \leq c(\tilde{\kappa})/\beta$ then, $\kappa = \{\kappa : \beta a_h^* = c(\kappa)\}$ and $b^* = a_h^*$. So given the assumption of $a_f^* \geq \breve{a}$, the above two solutions correspond to the region 1 and 2.

Scenario 2: $\breve{a} - b^* \leq a_f^* \leq \breve{a}$

In this scenario, the F would get the first best liquidity amount for the subsequent DM ($\breve{a}$) only after the bargaining. Hence, the bargaining problem is described by

$$\max_{\{\kappa, b^*\}} \{u(\kappa) - \beta b^*\},$$
$$s.t. \quad c(\kappa) \leq \beta b^* + \sigma [u(\tilde{q}) - \tilde{q}] - \sigma \left[u(q(a_f^*)) - \beta n(a_f^*)\right],$$

with $b^* \leq a_h^*$. First order conditions for this problem follows as:

$$\kappa : u'(\kappa) = \lambda_1 c'(\kappa), \tag{C.1}$$
$$b^* : -\beta + \lambda_1 \beta - \lambda_2 = 0, \tag{C.2}$$

where λ_1 and λ_2 are the associated lagrange multipliers for the above two constraints. Let us consider two possible cases.

<u>*When $\lambda_2 = 0$*</u>

If we let $\lambda_2 = 0$ then $b^* < a_h^*$ must hold. Since $a_f^* + b^* \geq \breve{a}$ by assumption in this scenario, the e.q.(C.1) ensures $\lambda_1 = 1 \Rightarrow \kappa = \tilde{\kappa}$. Moreover, the participation constraint also binds due to $\lambda_1 = 1$. Hence,

$$c(\tilde{\kappa}) = \beta b^* + \sigma [u(\tilde{q}) - \tilde{q}] - \sigma \left[u(q(a_f^*)) - \beta n(a_f^*)\right]. \tag{C.3}$$

It is understood that the maximum value of b^* must equal to $c(\tilde{\kappa})/\beta$ since e.q.(C.3) implies $b^* \propto a_f^*$ and $\max\{a_f^*\} = \breve{a}$ by the assumption. We also need to make sure that these solutions satisfy conditions imposed in this scenario. First, in order to ensure $b^* < a_b^*$ implied by $\lambda_2 = 0$, one needs the following condition based on e.q.(C.3).

$$c(\tilde{\kappa}) - \sigma\left[u(\tilde{q}) - \tilde{q}\right] + \sigma\left[u(q(a_f^*)) - \beta n(a_f^*)\right] < \beta a_h^*. \tag{C.4}$$

On top of that, one would also need to verify $b^* \geq \check{a} - a_f^*$ imposed by the scenario 2 assumption which leads to

$$c(\tilde{\kappa}) - \sigma\left[u(\tilde{q}) - \tilde{q}\right] + \sigma\left[u(q(a_f^*)) - \beta n(a_f^*)\right] \geq \beta(\check{a} - a_f^*). \tag{C.5}$$

Equations (C.4) and (C.5) hold if $a_h^* + a_f^* > \check{a}$ which therefore needs to be imposed. Next, a_f^* must be bounded from below due to the following reason. Combining the e.q.(C.4) and (C.5) generates

$$\sigma\left[u(\beta a_f^*) - \beta a_f^*\right] + \beta a_f^* \geq \beta\check{a} + \sigma\left[u(\check{q}) - \check{q}\right] - c(\tilde{\kappa}). \tag{C.6}$$

This e.q.(C.6) confirms that $\exists \bar{a}_f^*$ such that

$$\sigma\left[u(\beta a_f^*) - \beta a_f^*\right] + \beta a_f^* = \beta\check{a} + \sigma\left[u(\check{q}) - \check{q}\right] - c(\tilde{\kappa}). \tag{C.7}$$

Note here the condition $a_h^* + a_f^* > \check{a}$ imposed above is redundant as long as e.q.(C.4) holds and $a_f^* > \bar{a}_f^*$ since the LHS of inequality e.q.(C.4) is increasing in a_f^* while $\check{a} - a_f^*$ falls with a_f^*. It is also important to notice that the relative size of cost of producing *capital goods* during the FIM critically determines the sign of $\bar{a}_f^*$. If $c(\tilde{\kappa})$ turns out to be greater than $\beta\check{a} + \sigma\left[u(\check{q}) - \check{q}\right]$ then it is obvious that $\bar{a}_f^*$ must be a negative value otherwise $\bar{a}_f^*$ becomes positive. Hence, the bargaining solution depends on the set of parameter space for $c(\tilde{\kappa}), u(\tilde{q})$, and $\check{a}$. For now, let us restrict ourselves to the case, $c(\tilde{\kappa}) < \beta\check{a} + \sigma\left[u(\tilde{q}) - \tilde{q}\right]$ and consider the other case later. To sum up, for the solution to be $\kappa = \tilde{\kappa}$ and $b^* < a_h^*$ such that e.q.(C.3) holds

1. $\bar{a}_f^* \leq a_f^* \leq \check{a}$,
2. $a_h^* \geq \left\{c(\tilde{\kappa}) - \sigma\left[u(\tilde{q}) - u(\beta a_f^*) + \tilde{q} - \beta a_f^*\right]\right\}/\beta$.

which corresponds to the region 3 solution.

When $\lambda_2 > 0$

If we let $\lambda_2 > 0$ then, $b^* = a_h^*$ must hold. Since $a_f^* + b^* \geq \check{a}$ by assumption in this scenario, $a_f^* + a_h^* \geq \check{a}$ and the e.q.(C.1) ensures $\lambda_1 > 1 \Rightarrow \kappa < \tilde{\kappa}$. Moreover, the participation constraint also binds due to $\lambda_1 > 1$. Hence the solution for κ must satisfy

$$\begin{aligned} c(\kappa) &= \beta a_h^* + \sigma \left[u(\tilde{q}) - \tilde{q}\right] - \sigma \left[u(q(a_f^*)) - q(a_f^*)\right], \\ c(\tilde{\kappa}) &> \beta a_h^* + \sigma \left[u(\tilde{q}) - \tilde{q}\right] - \sigma \left[u(q(a_f^*)) - q(a_f^*)\right], \\ a_h^* &\leq \frac{c(\tilde{\kappa}) - \sigma \left[u(\tilde{q}) - u(\beta a_f^*) + \tilde{q} - \beta a_f^*\right]}{\beta}. \end{aligned} \tag{C.8}$$

Furthermore, combining e.q.(C.8) with $a_h^* \geq \breve{a} - a_f^*$ yields

$$\begin{aligned} c(\kappa) - \sigma \left[u(\tilde{q}) - \tilde{q}\right] + \sigma \left[u(q(a_f^*)) - q(a_f^*)\right] &\geq \beta(\breve{a} - a_f^*) \\ \sigma \left[u(\beta a_f^*) - \beta a_f^*\right] + \beta a_f^* &\geq \beta \breve{a} + \sigma \left[u(\breve{q}) - \breve{q}\right] - c(\kappa). \end{aligned} \tag{C.9}$$

From e.q.(C.7) and (C.9), it is understood that $a_f^* > \bar{a}_f^*$ in this case 2 as well. Thus for the solution to be $b^* = a_h^*$ and $\kappa < \tilde{\kappa}$ such that e.q.(C.8) holds

1. $\bar{a}_f^* \leq a_f^* \leq \breve{a}$,
2. $a_h^* + a_f^* \geq \breve{a}$,
3. $a_h^* \leq \left\{c(\tilde{\kappa}) - \sigma \left[u(\tilde{q}) - u(\beta a_f^*) + \tilde{q} - \beta a_f^*\right]\right\} / \beta$,

which corresponds to the region 4.

Scenario 3: $a_f^* + b^* \leq \breve{a}$

In this scenario, the F would never get the first best liquidity amount for the subsequent DM, i.e., $\breve{a}$, even after the bargaining. Hence, the bargaining problem is described by

$$\max_{\{\kappa, b^*\}} \left\{u(\kappa) - \beta b^*\right\},$$
$$s.t. \quad c(\kappa) \leq \beta b^* + \sigma \left[u(q(a_f^* + b^*)) - \beta n(a_f^* + b^*)\right] - \sigma \left[u(q(a_f^*)) - \beta n(a_f^*)\right],$$

with $b^* \leq a_h^*$. First order conditions for this problem follows as:

$$\kappa : u'(\kappa) = \lambda_1 c'(\kappa), \tag{C.10}$$
$$b^* : -\beta + \lambda_1 \left[\beta + \sigma u'(q(a_f^* + b^*))\beta - \sigma \beta\right] - \lambda_2 = 0, \tag{C.11}$$

where λ_1 and λ_2 are associated Lagrange multipliers for the above two constraints. Let us consider two possible cases.

<u>When $\lambda_2 = 0$</u>

If we let $\lambda_2 = 0$ then, the second constraint becomes slack, i.e., $b^* < a_h^*$. Also from e.q.(C.11) it is obvious that $\lambda_1 < 1 \Rightarrow \kappa > \tilde{\kappa}$. Again the first constraint binds due to positive value of λ_1 and therefore, the following must hold

$$c(\kappa) = \beta b^* + \sigma\left[u(q(a_f^* + b^*)) - \beta n(a_f^* + b^*)\right] - \sigma\left[u(q(a_f^*)) - \beta n(a_f^*)\right] > c(\tilde{\kappa}). \tag{C.12}$$

As before, we again need to make sure that these solutions satisfy conditions imposed in this scenario. First, in order to ensure $b^* < a_h^*$ stemming from $\lambda_2 = 0$, one needs the following condition based on e.q.(C.12)

$$\begin{aligned} c(\kappa) - \sigma\left[u(q(a_f^* + b^*)) - \beta n(a_f^* + b^*)\right] + \sigma\left[u(q(a_f^*)) - \beta n(a_f^*)\right] < \beta a_h^* \\ c(\kappa) - \sigma\left[u(q(a_f^* + b^*)) - u(q(a_f^*))\right] + \sigma\beta b^* < \beta a_h^*. \end{aligned} \tag{C.13}$$

On top of that, one would also need to verify $b^* \leq \breve{a} - a_f^*$ imposed by the scenario 3 assumption which gives out

$$c(\kappa) - \sigma\left[u(q(a_f^* + b^*)) - \beta n(a_f^* + b^*)\right] + \sigma\left[u(q(a_f^*)) - \beta n(a_f^*)\right] < \beta(\breve{a} - a_f^*) \tag{C.14}$$

$$\sigma u(\beta a_f^*) + (1-\sigma)\beta a_f^* < \beta\breve{a} + \sigma\left[u(q(a_f^* + b^*)) - q(a_f^* + b^*)\right] - c(\kappa). \tag{C.15}$$

Now the question is whether $a_h^* + a_f^* < \breve{a}$ or not. From e.q.(C.12), it is easy to see that

$$\sigma u(q(a_f^*)) = (1-\sigma)\beta b^* + \sigma u(q(a_f^* + b^*)) - c(\kappa), \tag{C.16}$$

which confirms that $\{b^*, \kappa\}$ is not uniquely determined, and yet positively related ($b^* \propto \kappa$). This in turn ensures that a_h^* must be bounded from below for the following reason. Due to $b^* \propto \kappa$, $c(\kappa) > c(\tilde{\kappa})$ and e.q.(C.16), minimum value for b^*, b_m^* is such that

$$c(\tilde{\kappa}) = \sigma\left[u(q(a_f^* + b_m^*)) - u(q(a_f^*))\right] + (1-\sigma)\beta b_m^*. \tag{C.17}$$

By the implicit function theorem, e.q.(C.17) confirms $\partial b_m^* / \partial a_f^* > 0$

$$\frac{\partial b_m^*}{\partial a_f^*} = -\frac{\frac{\partial G}{\partial a_f^*} \to \ominus}{\frac{\partial G}{\partial b_m^*} \to \oplus} > 0,$$

where $G(b_m^*, a_f^*) = \sigma\left[u(q(a_f^* + b_m^*)) - u(q(a_f^*))\right] + (1-\sigma)\beta b_m^* - c(\tilde{\kappa})$ and

$$\begin{aligned} \frac{\partial G}{\partial a_f^*} &= \sigma\beta\left[u'(q(a_f^* + b_m^*)) - u'(q(a_f^*))\right] < 0 \quad \text{due to the concavity assumption of } u(\cdot), \\ \frac{\partial G}{\partial b_m^*} &= \sigma\beta\left[u'(q(a_f^* + b_m^*)) - 1\right] + \beta b^* > 0 \quad \text{due to } a_f^* + b^* < \breve{a}. \end{aligned}$$

Thus $\min\{a_h^*\}$ ($a_{h,Min}^*$) must be an increasing function of a_f^*. In addition, when $a_f^* = 0$ the $a_{h,Min}^*$

must satisfy the following

$$c(\tilde{\kappa}) = \sigma u(q(a^*_{h,Min})) + (1-\sigma)\beta a^*_{h,Min}. \tag{C.18}$$

Since we earlier restricted the parameter space into *Case 1* such that $c(\tilde{\kappa}) < \sigma\left[u(\tilde{q}) - \tilde{q}\right] + \beta\breve{a}$, one can easily verify that $a^*_{h,Min} < \breve{a}$. Lastly we need to verify that when $a^*_f = \bar{a}^*_f$, $a^*_{h,Min}$ is such that $a^*_{h,Min} + a^*_f = \breve{a}$ so that the feasible domain for a^*_f in this scenario must be bounded from above, i.e., $\bar{a}^*_f$. This can be done easily by comparing e.q.(C.17) and (C.3). Considering the knife-edge case between region 3 and this region, e.q.(C.3) and (C.17) respectively gives out

$$c(\tilde{\kappa}) - \sigma u(\tilde{q}) + \sigma u(\beta\bar{a}^*_f) + \sigma\beta a^*_h = \beta a^*_h, \tag{C.19}$$

$$c(\tilde{\kappa}) - \sigma u(q(\bar{a}^*_f + b^*_m)) + \sigma u(\beta\bar{a}^*_f) + \sigma\beta b^*_m = \beta b^*_m. \tag{C.20}$$

These two equations become identical when $b^*_m = a^*_h$ and therefore, $a^*_{h,Min} + a^*_f = \breve{a}$ must hold at this knife-edge case of $a^*_f = \bar{a}^*_f$. To sum up, for the indeterminate combination of $(\kappa, b^*) = \left\{(\kappa, b^*) : \kappa > \tilde{\kappa}, b^* < a^*_h, b^* < \breve{a} - a^*_f, c(\kappa) = (1-\sigma)\beta b^* + \sigma\left[u(\beta(a^*_f + b^*) - u(\beta a^*_f)\right]\right\}$ to be the solution, the following restrictions on a^*_h and a^*_f must hold true

1. $a^*_h \geq a^*_{h,Min}$,
2. $a^*_f \leq \bar{a}^*_f$,

which corresponds to region 5.

<u>*When $\lambda_2 > 0$*</u>

If we let $\lambda_2 > 0$ then, $b^* = a^*_h$ must hold. Moreover, the participation constraint also binds due to $\lambda_1 > 0$. Hence the solution for κ must satisfy

$$c(\kappa) = (1-\sigma)\beta a^*_h + \sigma\left[u(q(a^*_f + a^*_h)) - u(q(a^*_f))\right]. \tag{C.21}$$

Now the question is whether κ here is bigger or less than $\tilde{\kappa}$. As a matter of fact, it can be easily shown that $c(\kappa) < c(\tilde{\kappa})$ in this case. First, the comparison between e.q.(C.21) and (C.17) confirm that $c(\kappa) \leq c(\tilde{\kappa})$ if $a^*_f \leq \bar{a}^*_f$. Second, if $a^*_f \geq \bar{a}^*_f$ and $a^*_f + a^*_h < \breve{a}$ then e.q.(C.21) tells us that the max of $c(\kappa)$ ($c(\kappa^{max})$) occurs at the point where $a^*_f = \bar{a}^*_f$ and $a^*_h = \breve{a} - \bar{a}^*_f$ due to again the concavity assumption on $u(\cdot)$. Thus plugging $a^*_f = \bar{a}^*_f$ and $a^*_h = \breve{a} - \bar{a}^*_f$ into e.q.(C.21) would yield the condition for $c(\kappa^{max})$ as

$$\begin{aligned} c(\kappa^{max}) &= (1-\sigma)\beta(\breve{a} - \bar{a}^*_f) + \sigma\left[u(\tilde{q}) - u(q(\bar{a}^*_f))\right] \\ &= (1-\sigma)\beta a^*_h + \sigma\left[u(\tilde{a}) - u(q(\bar{a}^*_f))\right], \end{aligned} \tag{C.22}$$

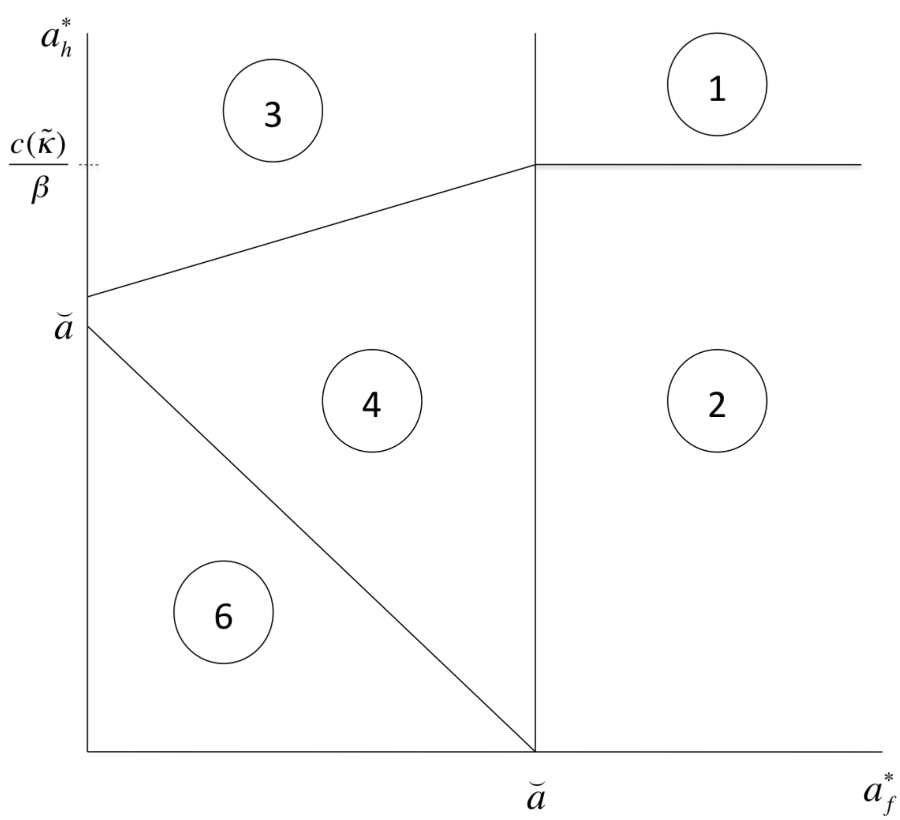
which is same as e.q.(C.19). This completes the proof that $c(\kappa)$ regardless of a_f^* domain becomes bounded from above, $c(\tilde{\kappa})$ in this case. To sum up, for the $b^* = a_h^*$ and κ such that e.q.(C.21) holds to be the solution, the following condition should be met

1. $a_h^* \leq a_{h,Min}^*$,
2. $a_f^* \leq \breve{a} - a_h^*$.

which corresponds to region 6.

Lastly, let us consider the parameter space such that $\beta\breve{a} + \sigma\big[u(\tilde{q}) - \tilde{q}\big] \leq c(\tilde{\kappa})$. In this case, $\bar{a}_f^*$ becomes negative. This essentially eliminates the indeterminate solution region 5. The reason for this disappearance is quite intuitive. Recall the participation constraint for the F in the FIM bargaining problem. The F's liquidity evaluation of foreign assets, i.e., $\sigma u(q(a_f^* + b^*)) - \sigma\beta n(a_f^* + b^*) - \sigma\left[u(q(a_f^*)) - \beta n(a_f^*)\right]$, even when the her initial foreign asset holdings are zero should reach an upper bound. Once the disutility of producing $\tilde{\kappa}$, i.e., $c(\tilde{\kappa})$, exceeds this boundary, the F would never be willing to produce more than $\tilde{\kappa}$ and the terms of trade would never settle at the point where $\kappa > \tilde{\kappa}$ even if a_f^* falls into zero as shown in Figure 1. All the other conditions regarding the remaining regions stay same. The following summarizes and graphically illustrates the bargaining solution under this new parameter space.

Supplementary Figure 1: Regions of the FIM Bargaining Solution $(\beta\breve{a} + \sigma\left[u(\tilde{q}) - \tilde{q}\right] \leq c(\tilde{\kappa}))$



$$
\text{If} \quad \begin{cases} a_h^* \geq \frac{c(\tilde{\kappa})}{\beta} \\ a_f^* \geq \breve{a} \end{cases} \quad \text{then} \quad \begin{cases} \kappa = \tilde{\kappa} \\ b^* = \frac{c(\tilde{\kappa})}{\beta} \end{cases} \quad \Rightarrow \text{Region 1},
$$

$$
\text{If} \quad \begin{cases} a_h^* \leq \frac{c(\tilde{\kappa})}{\beta} \\ a_f^* \geq \breve{a} \end{cases} \quad \text{then} \quad \begin{cases} \kappa = \{\kappa : \beta a_h^* = c(\kappa)\} \\ b^* = a_h^* \end{cases} \quad \Rightarrow \text{Region 2},
$$

$$
\text{If} \quad \begin{cases} a_h^* \geq \frac{c(\tilde{\kappa}) - \sigma\left[u(\tilde{q}) - u(\beta a_f^*) + \tilde{q} - \beta a_f^*\right]}{\beta} \\ a_f^* \leq \breve{a} \end{cases} \quad \text{then} \quad \begin{cases} \kappa = \tilde{\kappa} \\ b^* = \frac{c(\tilde{\kappa}) - \sigma\left[u(\tilde{q}) - u(\beta a_f^*) + \tilde{q} - \beta a_f^*\right]}{\beta} \end{cases} \quad \Rightarrow \text{Region 3},
$$

$$
\text{If} \quad \begin{cases} a_h^* \leq \frac{c(\tilde{\kappa}) - \sigma\left[u(\tilde{q}) - u(\beta a_f^*) + \tilde{q} - \beta a_f^*\right]}{\beta} \\ \breve{a} - a_h^* \leq a_f^* \leq \breve{a} \end{cases} \quad \text{then} \quad \begin{cases} \kappa = \big\{\kappa : c(\kappa) = \beta a_h^* \\ \quad +\sigma\left[u(\tilde{q}) - u(\beta a_f^*) - \tilde{q} + \beta a_f^*\right]\big\} \\ b^* = a_h^* \end{cases} \quad \Rightarrow \text{Region 4},
$$

$$
\text{If} \quad a_f^* + a_h^* \leq \breve{a} \quad \text{then} \quad \begin{cases} \kappa = \big\{\kappa : c(\kappa) = \beta a_h^* + \sigma\left[u(\beta(a_f^* + a_h^*)) - u(\beta a_f^*)\right] \\ \quad -\sigma\left[\beta(a_f^* + a_h^*) - \beta a_f^*\right]\big\} \\ b^* = a_h^* \end{cases} \quad \Rightarrow \text{Region 6}.
$$

This completes the proof. *Q.E.D*

Proof of Lemma 3.

Consider the case where $\widehat{a}_f^* > \breve{a}$. It becomes obvious from Lemma 1 that the terms of trade in the F's local DM are fixed regardless of the amount of foreign assets the F chooses to hold. Hence by taking the first order condition of e.q.(12) with respect to $\widehat{a}^*$, we obtain

$$
J_{\widehat{a}^*}^F(\widehat{a}^*) = -\psi^* + \beta \leq 0,
$$

where the weak inequality sign comes from Lemma (7). From this one can easily verify that the optimal choice of foreign asset holdings for the F can be no greater than or equal to $\breve{a}$ unless $\psi^* = \beta$. We now consider the second case where $\widehat{a}_f^* \leq \breve{a}$. Again following from the bargaining solution in Lemma (1) we have a FOC as

$$
J_{\widehat{a}^*}^F(\widehat{a}^*) = -\psi^* + \beta + \sigma\beta\left\{u'(q(a_f^*)) - 1\right\}.
$$

This justifies the optimality condition in Lemma 3. For the uniqueness of $\widetilde{a_f^*}$, we need following observations. Given the strict concavity assumption of agent's utility function, it is easy to understand that the second derivative of the F's objective function with respect to $\widehat{a}^*$ is strictly negative, i.e., $J_{\widehat{a}^*\widehat{a}^*}^F(\widehat{a}^*) < 0$ for all $\widehat{a}^* \in (0, \breve{a}]$. Furthermore, one can also easily show that the following two conditions must hold in the limit.

$$\lim_{\widehat{a}^* \to 0} J^F_{\widehat{a}^*}(\widehat{a}^*) > 0,$$

$$\lim_{\widehat{a}^* \to \check{a}^-} J^F_{\widehat{a}^*}(\widehat{a}^*) \leq 0.$$

Combining all these results above, we can finally conclude that the optimal choice of $\widetilde{a}^*_f$ is unique, and it satisfies $\widetilde{a}^*_f \in (0, \check{a})$ when $\psi^* < \beta$. On the other hand, if ψ^* happens to be same as β then, the F's optimal foreign asset holdings could be either same as $\check{a}$ or anything bigger than that. *Q.E.D*

Proof of Lemma 4.

With regard to the optimal home asset holdings ($\widetilde{a}$), one can refer to the proof of Lemma 3 since the exactly same line of reasoning applies. From Lemma 2 and Lemma 6, one can infer the parameter space of (ψ^*, a^*_f) that is consistent with the optimal choice of $\widetilde{a}^*_h$ in each of the six regions.

Region 1: First, from Lemma 6, the optimality requires that $\psi^* = \beta$. Second, Lemma 2 restricts $\widetilde{a}^*_h$ to be greater than or equal to $c(\tilde{\kappa})/\beta$.

Region 2: The optimality condition based on Lemma 6 asks $\psi^* - \beta = \chi_h \beta \left\{u'(\kappa)/c'(\kappa) - 1\right\}$. Since the Lemma 2 implies $\kappa < \tilde{\kappa}$ in this region, the optimality should be consistent with $\psi^* > \beta$.

Region 3: The optimal condition based on Lemma 6 implies $\psi^* = \beta$. At the same time, Lemma 2 pins down the $\widetilde{a}^*_h$ such that $\widetilde{a}^*_h = \mathbb{R}_{++} \geq c(\tilde{\kappa})/\beta - \sigma\left[u(\tilde{q}) - u(\beta\widehat{a}^*_f) + \tilde{q} - \beta\widehat{a}^*_f\right]/\beta$.

Region 4: Lemma 2 restricts $\widetilde{a}^*_h$ to be less than $c(\tilde{\kappa})/\beta$, and hence implies $\kappa < \tilde{\kappa}$. At the same time, the optimality from Lemma 6 requires $\psi^* - \beta = \chi_h\beta\left\{u'(\kappa)/c'(\kappa) - 1\right\}$. Combining the two results, it is obvious that the optimality should be consistent with $\psi^* > \beta$. Nevertheless, the upper bound of ψ^* ($\overline{\psi^*}$) that is consistent with the optimality should exist. This condition is attributed to the fact that the Lemma 2 also bounds $\widetilde{a}^*_h$ from below ($\check{a} - a^*_f$). If ψ^* grows too big, the optimal amount of $\widetilde{a}^*_h$ defined in Lemma 2 may fall below $\check{a} - a^*_f$. In order to prevent this, $\overline{\psi^*}$ should be such that it satisfies the optimality, i.e., $\overline{\psi^*} - \beta = \chi_h\beta\left\{u'(\kappa)/c'(\kappa) - 1\right\}$) given κ that guarantees the minimum value of $\widetilde{a}^*_h$, i.e., $c(\kappa) = \sigma\left[u(\tilde{q}) - \tilde{q}\right] - \sigma\left[u(q(\widehat{a}^*_f)) - q(\widehat{a}^*_f)\right] + \beta(\hat{a} - \widehat{a}^*_f)$.

To the right side of $\bar{a}^*_f$ in Region 6: Similar to the region 4 case, Lemma 2 restricts $\widetilde{a}^*_h$ such that $\kappa < \tilde{\kappa}$. Given the optimal condition of $\psi^* - \beta = \chi_h\beta\left\{u'(\kappa)/c'(\kappa)\left\{(1-\sigma) + \sigma u'(\beta\widehat{a}^*_f)\right\} - 1\right\}$ from Lemma 6, the optimality should imply $\psi^* > \beta$. However, the region 4 case shows that foreign asset price range of $\beta < \psi^* < \overline{\psi^*}$ should lead to the $\widetilde{a}^*_h$, which dominates the one implied by the optimality in this region. For this reason, only $\psi^* > \overline{\psi^*}$ is compatible with the optimal choice in this region.

Region 5: From Lemma 6, the optimality requires that $\psi^* = \beta$. Moreover, proof for Lemma 2 restricts $\widetilde{a}^*_h$ to be greater than or equal to $a^*_{h,Min}$.

To the left side of $\bar{a}^*_f$ in Region 6: The optimality condition based on Lemma 6 asks $\psi^* - \beta =$

$\chi_h \beta \left\{ u'(\kappa)/c'(\kappa) \left\{ (1-\sigma) + \sigma u'(\beta \widehat{a}_f^*) \right\} - 1 \right\}$. Since the Lemma 2 implies $\kappa < \tilde{\kappa}$ in this region, the optimality should be consistent with $\psi^* > \beta$.
Rearranging the results above should suffice to explain the H's optimal choice of foreign asset holdings. This completes the proof. *Q.E.D*

In what follows, we provide intuitive interpretation of important properties of the H's optimal portfolio choice. First of all, her optimal home asset demand is trivial because she would only take the local DM's bargaining protocol into consideration. As a matter of fact, it is identical to the F's optimal foreign asset holdings since both H and F face a symmetric market structure in their own local DMs, i.e., same degree of search frictions prevail, and only local assets are used as media of exchange.

However, H's optimal choice for foreign asset holdings would be nontrivial. Suppose the foreign asset price is at the fundamental level ($\psi^* = \beta$) for instance. Since the cost of carrying foreign assets becomes zero, it would not be optimal for H to be in a region where her assets would not allow her to afford the optimal quantity of κ. In short, if $\psi^* = \beta$ then, H would never choose a portfolio in the interior of regions 2, 4 or 6 of Figure 2.

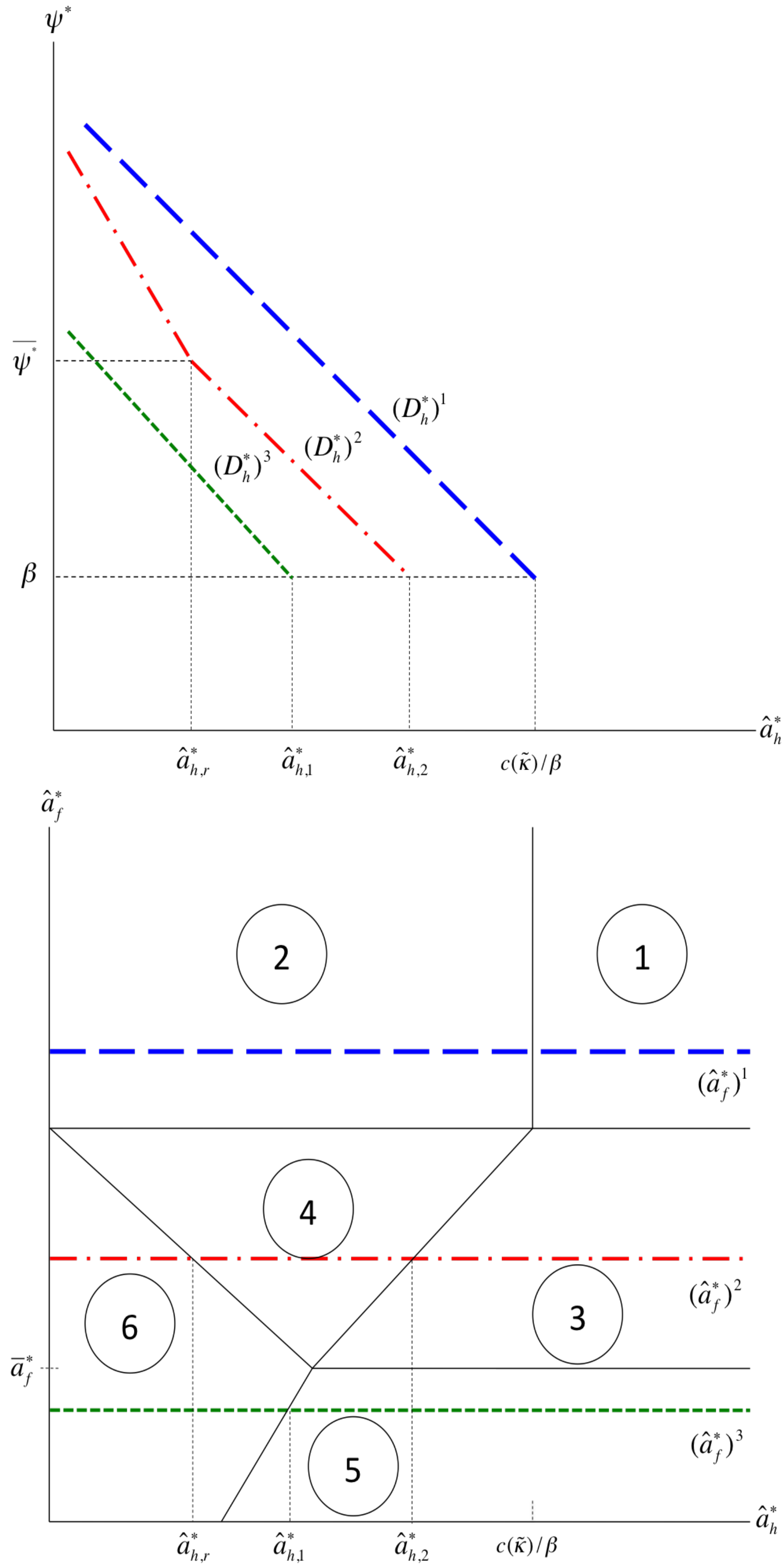
In contrast, if $\psi^* > \beta$, carrying the asset becomes costly. The optimal choice of H is then pinned down by the first-order conditions and, graphically, it lies within regions of either 2, 4 or 6 depending on her beliefs upon a_f^*. For instance, suppose H's belief on the F's foreign asset holdings happens to be greater than the first best liquidity amount ($\check{a}$). In this case, Lemma 6 confirms that the FOC associated with the region 2 always pins down the H's optimal choice of foreign asset holdings.

Interesting case happens when H believes that $\widehat{a}_f^*$ lies in between $\bar{a}_f^*$ and $\check{a}$. In this scenario, the relative size of the foreign asset price becomes crucial. When ψ^* is too high, i.e., $\psi^* > \overline{\psi^*}$, the cost of carrying asset becomes too burdensome for H. Thus, she would typically choose to hold less foreign assets and settle in the interior of region 6. Then, the associated FOC determines the optimal level of foreign asset holdings. On the other hand, if the ψ^* stays in a rather moderate range, i.e., $\beta < \psi^* < \overline{\psi^*}$, then, she would increase her foreign asset holdings so that her marginal benefit falls and equalizes to a new and diminished level of marginal cost, i.e., she would end up within the region 4.

To graphically sum up intuition, the foreign asset demand by H, D_h^* is plotted in Figure 2 against the price, ψ^*.[1] In this graph, H's belief on the level of F's foreign asset holdings is kept fixed at the values $(\widehat{a}_f^*)^1$, $(\widehat{a}_f^*)^2$ and $(\widehat{a}_f^*)^3$. These values are indicated in the lower panel of Figure 2, which replicates Figure 2. The vertical alignment of the two plots enables one to find which regions in terms of Figure 2 H finds herself in, for any choice of $\widehat{a}_h^*$, given the value of $(\widehat{a}_f^*)^j$, $j = 1, 2$ and 3. In essence, the greater F's foreign asset holdings are the more H demands the asset, i.e., $(D_h^*)^1 > (D_h^*)^2 > (D_h^*)^3$. This is because of the fact that F who holds more foreign as-

[1] $a_{h,r}^*$ in Figure 2 is defined as follows. $a_{h,r}^* = \left\{ a_h^* : c(\tilde{\kappa}) = \sigma \left[u(q(a_f^* + a_h^*)) - u(q(a_f^*)) \right] + (1-\sigma)\beta a_h^* \right\}$.

Supplementary Figure 2: Home Agent's Foreign Asset Demand Given Different Levels of $\widehat{a}_f^*$



sets becomes less desperate during the FIM trade. Thus, H would have to give up more foreign assets to induce F to accept the offer in the FIM bargaining.

Another important feature of the graph is the kinked demand curve for the moderate range of $\widehat{a}_f^*$, i.e., $(a_f^*)^2$. $(D_h^*)^2$ exhibits a kink at a threshold level of $\overline{\psi^*}$. To illustrate this property, one needs to recall the regime switch between region 4 and 6 in the neighborhood of $\overline{\psi^*}$ in the previous paragraph. Imagine a case where the foreign asset price steadily rises from its fundamental value β. Once the ψ^* pushes H from the region 4 into 6, she would deal with more desperate F during the FIM bargaining. This would in turn make her less sensitive to the change in the foreign asset prices compared to the case in the region 4. Another way of putting it is that F's willingness to provide more κ in exchange for the same amount of foreign assets would somewhat offset the effects of change in the cost of carrying assets. In short, H's elasticity of asset demand with respect to ψ^* should be lower in the region 6 than 4, consistent with the direction of a kink in $(D_h^*)^2$.

Proof of Lemma 5.

When T^* is plentiful: $T^* \geq \check{a} + c(\tilde{\kappa})/\beta$

Figure 2 confirms that the region 1, 2, 3, and 5 could be all potentially possible equilibrium region. It is obvious from Lemma 3 and 4 that $\psi^* = \beta$ if the equilibrium happens to occur in either of these regions.

(i): Now suppose the equilibrium (a_h^*, a_f^*) lies in the region 2. Then Lemma 3 tells that $a_f^* > \check{a}$ must be consistent with $\psi^* = \beta$. Yet, $\partial J_2^H(\widehat{a}_h, \widehat{a}_h^*)/\partial \widehat{a}_h^*$ from the Lemma 6 implies that $\kappa = \tilde{\kappa}$ which is a contradiction to Lemma 2. Thus the region 2 can not be the equilibrium region.

(ii): Suppose the equilibrium lies in either of the region 3 or 5. Then the Lemma 3 tells that $\psi^* > \beta$ but again from $\partial J_3^H(\widehat{a}_h, \widehat{a}_h^*)/\partial \widehat{a}_h^*$ or $\partial J_3^H(\widehat{a}_h, \widehat{a}_h^*)/\partial \widehat{a}_h^*$ from the Lemma 6 indicates that only $\psi^* = \beta$ must be consistent with the H's optimality. Hence, the region 3 can not be the equilibrium region either.

(iii): Suppose the equilibrium lies in the region 1. Then the Lemma 3 implies $\psi^* = \beta$. At the same time, $\partial J_1^H(\widehat{a}_h, \widehat{a}_h^*)/\partial \widehat{a}_h^*$ from the Lemma 6 also confirms that $\psi^* = \beta$ is consistent with the H's optimality. Hence, the equilibrium can be achieved in this regions subject to:

1. $a_h^* \geq c(\tilde{\kappa})/\beta$
2. $a_f^* \geq \check{a}$
3. $a_h^* + a_f^* = T^*$.

As long as any combination of (a_h^*, a_f^*) meets the above three conditions, the equilibrium can be achieved and therefore, the indeterminacy arises in this case.

When T^* lies within a moderate range: $\breve{a} \leq T^* < \breve{a} + c(\tilde{\kappa})/\beta$

It is understood that the region 2, 3, and 5 can not be the equilibrium region for the same reason as in the case of $T^* \geq \breve{a} + c(\tilde{\kappa})/\beta$. This only leaves us with the region 4 as the only feasible equilibrium region. Indeed the Lemma 3 and 4 restrict the foreign asset price to be greater than the fundamental value in this region. Specifically, the two optimal conditions at the market clearing situation are:

$$\psi^* - \beta = \sigma\beta\left[u'(\beta a_f^*) - 1\right], \tag{C.23}$$

$$\psi^* - \beta = \chi_h\beta\left[\frac{u'(\kappa)}{c'(\kappa)} - 1\right], \tag{C.24}$$

$$where \quad c(\kappa) = \beta(T^* - a_f^*) + \sigma\left[u(\tilde{q}) - u(\beta a_f^*) - \tilde{q} + \beta a_f^*\right].$$

From e.q(C.23) and (C.24) the following must be satisfied in equilibrium as well.

$$\frac{\chi_h}{\sigma} = \frac{u'(\beta a_f^*) - 1}{u'(\kappa)/c'(\kappa) - 1}. \tag{C.25}$$

Finally let us prove if $\exists! a_f^* \in (\bar{a}_f^*, \breve{a})$. By rearranging e.q(C.25), one can define $G(a_f^*)$ as:

$$G(a_f^*) \equiv \sigma\left\{u'(\beta a_f^*) - 1\right\} - \chi_h\left\{\frac{u'(\kappa)}{c'(\kappa)} - 1\right\} = 0, \tag{C.26}$$

$$where \quad c(\kappa) = \beta(T^* - a_f^*) + \sigma\left[u(\tilde{q}) - u(\beta a_f^*) - \tilde{q} + \beta a_f^*\right]. \tag{C.27}$$

First, by taking the $G(a_f^*)$ to the limit the following must hold.

$$\lim_{a_f^* \to \bar{a}_f^{*+}} G(a_f^*) = \sigma\left\{u'(\beta\bar{a}_f^*) - 1\right\} - \chi_h\left\{\frac{u'(\tilde{\kappa})}{c'(\tilde{\kappa})} - 1\right\} \tag{C.28}$$

$$= \oplus - 0 > 0,$$

$$\lim_{a_f^* \to \breve{a}^-} G(a_f^*) = \sigma\left\{u'(\beta\breve{a}) - 1\right\} - \chi_h\left\{\frac{u'(\kappa)}{c'(\kappa)} - 1\right\} \tag{C.29}$$

$$= 0 - \oplus < 0,$$

where the second term in e.q(C.29) becomes a negative value since the κ in the region 4 happens to be less than $\tilde{\kappa}$ according to the Lemma 2.

$$G'(a_f^*) = \sigma\beta\underbrace{u''(\beta a_f^*)}_{\ominus} \tag{C.30}$$

$$-\chi_h\left\{\underbrace{u''(\kappa)}_{\ominus}\underbrace{\frac{\partial\kappa}{\partial a_f^*}}_{\ominus}c'(\kappa)^{-1} - u'c'(\kappa)^{-2}\underbrace{c''(\kappa)}_{\oplus}\underbrace{\frac{\partial\kappa}{\partial a_f^*}}_{\ominus}\right\} < 0.$$

Finally, all is left to guarantee the uniqueness of equilibrium a_f^* is to show that $G'(a_f^*) < 0$ as shown in e.q. C.30 where $\partial\kappa/\partial a_f^* = -\{(1-\sigma)\beta + \sigma u'(\beta a_f^*)\}/c'(\kappa) < 0$ from the e.q(C.27). This proves $G'(a_f^*) < 0$ and therefore, the uniqueness of the equilibrium in the region 4 is established.

When T^* is scarce: $T^* \leq \breve{a}$

Potentially the equilibrium a_h^*, a_f^* can be in either region 5 and 6. Again It is obvious that the region 5 can not be the equilibrium region for the same reason in the previous two cases. This only leaves us with the region 6 as the only feasible equilibrium region. Indeed the Lemma 3 and 4 restrict the foreign asset price to be greater than the fundamental value in this region. Specifically, the two optimal conditions are:

$$\psi^* - \beta = \sigma\beta\left[u'(\beta a_f^*) - 1\right], \tag{C.31}$$

$$\psi^* - \beta = \chi_h\beta\left[u'(\kappa)/c'(\kappa)\left\{(1-\sigma) + \sigma u'(\beta T^*)\right\} - 1\right], \tag{C.32}$$

$$where \quad c(\kappa) = \beta(T^* - a_f^*) + \sigma\left[u(\beta T^*) - u(\beta a_f^*) + \beta(T^* - a_f^*)\right].$$

From e.q(C.31) and (C.32) the following must be satisfied in equilibrium as well.

$$\frac{\chi_h}{\sigma} = \frac{u'(\beta a_f^*) - 1}{u'(\kappa)/c'(\kappa)\left\{(1-\sigma) + \sigma u'(\beta T^*)\right\} - 1}. \tag{C.33}$$

Finally let us prove if $\exists! a_f^* \in (0, \breve{a})$. By rearranging e.q(C.33), one can define $Z(a_f^*)$ as

$$Z(a_f^*) \equiv \sigma\Big\{u'(\beta a_f^*) - 1\Big\} - \chi_h\left\{\frac{u'(\kappa)}{c'(\kappa)}\Big\{(1-\sigma) + \sigma u'(\beta T^*)\Big\} - 1\right\} = 0, \tag{C.34}$$

$$where \quad c(\kappa) = \beta(T^* - a_f^*) + \sigma\Big[u(\beta T^*) - u(\beta a_f^*) + \beta(T^* - a_f^*)\Big]. \tag{C.35}$$

First, by taking the $Z(a_f^*)$ to the limit the following must hold.

$$\begin{aligned}\lim_{a_f^* \to 0^+} Z(a_f^*) &= \sigma\Big\{u'(\beta 0) - 1\Big\} - \chi_h\left\{\frac{u'(\kappa)}{c'(\kappa)}\Big\{(1-\sigma) + \sigma u'(\beta T^*)\Big\} - 1\right\} \\ &= \infty - \oplus > 0,\end{aligned} \tag{C.36}$$

$$\begin{aligned}\lim_{a_f^* \to \breve{a}^-} Z(a_f^*) &= \sigma\Big\{u'(\tilde{q}) - 1\Big\} - \chi_h\left\{\frac{u'(\kappa)}{c'(\kappa)}\Big\{(1-\sigma) + \sigma u'(\beta T^*)\Big\} - 1\right\} \\ &= 0 - \oplus < 0,\end{aligned} \tag{C.37}$$

where the second term in e.q(C.37) becomes a negative value since the κ in the region 6 happens to be less than $\tilde{\kappa}$ according to the Lemma 2. Finally all is left to guarantee the uniqueness of equilibrium a_f^* is to show that $Z'(a_f^*) < 0$. By taking the first derivative of $Z(a_f^*)$ function with respect to a_f^*, the following equation must hold true.

$$\begin{aligned} Z'(a_f^*) =& \sigma\beta \underbrace{u''(\beta a_f^*)}_{\ominus} \\ & - \chi_h \underbrace{\{(1-\sigma)+\sigma u'(\beta T^*)\}}_{\oplus} \left\{ \underbrace{u''(\kappa)}_{\ominus} \underbrace{\frac{\partial \kappa}{\partial a_f^*}}_{\ominus} c'(\kappa)^{-1} - u'c'(\kappa)^{-2} \underbrace{c''(\kappa)}_{\oplus} \underbrace{\frac{\partial \kappa}{\partial a_f^*}}_{\ominus} \right\} < 0, \end{aligned} \tag{C.38}$$

where $u'(\beta T^*) \leq u'(\tilde{q}) = 1$ and $\partial\kappa/\partial a_f^* = -\{(1-\sigma)\beta + \sigma u'(\beta a_f^*)\}/c'(\kappa) < 0$ from the e.q(C.35). This proves $Z'(a_f^*) < 0$ and therefore, the uniqueness of the equilibrium in the region 4 is established. *Q.E.D*.

Proof of Lemma 6.

From the *Case 1* in Lemma 2, it is easy to check that the terms of trade in the FIM in regions 1,3, and 5 have nothing to do with $\widehat{a}_h^*$. Thus the third line e.q.(10) basically becomes a constant term. This makes the first derivative of J^H with respect to $\widehat{a}_h^*$ simply equal to $-\psi^* + \beta$. On the contrary, the H would experience the liquidity shortage in region 2,4, and 6. Therefore she would have to give up all of her foreign asset holdings during the FIM bargaining. This would in turn cause κ to depend on $\widehat{a}_h^*$ as well. Thus the partial derivatives in these regions should take a form as

$$\frac{\partial J_i^H(\widehat{a}_h, \widehat{a}_h^*)}{\partial \widehat{a}_h^*} = -\psi^* + \beta + \chi_h\beta \left\{ u'(\kappa(\cdot)) \frac{\partial\kappa(\cdot)}{\partial \widehat{a}_h^*} - \beta \frac{\partial b^*(\cdot)}{\partial \widehat{a}_h^*} \right\} \quad i = 2, 4, 6,$$

$\partial b^*(\cdot)/\partial \widehat{a}_h^* = 1$ since $b^* = a_h^*$ for all regions of 2,4, and 6. Applying the Implicit Function Theorem to the FIM bargaining protocol described in *Case 1* of Lemma 2 indicates

$$\frac{\partial \kappa(\cdot)}{\partial \widehat{a}_h^*} = \begin{cases} -\frac{-\beta}{c'(\kappa)} & \text{if} \quad i = 2, 4, \\ -\frac{-\beta - \sigma u'(\beta(a_f^* + a_h^*))\beta + \sigma\beta}{c'(\kappa)} & \text{if} \quad i = 6. \end{cases}$$

This completes the proof. *Q.E.D*

Proof of Lemma 7.

The budget constraint of the centralized market implies that the H can exploit more labor units in period t by dL_t^h and get either $\psi_{t+1} dB_{t+1}^h$ units of home bonds or $\psi_{t+1}^* dR_{t+1}^h$ units of foreign reserves. In the next period the H can therefore decrease the amount labor units exploited by $dL_{t+1}^h = dB_{t+1}^h$ or dR_{t+1}^h. The net utility gain of doing this strategy is

$$\begin{aligned} d\mathcal{U}_t^h = -dL_t^h + \beta dL_{t+1}^h =& - dB_{t+1}^h \left[\psi_{t+1} - \beta\right] \\ =& - dR_{t+1}^h \left[\psi_{t+1}^* - \beta\right]. \end{aligned}$$

So, either of $\beta > \psi_{t+1}$ or $\beta > \psi^*_{t+1}$ implies that $d\mathcal{U}_t > 0$ which would in turn cause for infinite labor demand every period. Therefore in any any equilibrium $\beta \leq \psi$ and $\beta \leq \psi^*$. This can be applied to the F exactly in the same way. By the similar budget constraint of the F in the centralized market, F can also exploit more labor units in period t by dL^f_t and get dR^f_{t+1} units of foreign assets. In the next period the F can therefore decrease the amount labor units exploited by $dL^f_{t+1} = dR^f_{t+1}$. The net utility gain of doing this strategy is

$$d\mathcal{U}^f_t = -dL^f_t + \beta dL^f_{t+1} = -dR^f_{t+1}\left[\psi^*_{t+1} - \beta\right].$$

Again, this confirms that in any any equilibrium $\beta \leq \psi^*$. *Q.E.D*

Proof of Proposition 1.

We first prove for the effects of T^* on various equilibrium objects. To that end, we separately provide proofs for each case, i,e., $\breve{a} < T^* \leq \breve{a} + c(\tilde{\kappa})/\beta$ and $T^* < \breve{a}$.

When $\breve{a} < T^* \leq \breve{a} + c(\tilde{\kappa})/\beta$

It is obvious that the home country would on aggregate hold on to home assets exceeding the first best amount $\breve{a}$. Then from Lemma 4, $\psi = \beta$ must hold in equilibrium. Since the proof for the Lemma 5 confirms $\psi^* > \beta$ for the case $\breve{a} < T^* \leq \breve{a} + c(\tilde{\kappa})/\beta$, it should be easy to see $\psi^* > \psi$. For comparative statics, now recall the e.q(C.23) and (C.24). Instead of performing the total differentiation to the optimal conditions, one can simply conduct a thought experiment using the e.q.(C.23) and (C.24). Starting with an equilibrium situation, suppose T^* all of sudden increases. Then by e.q.(C.23), ψ^* must remain unchanged and also by e.q(C.24) κ must remain same. But by the $c(\kappa)$ function in e.q(C.24), κ must also go up which is a contradiction. Now let us suppose a^*_f falls and a^*_h rises. Then by e.q(C.23) ψ^* must increase which in turn imply a fall in κ by the e.q(C.24). Yet the $c(\kappa)$ function in e.q(C.24) again forces κ to increase, which contradicts the fall in κ by the e.q(C.24). Therefore these thought experiments leaves us with nothing but $\partial a^*_f/\partial T^* > 0$ and $\partial \psi^*/\partial T^* < 0$. Having the effects of ΔT^* on ψ^* and a^*_f established, one can further pursue the same experiments with the a^*_h and κ. Since $\partial \psi^*/\partial T^* < 0$ for sure, the e.q(C.24) also makes κ rise in response to the increase in κ. Consequently, the increase in κ and the $c(\kappa)$ function in the e.q(C.24) also forces a^*_h to go up in equilibrium as κ goes up. To sum up, it must be also true that in equilibrium $\partial a^*_h/\partial T^* > 0$ and $\partial \kappa/\partial T^* > 0$.

When $T^* < \breve{a}$

it is easily understood why both ψ^* and ψ exceed the β in equilibrium. For the $\psi^* > \psi$ in equilibrium, one could simply recall the e.q(C.31) and the optimal condition for the home asset holdings by the home agent in Lemma 4 as:

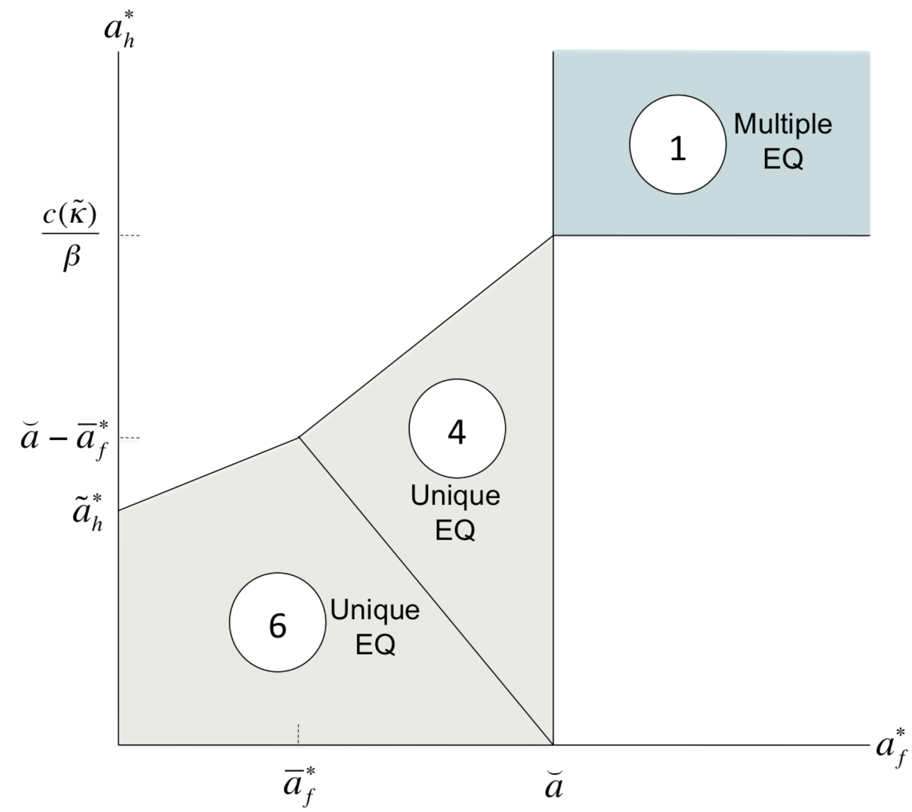
$$\psi - \beta = \sigma\beta\left\{u'(\beta T) - 1\right\},$$
$$\psi^* - \beta = \sigma\left\{u'(\beta(T^* - a^*_h)) - 1\right\}.$$

Since $\beta T^* > \beta(T^* - a_h^*)$ when $a_h^* \in (0, T^*)$, $\psi^* > \psi$ must hold in equilibrium. For comparative statics, the exactly same kind of experiments in the preceding case could be conducted. Recall the e.q(C.31) and (C.32). Let us imagine a situation where κ rises from the initial steady state. Suppose $\Delta a_f^* = 0$ and a_h^* goes up in response. Then by the e.q(C.31), $\partial\psi^*/\partial T^* = 0$ ad by the e.q(C.32), $\partial\kappa/\partial T^* < 0$. But these would mean in accordance with the $c(\kappa)$ function in the e.q(C.32) that a_h^* must fall which is a contradiction. Now suppose a_f^* goes up and a_h^* decreases instead. Then by the same cost function, κ must increase and at the same time ψ^* should fall. However by the e.q(C.31) the ψ^* must rise so again the contradiction arises. Hence $\partial a_f^*/\partial T^* > 0$ and $\partial\psi^*/\partial T^* < 0$ must be true just like the preceding example. Nevertheless the effects of ΔT^* on the κ and a_h^* are this time ambiguous. The e.q(C.32) reveals that given the increase in κ the rise in a_f^* and the fall in ψ^* can not guarantee the signs of $\partial a_h^*/\partial T^*$ and $\partial\kappa/\partial T^*$. Basically this has been caused by the effect of the terms inside the square bracket in the RHS of e.q(C.32), i.e., $(1-\sigma)+\sigma u'(\beta T^*)$, which generates additional downward pressure for the expected surplus from carrying the asset in the case of rising κ. Therefore without this term, this thought experiment would have resulted in exactly same results as the preceding case especially the $\partial a_h^*/\partial T^* > 0$ and $\partial\kappa/\partial T^* > 0$. But since this term is present, the upward pressure for the a_h^* would be somewhat mitigated. Depending on the parameter values, the precise effect would vary. At least though it is obvious that the positive effect of κ changes on the a_h^* in this scarce κ case is smaller than the one in the less scarce case of $\check{a} < T^* \leq \check{a} + c(\tilde{\kappa})/\beta$.

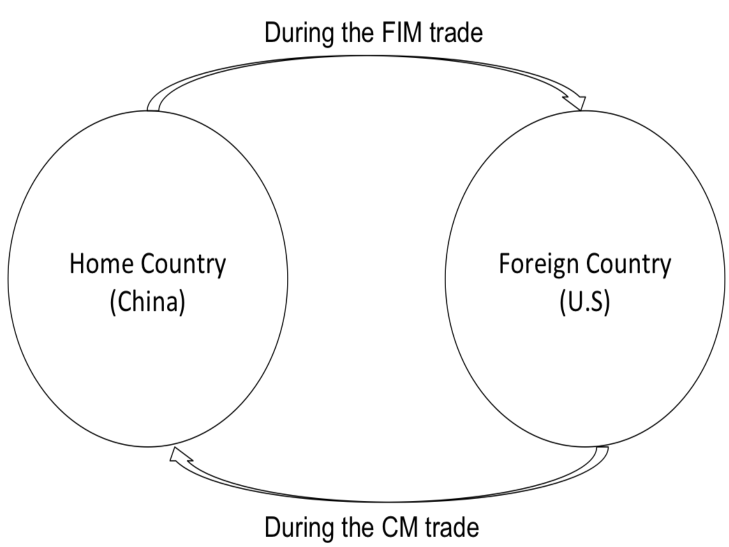
Next, we provide proofs for the effects of χ_h on the various equilibrium objects. An easy proof could be done by a similar thought experiment as in Proposition 1. Recall the e.q(C.31) and (C.32). Suppose χ_h increases and as a result ψ^* remains same and κ goes up. Then by the cost function in e.q(C.32), it must be true that a_h^* rises while a_f^* falls. But then since a_f^* falls the e.q(C.31) implies an increase in ψ^* which is a contradiction. Now suppose κ remains same while ψ^* increases but this generates an immediate contradiction from the e.q(C.32). Next suppose the κ falls down and ψ^* increases instead. But again from the cost function, a_h^* must decrease which would automatically imply an increase in the level of a_f^* by the market clearing condition. This combined with the e.q(C.31) would simply mean a fall in the equilibrium level of ψ^* which is again a contradiction. Lastly now suppose χ_h increases along with κ and ψ^*. Then since κ goes up it is easily understood that a_h^* increases while a_f^* falls from the cost function. Again since a_f^* falls the e.q(C.31) implies an increase in ψ^* which is consistent with the assumption. This completes the proof. *Q.E.D*

D Supplementary figures and tables

Supplementary Figure 3: Aggregate Regions of (a_h^*, a_f^*) in Equilibrium



Supplementary Figure 4: Flows of Foreign Assets in Equilibrium



Supplementary Table 1: **Countries Included in the Sample**

East and Central Asia	
Bangladesh	Malaysia*
Cambodia	Mongolia
China, P.R.: Mainland*	Nepal
China: Hong Kong S.A.R.*	Pakistan
India*	Philippines*
Indonesia*	Singapore*
Korea, Rep.*	Sri Lanka
Kazakhstan	Tajikistan
Kyrgyz Republic	Thailand*
Lao People's Dem. Rep.	Vietnam
Oil-producing countries	
Algeria	Lebanon
Bahrain	Libya
Jordan	Morocco
Egypt	Oman
Israel*	Saudi Arabia
Kuwait	Tunisia
Latin America	
Argentina*	Honduras
Bolivia	Mexico*
Brazil*	Nicaragua
Chile*	Panama*
Colombia*	Paraguay
Costa Rica	Peru*
Dominican Republic	Uruguay
El Salvador	Venezuela, Rep.*
Guatemala	
East Europe & Others	
Albania	Latvia
Armenia	Lithuania
Belarus	Macedonia
Bosnia and Herzegovina	Poland*
Bulgaria	Romania
Croatia*	Russia*
Czech Republic	Slovak Republic
Estonia	Slovenia
Georgia	South Africa*
Moldova	Turkey
Hungary*	Ukraine

* indicates countries that private venture capital data are available

Supplementary Table 2: **Robustness Check I, Alternative Financial Openness (LMF)**

Simultaneous Equations Model (3SLS)			
	(1)	(2)	(3)
Dependent var.		Reserves/GDP	
OTC Inflows	0.5851**	0.6121**	0.6345***
	(0.2692)	(0.2667)	(0.2383)
Financial Openness	0.0039	0.0050	
	(0.0113)	(0.0108)	
Trade openness	0.0285***	0.0266***	0.0290***
	(0.0096)	(0.0097)	(0.0101)
POP	-0.0511***	-0.0506***	-0.0522***
	(0.0138)	(0.0137)	(0.0141)
M2/GDP	0.2245***	0.2228***	0.2268***
	(0.0150)	(0.0150)	(0.0150)
TOT	0.0010***	0.0009***	0.0010***
	(0.0003)	(0.0003)	(0.0003)
Total External Debt/GDP	-0.0105	-0.0101	-0.0066
	(0.0151)	(0.0150)	(0.0092)
Short-term external Debt/GDP	0.0452	0.0440	0.0454
	(0.0429)	(0.0426)	(0.0415)
Exchange rate_volatility	-0.0493	-0.0490	-0.0494
	(0.0326)	(0.0325)	(0.0334)
Pegs	-0.0072	-0.0071	-0.0072
	(0.0065)	(0.0064)	(0.0064)
R^2	0.482	0.478	0.475
Dependent var.		OTC Inflows	
Reserves/GDP	0.0703*	0.0857**	0.0752*
	(0.0424)	(0.0412)	(0.0419)
Financial openness	0.0046		0.0045
	(0.0046)		(0.0045)
Trade openness	0.0237***	0.0269***	0.0233***
	(0.0064)	(0.0063)	(0.0063)
GDPPC	0.0021***	0.0022***	0.0021***
	(0.0006)	(0.0006)	(0.0006)
R^2	0.134	0.122	0.133
Observations	985	985	985

Bootstrap standard errors are in parentheses.*** p<0.01, ** p<0.05, * p<0.1
Country fixed effects, year fixed effects, and currency and banking crisis dummies are included as exogneous variables in the system.

Supplementary Table 3: **Robustness Check III, Another OTC measure**

Simultaneous Equations Model (3SLS)			
	(1)	(2)	(3)
Dependent var.		Reserves/GDP	
OTC Inflows	0.6413***	0.6454***	0.6646***
	(0.2286)	(0.2280)	(0.2258)
Financial Openness	0.0007	0.0019	
	(0.0022)	(0.0020)	
Trade Openness	0.0265**	0.0257**	0.0259**
	(0.0119)	(0.0121)	(0.0116)
POP	-0.0539***	-0.0535***	-0.0545***
	(0.0146)	(0.0146)	(0.0143)
M2/GDP	0.2327***	0.2320***	0.2322***
	(0.0127)	(0.0128)	(0.0128)
TOT	0.0010***	0.0010***	0.0010***
	(0.0003)	(0.0003)	(0.0003)
Total External Debt/GDP	-0.0055	-0.0052	-0.0056
	(0.0093)	(0.0093)	(0.0093)
Short-term external Debt/GDP	0.0444	0.0435	0.0452
	(0.0333)	(0.0334)	(0.0328)
Exchange rate_volatility	-0.0467	-0.0464	-0.0467
	(0.0316)	(0.0315)	(0.0316)
Pegs	-0.0094*	-0.0093*	-0.0091*
	(0.0049)	(0.0049)	(0.0051)
R^2	0.486	0.486	0.483
Dependent var.	OTC Inflows: Debt liability only		
Reserves/GDP	0.0716**	0.0759**	0.0743**
	(0.0291)	(0.0305)	(0.0295)
Financial Openness	0.0044***		0.0044***
	(0.0011)		(0.0011)
Trade Openness	0.0300***	0.0319***	0.0297***
	(0.0057)	(0.0060)	(0.0057)
GDPPC	0.0010*	0.0012**	0.0010*
	(0.0005)	(0.0006)	(0.0005)
R^2	0.137	0.123	0.136
Observations	985	985	985

Bootstrap standard errors are in parentheses.*** p<0.01, ** p<0.05, * p<0.1
Country fixed effects, year fixed effects, and currency and banking crisis dummies are included as exogneous variables in the system.